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Development of a Consistently Formulated General Order Nodal Method for Solving the Three-Dimensional Multi-Group Neutron Kinetic Equations

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ABSTRACT

A new general high order consistent nodal method for solving the 3-D multigroup neutron kinetic equations in (x-y-z) geometry has been derived by expanding the flux in a multiple polynomial series for the space variables without the quadratic fit approximations of the transverse leakage and for the time variable and using a weighted-integral technique. The derived equation set is consistent mathematically, and therefore, we can expect very accurate solutions and less computing time since we can use coarse meshes in time variable as well as in spatial variables and the solution would converge exactly in fine mesh limit.

I. INTRODUCTION

Various coarse-mesh nodal methods have been developed and implemented for the numerical solution of the neutron diffusion and transport problems[1]. Numerical testing of these methods and comparison of their results to those obtained by the conventional methods have established the high accuracy and computational efficiency of nodal methods. However, in nodes the transverse currents at their surfaces not varying smoothly, modern nodal methods using transverse integration procedure suffer from substantial errors and convergence difficulties[2]. These problems arise mainly from the non-self-consistent quadratic fit approximations[3] of the transverse leakage. In transient problems, the methods also suffer from low computational efficiency.

The major disadvantage in using coarse meshes with the lowest order modern nodal methods is the reduced resolution; accurate point values of the flux distribution are somewhat difficult to recover from the converged solution, especially in highly distorted flux regions. Originally, higher order methods have been suggested[4] as a remedy to this problem, and have been implemented[2-3] in solving the steady state diffusion equation to improve the accuracy of the nodal solution, of which the numerical evidence showed very good accuracy. For the time dependence of the equations, the methods still use the conventional finite difference schemes.

In the present work, a new general high order consistent nodal method expanding the flux in a multiple polynomial series for the space variables without the quadratic fit approximations of the transverse leakage and for the time variable, using a weighted-integral technique, and then determining the expansion coefficients numerically is derived for solving 3-D multigroup kinetic equations in (x-y-z) geometry.

II. MATHEMATICAL FORMULATION

In a starting point deriving the nodal method, the time-dependent diffusion equation is expressed by dimensionless variables and modified with a trick[5] for improving computational efficiency in dealing the feedback effects and transient perturbation as

$$\frac{2}{\Delta t_n v_g}\frac{\partial\phi_g(u,v,w,\tau)}{\partial\tau} - \frac{4D_g}{\Delta x_i^2}\frac{\partial^2\phi_g(u,v,w,\tau)}{\partial u^2} - \frac{4D_g}{\Delta y_j^2}\frac{\partial^2\phi_g(u,v,w,\tau)}{\partial v^2} - \frac{4D_g}{\Delta z_k^2}\frac{\partial^2\phi_g(u,v,w,\tau)}{\partial w^2} + \hat{\Sigma}_{rg}\,\phi_g(u,v,w,\tau) = S_g(u,v,w,\tau), \quad (1)$$

where

$$S_g(u,v,w,\tau) \equiv \{\hat{\Sigma}_{rg} - \Sigma_{rg}(\tau)\}\,\phi_g(u,v,w,\tau) + \chi_g\,(1-\beta)\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\tau)\,\phi_{g'}(u,v,w,\tau) + \sum_{\substack{g'=1\\ g'\neq g}}^{G}\Sigma_{sg'g}(\tau)\,\phi_{g'}(u,v,w,\tau) + \sum_{p=1}^{6}\chi_{gp}\,\lambda_p\,C_p(u,v,w,\tau). \quad (2)$$

All the coefficients on the left hand side in Eq. (1) are node averaged constants, and the in-node time dependence of the removal cross section has been transferred into the time-dependent effective source term of the right hand side.

Through the integration procedure of spatial variables, we can reduce the diffusion equation to time-dependent ordinary differential equations. For the u, v and w moments of the τ-dependent flux, we multiply Eq. (1) by $P_{n_u}(u)$, $P_{n_v}(v)$ and $P_{n_w}(w)$, the Legendre polynomials in u, v and w variables, respectively, and integrate over $-1 \le u, v, w \le 1$ in the node to arrive at the time-dependent generalized spatial moment equation as

$$\frac{2}{\Delta t\ v}\frac{d\phi_{n_u n_v n_w}(\tau)}{d\tau} + \hat{\Sigma}_r\ \phi_{n_u n_v n_w}(\tau) = A_{\tau n_u n_v n_w}(\tau), \tag{3}$$

where

$$\phi_{n_u n_v n_w}(\tau) = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} du\ dv\ dw\ P_{n_u}(u)\ P_{n_v}(v)\ P_{n_w}(w)\ \phi(u,v,w,\tau), \tag{4}$$

$$A_{\tau n_u n_v n_w}(\tau) \equiv S_{n_u n_v n_w}(\tau) - L_u(n_u,n_v,n_w,\tau) - L_v(n_u,n_v,n_w,\tau) - L_w(n_u,n_v,n_w,\tau), \tag{5}$$

$$L_u(n_u,n_v,n_w,\tau) \equiv \frac{4}{\Delta x^2}\{J_u{}^R{}_{n_v n_w}(\tau) + (-1)^{n_u+1}\ J_u{}^L{}_{n_v n_w}(\tau) - \sum_{m=0}^{[(n_u-1)/2]} (2n_u-4m-1)\ J_{u(n_u-2m-1)n_v n_w}(\tau)\}, \tag{6}$$

$$J_u{}^R{}_{n_v n_w}(\tau) = \int_{-1}^{1} dv\ P_{n_v}(v) \int_{-1}^{1} dw\ P_{n_w}(w)\ J_u(1,v,w,\tau)$$
$$= -D \int_{-1}^{1} dv\ P_{n_v}(v) \int_{-1}^{1} dw\ P_{n_w}(w) \frac{\partial\phi(1,v,w,\tau)}{\partial u}, \tag{7}$$

and group index g is omitted for clarity.

The coupling terms, L's, arise from the intergration of u-, v- and w-channel derivatives, respectively. $A_{\tau n_u n_v n_w}(\tau)$ is the weighted-integral effective source term involving the fission, scatter and 3-directional transverse leakages. J's are the net current moments at the right (R) or left (L) surface or the in-node current moment.

Solving Eq. (3) and expanding the τ-dependent terms by Legendre polynomial, we obtain the equation for the n_u, n_v, n_w and n_τ moment fluxes or time edge moment fluxes,

$$\phi^E{}_{n_u n_v n_w} = \mathrm{Exp}(-2\sigma)\ \phi^I{}_{n_u n_v n_w} + \sum_{n_{\tau'}=0} F_{n_{\tau'}}\ A_{\tau n_u n_v n_w n_{\tau'}}, \tag{8}$$

and

$$\phi_{n_u n_v n_w n_\tau} = G_{n_\tau}\ \phi^I{}_{n_u n_v n_w} + \sum_{n_{\tau'}=0} H_{n_\tau n_{\tau'}}\ A_{\tau n_u n_v n_w n_{\tau'}}, \tag{9}$$

where $F_{n_\tau'}$, G_{n_τ}, and $H_{n_\tau n_\tau'}$, are the coefficient functions which can be analytically evaluated. The superscripts I and E denote the initial and end points on a discretized time interval. The $\phi^I_{n_u n_v n_w}$, $\phi^E_{n_u n_v n_w}$ and $\phi_{n_u n_v n_w n_\tau}$ are time edges and in-node flux moments.

The *equation for the* v, *w and* τ *moments of the* u-dependent *flux is* expressed by similar procedure as

$$-\frac{4D}{\Delta x^2}\frac{d^2\phi_{n_v n_w n_\tau}(u)}{du^2} + \hat{\Sigma}_r\,\phi_{n_v n_w n_\tau}(u) = A_{u n_v n_w n_\tau}(u), \tag{10}$$

where $A_{u n_v n_w n_\tau}(u)$ similar to $A_{\tau n_u n_v n_w}(\tau)$ in Eq. (3) is the effective source term involving the fission, scatter, 2-directional transverse leakages and time-edge fluxes.

Expanding the effective source term in u-direction, $A_{u n_v n_w n_\tau}(u)$ by the Legendre polynomial and solving Eq. (10) with substituting the expansion of $A_{u n_v n_w n_\tau}(u)$, we obtain the equation for the u-dependent, n_v, n_w and n_τ moment fluxes, $\phi_{n_v n_w n_\tau}(u)$. The same procedure yields the expressions for $\phi_{n_u n_w n_\tau}(v)$ and $\phi_{n_u n_v n_\tau}(w)$ in v- and w-channel, respectively.

The solution for the flux moments given by Eq. (8) or (9) depends on the availability of the net current moments at interfaces of spatial nodes and of flux moments at edges of time mesh. The spatial coupling parameters which are defined in terms of net current moments across a surface can be generated from the continuity condition for the flux moments at interfaces. When this condition and the equation for flux moments, $\phi_{n_v n_w n_\tau}(u)$, $\phi_{n_u n_w n_\tau}(v)$ and $\phi_{n_u n_v n_\tau}(w)$ are used at a given interface, the spatial dependent flux moments are eliminated and an equation relating the three net current moments at three consecutive interfaces is obtained. For example, for u-direction,

$$E_{u,i-1}\,J_u^L{}_{n_v n_w n_\tau,i-1} + E_{u,i}\,J_u^L{}_{n_v n_w n_\tau,i} + E_{u,i+1}\,J_u^L{}_{n_v n_w n_\tau,i+1} = Q_{u n_v n_w n_\tau,i}. \tag{11}$$

The equation set can be globally expressed as a tri-diagonal system of equations relating each set of net current moments for the one-dimensional block under consideration. The coefficients, $E_{u,i-1}$, $E_{u,i}$, and $E_{u,i+1}$ are the same for net current moments of all orders. The one-dimensional block equations for the current moments are coupled via their right sides, $Q_{u n_v n_w n_\tau,i}$, which are effective source moments. When the derived equation set is truncated at finite moment series for a desired order moment, they can be solved by the standard iterative-method.

III. SUMMARY

A consistently formulated nodal method for solving the diffusion kinetics problems successfully has been derived by using Legendre polynomial expansion and a weighted-integral technique. The method is based on the nodal balance equations written in terms of the net current moments across the surfaces of node and of the flux moments at edges of time mesh. It uses a mathematically consistent polynomial expansion for the spatial and time-variables, which renders the use of an approximation for the transverse leakages no necessary and makes the use of coarse mesh in the time variable as well as in the spatial variables possible, therefore, we can expect very accurate solutions and less computing time, and the solution would converge exactly when the mesh width is decreased or the approximation orders are increased. The result establishes the missing link between nodal methods and conventional finite difference methods. The method also has been written in a way of avoiding the expensive recalculation and storage requirement of time-dependent coefficients by feedback effects and perturbation insertions. It would be a computational frame that could warrant accuracy, efficiency and flexibility in solving the reactor physics problems. The method will in succeeding work be tested against various nodal orders in time and space variables for ensuring their effectiveness.

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