Proceedings of the Korean Nuclear Society Spring Meeting
Cheju, Korea, May 1996

Directional Wigner-Ville Distribution and Its Application for Rotating-Machinery Condition Monitoring

Dong Wan Kim, Jae Hong Ha, Hae Gon Shin, Yoon Hee Lee, Young Baik Kim
Korea Atomic Energy Research Institute

Abstract

Vibration analysis is one of the most powerful tools available for the detection and isolation of incipient faults in mechanical systems. The methods of vibration analysis in use today and under continuous study are broad band vibration monitoring, time domain analysis, and frequency domain analysis. In recent years, great interest has been generated concerning the use of time-frequency representation and its application for a machinery diagnostics and condition monitoring system. The objective of the research described in this paper was to develop a new diagnostic tool for the rotating machinery. This paper introduces a new time-frequency representation, Directional Wigner-Ville Distribution, which analyses the time-frequency structure of the rotating machinery vibration.

I. Introduction

In vibration analysis of cycling-machinery system, vibration indicates the detailed motion of the mechanical elements of a machine as a characteristic frequency. For example, the up and down motion of a piston in an engine system produces the frequency corresponding to the vibration characteristics, which indicate the reciprocating movement and combustion of engine system[4]. The conventional method of vibration analysis is the use of the frequency domain signal analysis by Fourier transformation with the aid of time domain analysis. However, these conventional procedures require the experimental investigator and do not always decide cause of vibration due to the short of time-frequency domain representation. Recently, Wigner-Ville Distribution that is time-frequency analysis of a signal was used. However, this Wigner-Ville Distribution does not show the mechanical elements motion in vibration analysis. Using the Wigner-Ville Distribution as a guide for time-frequency representation, this paper author proposed a new time-frequency representation, the Directional Wigner-Ville Distribution. In rotating machinery vibration analysis, the proposed Directional Wigner-Ville Distribution shows the direction of shaft rotating motion using two orthogonal proximity probe signals, if the rotating direction of the shaft is known, on the time-frequency plan. Knowing this direction of shaft rotating motion, the shaft rotating precession that is the basic characteristic for comprehensive rotating machinery diagnostic analysis can be decided. With this basic function, the Directional Wigner-Ville Distribution also shows the spectral analysis of a vibration signal that is a powerful technique for fault diagnosis because the different faults produce a vibration with different frequency components. For better evaluation of Directional Wigner-Ville Distribution, the author summarizes the definition of Wigner-Ville Distribution and its frequency characteristics. Then, the Directional Wigner-Ville Distribution will be described.

II. Wigner Distribution

The Base idea of Wigner Distribution was introduced in 1932 by Wigner in quantum mechanics. Although the idea was reintroduced in 1948 by Ville, it has received little attention in signal analysis field at this time. A review of the history of the Wigner-Ville distribution has been given by Bruijn, who also gave a mathematical basis for this new signal transformation[5]. The Wigner Distribution for a real signal was defined by;

$$W_f(t,\omega)=\int_{-\infty}^{\infty} f\left(t+\frac{\tau}{2}\right) f^*\left(t-\frac{\tau}{2}\right) e^{-j\omega\tau} d\tau \qquad (1)$$

Where $f(t)$ is the time history, the Wigner Distribution of which is to be calculated, t is time, ω is frequency, the superscript asterisk denotes complex conjugation and $f(t+\frac{\tau}{2})f^*(t-\frac{\tau}{2})$ may be called the Wigner kernel. There are a number of properties of Wigner Distribution that are important to use the Wigner Distribution for signal analysis. For example, the averaging of frequency with the Wigner Distribution of an analyzed signal at a particular time, t, gives the instantaneous frequency of the signal and the averaging of time at a particular frequency, ω, yields group delay of the signal at that frequency, respectively;

$$\omega_I(t)=\frac{\int_{-\infty}^{\infty}\omega W_f(t,\omega)\,d\omega}{\int_{-\infty}^{\infty}W_f(t,\omega)\,d\omega}, \quad \tau_G(\omega)=\frac{\int_{-\infty}^{\infty}t W_f(t,\omega)\,dt}{\int_{-\infty}^{\infty}W_f(t,\omega)\,dt}. \qquad (2)$$

Furthermore, the projections of the Wigner Distribution of an analyzed signal on the frequency axis and on the time axis provides the energy density spectrum and the instantaneous power. There are a lot of attractive properties in Wigner Distribution[8]. However, the distribution of a real signal defined by Eq.(1) does not represent the true time-frequency pattern due to some interaction components.

III. Wigner-Ville Distribution

In this section, the author describes the Wigner-Ville Distribution that eliminates the interaction problem, named aliasing, of Wigner Distribution with the Nyquist sampling frequency rate. In Wigner-Ville Distribution definition, there are implications regarding the choice of an analytic signal, which eliminate the aliasing problem of Wigner Distribution, and which follow from the definition of the Wigner Distribution[9]. This distribution satisfies mathematical completeness of the problem by accounting for all frequencies. In Wigner-Ville Distribution, the analytic signal is a complex signal that contains both real and imaginary components. The imaginary part is obtained by Hilbert transformation. The analytical signal of a real signal, $f(t)$, can be expressed by;

$$f_a(t)=f(t)+j\hat{f}(t), \quad (3)$$

with the Hilbert transform of $f(t)$ defined as;

$$\hat{f}(t)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(\tau)}{t-\tau}d\tau. \quad (4)$$

By taking Fourier transformation of the above analytic signal, the frequency characteristics of the Wigner Distribution at a fixed time can be calculated as[9]:

$$F_a(\omega)=\begin{cases}2F(\omega) & \omega>0\\ F(0) & \omega=0\\ 0 & \omega<0\end{cases} \quad (5)$$

Using this analytic signal instant of a real signal, the Wigner-Ville Distribution is defined as;

$$W_a=\int_{-\infty}^{\infty}f_a(t+\frac{\tau}{2})f_a^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau. \quad (6)$$

The Wigner-Ville Distribution has been recently introduced as the unique generalized distribution for time-frequency representation and its several properties of the Wigner-Ville Distribution was shown as the reference[5][10]. However, the Wigner-Ville Distribution alone does not function as an adequate estimator due to its high variance. The next section presents the computing problems and its solution to obtain a consistent, readable estimator of the time-varying spectrum, smoothed Wigner-Ville Distribution.

IV. Pseudo Wigner-Ville Distribution

In the above section, the Wigner-Ville Distribution was discussed briefly. Unfortunately, these desirable Wigner-Ville Distribution properties are offset by two major practical limitations. First, the entire signal enters into the calculation of the distribution at any point in time-frequency plane, precluding its online operation with long signals. Second, due to its nonlinearity, interference components arise between each pair of signal components, complicating its interpretation. In the finite length of the signal, the calculation of Wigner-Ville Distribution requires the use of a window. This leads to the following definition of the Pseudo Wigner-Ville Distribution from Equation (6);

$$W_P(t,\omega)=\int_{-\infty}^{\infty}f_a(t+\frac{\tau}{2})f_a^*(t-\frac{\tau}{2})w(t,\tau)e^{-i\omega\tau}d\tau. \quad (7)$$

Where, $W_P(t,\omega)$ is the Pseudo Wigner-Ville Distribution and $w(t,\tau)$ is the window function that expresses the finite length of the signal by the truncation of the signal in practice. Since time windowing acts as a smoother in the frequency domain, the Pseudo

Wigner-Ville Distribution suppresses the Wigner-Ville Distribution interference components that oscillate in the frequency direction. Due to amount of mathematical properties and application of Gaussian distribution window function to Pseudo Wigner-Ville Distribution[2][3][4][11], the author chosen the Gaussian window function as a sliding exponential window in the time-frequency domain and used the Gaussian window function in the Directional Wigner-Ville Distribution calculation described in the next section. In next section, the author defines the Directional Wigner-Ville Distribution that is a different interpretation of Wigner-Ville Distribution for rotating-machinery diagnostic.

V. Directional Wigner-Ville Distribution

The definition of Wigner-Ville Distribution with the analytic signal, $f_a(t)$, has been expressed in above section. We now extend Wigner-Ville Distribution algorithms for the analysis of two proximate probe signals, $x(t)$ and $y(t)$, which are separated as $90°$ in polar coordinates in rotating machinery. The proposed analytic signal composes of two proximate probe signals, $x(t)$ and $y(t)$, can be expressed as;

$$a_{xy}(t)=x(t)+j\hat{y}(t). \tag{8}$$

Where, the Hilbert transformation of a real signal, $\hat{y}(t)$, make the $90°$ phase change a real signal, $y(t)$, in the frequency domain without changing the amplitude of the real signal[13]. Using this analytic signal, the Directional Wigner-Ville Distribution is defined as;

$$W_D(t,\omega)=\int_{-\infty}^{\infty} a_{xy}(t+\tfrac{\tau}{2})\, a_{xy}^{*}(t-\tfrac{\tau}{2})\, e^{-j\omega\tau} d\tau \tag{9}$$

By taking Fourier transformation of both side of the proposed analytic signal. The frequency characteristics of the proposed Directional Wigner-Ville Distribution at a fixed time can be written:

$$A_{XY}(\omega)=\begin{cases} X(\omega)+Y(\omega) & \omega>0 \\ 0 & \omega=0 \\ X(\omega)-Y(\omega) & \omega<0 \end{cases} \tag{10}$$

The above equation means that, depend on the amplitude and phase of two input signals, the Directional Wigner-Ville Distribution has a different magnitude in the positive and negative frequency domain. These special structures of the Directional Wigner-Ville Distribution admit showing direction of rotating-machinery elements motion, forward whirl and backward whirl, depend on the amplitude and phase of two proximate probe signals. Also, this Directional Wigner-Ville Distribution can be returned to the Wigner Distribution and Wigner-Ville Distribution by applying the proper analytic signal to the Directional Wigner-Ville Distribution calculation.

VI. Application of The Directional Wigner-Ville Distribution

The Directional Wigner-Ville Distribution of several transient rotating machinery responses has been calculated and is discussed in this section. For a comprehensive understanding of Directional Wigner-Ville Distribution as a diagnosis tool, the author studied the rotating machinery undergoing full annular rubbing and partial annular rubbing condition. The rotating machinery consists of a rigid disk and a uniform flexible shaft supported by two bearings. In order to measure shaft vibration, two proximity probe sensors, horizontal(X) direction and vertical(Y) directions, were installed. Shown in Figure 1 is the Directional Wigner-Ville Distribution of rotating-machinery with 1750 RPM in steady-state, which shows that rotating-machinery has unbalance with different amplitudes in X and Y direction. Figure 2 shows the Directional Wigner-Ville Distribution of the rotating machinery that suffer full annular rubbing with backward whirl condition in a rotor shaft. The orbit of a full annular rub is generally circular and is as large as the bearing clearance with the rotating speed frequency components. Specially near the resonance frequency of the rotating machinery, rotating machinery has two split critical frequencies, the vertical direction resonance frequency and the horizontal direction resonance frequency, due to the different stiffness of the bearing system. When the rotating machinery is operating in the two resonance frequency zones, between the vertical direction resonance frequency and the horizontal direction resonance frequency, the rotor shaft undergoes the backward whirl with stress per revolution[12]. Figure 3 shows the changing rotating shaft orbits from the forward whirl to the backward whirl between the vertical direction resonance frequency and the horizontal direction resonance frequency. In the case of partial rubs, the rotating machinery shaft orbits have multiple frequency components, integer times the rotating speed frequency, with forward and backward whirl. Figure 4 shows the Direction Wigner-Ville Distribution of the shaft backward whirl with the multiple frequency components in the partial rubbing condition of rotating machinery. Although the backward whirl with the full annular rub condition that illustrated here is extremely rare, it is important to recognize this rubbing condition due to its destructive characteristics.

Remember that all of the Directional Wigner-Ville Distributions presented in above have been smoothed using a Gaussian window. It can be observed that Directional Wigner-Ville Distribution is sensitive to the amplitude and phase of vibration X and Y signals. Having inspected various examples, it seems that the Directional Wigner-Ville distribution is a good method for deciding whether the rotating-machinery shaft orbit is forward or backward whirl relation to the shaft rotating direction. Also, this Directional Wigner-Ville Distribution shows the time-varying shaft vibration frequency change. So, we can investigate the rotating-machinery shaft vibration frequency components and direction in the time-frequency domain using the Directional Wigner-Ville Distribution.

VII. Conclusions

This study was performed to develop a new technique for monitoring rotating machinery based on Wigner-Ville Distribution in an effort to reduce the safety problems caused by failure of rotating machinery. The approach chosen was to provide accurate estimates of the rotating machinery vibrations components and the direction of shaft rotating motion to be used in rotating machinery systems for diagnostic purposes. As seen in the above section, the developed Directional Wigner-Ville Distribution can provide a useful visualization of frequency components with time and allow the presence of various frequency components to be detected by inspection. The Directional Wigner-Ville Distribution can be used to portray both transient phenomena as well as stationary phenomena and therefore can be used for machinery condition monitoring with two input signals. However, there are remains one important issue concerning the application of Directional Wigner-Ville Distribution. As Wigner Distribution has an interaction problem, aliasing, the Directional Wigner-Ville Distribution also has an aliasing problem. Due to the many attractive properties of Directional Wigner-Ville Distribution, this Distribution should open up new application area by overcoming its limitations in the near future. Also, there is a future research work for automatically detecting the presentation of frequency components and quantifying the contribution of individual frequency components.

VIII. References

[1] W. Martin and P. Flandrin, Wigner-Ville Spectral Analysis of Nonstationary Process, IEEE Transaction on Acoustics, Speech, and Signal Processing, Vol.ASSP-33, No.6, Dec. 1985.

[2] You-Kyu Park and Yang-Han Kim, Bias error due to windows for the Wigner-Ville distribution estimation, Mechanical System and Signal processing, 1994.

[3] T. J. Wahl and J. S. Bolton, The Application of the Wigner Distribution to the Identification of Structure-Borne Noise Components, Journal of Sound and Vibration 163(1), 101-122, 1993.

[4] M. Chiollaz and B. Favre, Engine Noise Characterization with Wigner-Ville Time-Frequency Analysis, Mechanical Systems and Signal Processing, 375-400,1993.

[5] T.A.C.M Ciaasen and W.F.G. Mecklenbrauker, The Wigner Distribution - a tool for time-frequency signal analysis, Phillips J. Res. 35, 217-250, 1980.

[6] W. D. Mark, Spectral Analysis of the Convolution and Filtering of Non-Stationary Processing, Journal of sound and Vibration, Vol.11, PP.19-63, 1970.

[7] L. Salvatore and M. Savino, Time-Frequency Representations of Instantaneous Complex Power, Voltage and Current Spaces Vectors, ETEP, Vol.4, No.5, Sep/Oct, 1994.

[8] L. Cohen, On a Fundamental Property of the Wigner Distribution, IEEE Transaction on Acoustics, Speech, and Signal Processing, Vol. ASSP-35, No. 4, Apr. 1987.

[9] Boualem Boash, Note on the Use of the Wigner Distribution for Time-Frequency Signal Analysis, IEEE Trans. Acoustic, Speech and Signal Processing, Vol. 36, No. 9, Sep. 1998.

[10] T.A.C.M Clasen and W.F.G. Mechlenbrauker, The Wigner Distribution-a tool for time-frequency signal analysis PART II: Discrete-Time signal, Phillips. J. Res.35, 276-300, 1980.

[11] N. Yen, Time and Frequency Representation of Acoustic Signal by Means of the Wigner Distribution Function: Implementation and Interpretation, Journal of the Acoustical Society of America, Vol. 81, 1987.

[12] Chong-Won Lee, Vibration Analysis of Rotors, Pp. 7-11, Kluwer Academic Publishers, 1993.

[13] Rashid Ansari, IIR Discrete-Time Hilbert Transformers, IEEE Transaction on Acoustics, Speech, and Signal Processing, Vol. ASSP-35, No. 8, Aug. 1987.

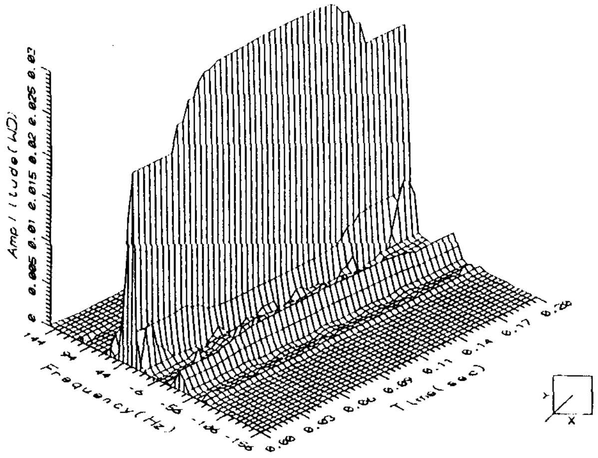

Fig.1 Directional Wigner-Ville Distribution
(1750 RPM, Forward Whirl)

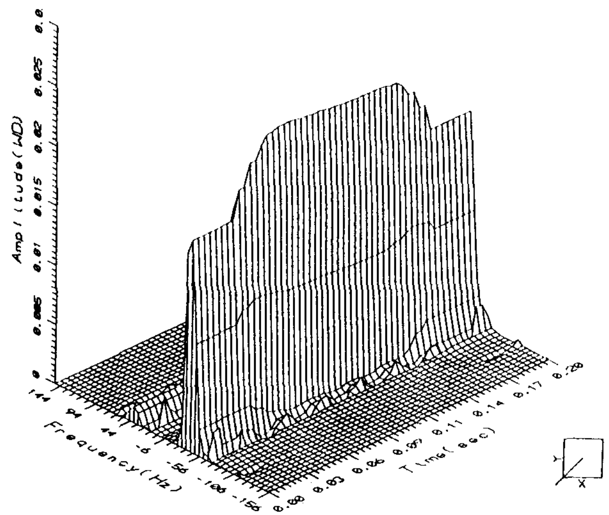

Fig.2 Directional Wigner-Ville Distribution
(1700 RPM, Backward Whirl, Full Annular Rub)

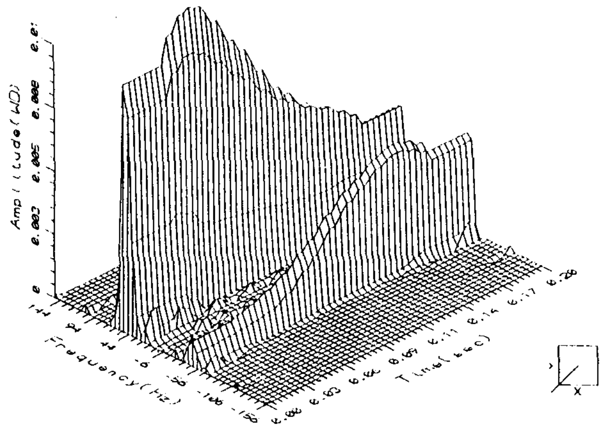

Fig.3 Directional Wigner-Ville Distribution
(2450 RPM, Forward to Backward Whirl)

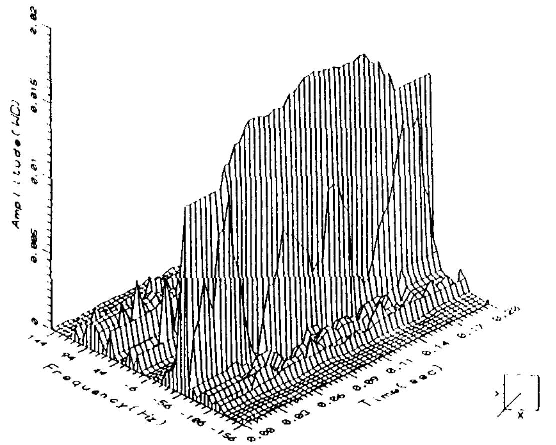

Fig.4 Directional Wigner-Ville Distribution
(2900 RPM, Backward Whirl, Multi-Point Rub)