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A Generalized Model for Homogenized Reflectors

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ABSTRACT

A new concept of equivalent homogenization is proposed. The concept employs new set of homogenized parameters: homogenized cross sections (XS) and interface matrix (IM), which relates partial currents at the cell interfaces. The idea of interface matrix generalizes the idea of discontinuity factors (DFs), proposed and developed by K.Koebke and K.Smith. The method of K.Smith can be simulated within framework of new method, while the new method approximates heterogeneous cell better in case of the steep flux gradients at the cell interfaces. The attractive shapes of new concept are: improved accuracy, simplicity of incorporation in the existing codes, equal numerical expenses in comparison to the K.Smith's approach. The new concept is useful for: (a) explicit reflector/baffle simulation; (b) control blades simulation; (c) mixed UO2/MOX core simulation.

The offered model has been incorporated in the finite difference code and in the nodal code PANBOX. The numerical results show good accuracy of core calculations and insensitivity of homogenized parameters with respect to in- core conditions.

I. *INTRODUCTION*

A new concept of equivalent homogenization has been developed to take into account internal asymmetry and heterogeneity of L-shaped reflector. We analyzed the fitness for explicit reflector simulation of two most popular homogenization techniques. We found out, that the first one, proposed by K.Koebke introduces undesirable symmetry in equivalent cell. The second one, proposed by K.Smith, has enough degrees of freedom in order to describe asymmetry of L- shaped reflector. However, the accuracy of this model can be improved without additional numerical expenses. The theory of K.Smith supposes continuity of current in order to ensure neutron conservation at the cell interfaces. In contrast to the concept of K.Smith, the offered concept admits discontinuity of net current. In fact, discontinuity of current suggests very useful additional degree of freedom for baffle simulation. The baffle or control blade can be considered as absorbing thin layer at the cell interfaces. Discontinuity of current simulates absorption in the layer. In spite of simplification of geometry of absorber, the thin layer approximates absorbing blade/baffle better than discontinuity factors of flux do. The procedure for derivation of equivalent parameters from 2D referenced solution has been developed.

The equivalent cell consists of homogeneous cell, covered by thin layer with the given response properties. The interface matrix simulates response of thin layer. Thus, the new set of homogenized parameters includes IM and XS.

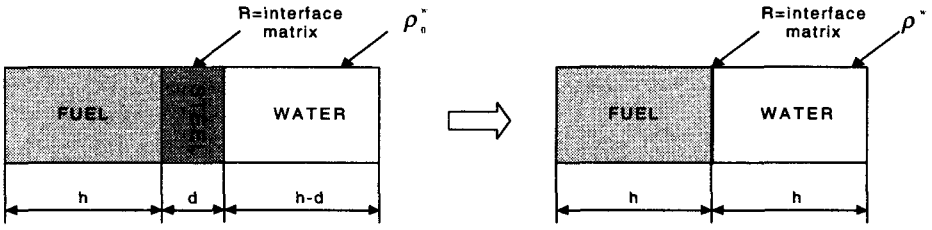
By the definition, IM preserves absorption rate of 2D baffle. From the other standpoint, the offered explicit model of reflector/baffle combines convenient 2D nodal/finite difference approach inside the homogeneous cell and 1D analytical solution for the baffle. The IM provides 1D analytical solution. The possibility of using 1D model can be derived from three fundamental facts: (1) baffle is optically thin for the fast neutrons; (2) for the thermal neutrons the transverse leakages Lx,Ly have a ratio of approximately 10:1; **(3) conservation of absorption rate for the baffle ensure accuracy of full-core model.** Note, 1D model provides good accuracy for a small variation of fast flux within the optically thin baffle (10% of diffusion length). Additionally, for fast neutrons the ratio Lx/Ly is about 1:3. For thermal neutrons the size of baffle is approximately equal to 1 diffusion length, while the ratio Lx/Ly is about 1:10. The small transverse leakage within the baffle can be neglected in the distance of 1 diffusion length . Hence, 1D model of baffle should provide more than enough accuracy. **Besides, corrected interface matrix (CIM) can be employed instead of IM to preserve response of cell. The CIM is derived from assumption that the set CIM+XS+NODAL METHOD should preserve referenced incoming and outgoing currents at the boundary of cell.**

II. *HOMOGENIZATION PROCEDURE*

The procedure derives XS and IM from the known high-order 2D solution for fragment of core. We can employ HELIOS/MCNP/CASMO code in order to get high-order solution. The procedure uses standard method for calculating XS of pure water (or water with boron acid) and the special procedure for IM calculation, presented in the next section.

The starting point of homogenization technique is an assumption that the baffle (or control blade) can be compressed up to thin layer. The picture 1 shows transformation of cell. The transformed cell conserves weight of water. Also, IM by the definition conserves absorption rate for L-shaped/slab baffle in the following sense. The thin layer conserves absorption rate of the baffle under condition of conservation of surface-averaged flux and surface-averaged current at the core-baffle interface. The original absorption rate for the baffle and partial currents at the core/baffle interface are supposed known from HELIOS program, executed for 2D heterogeneous fragment of core. Hence, the thin layer simulate

accurately absorption rate of 2D referenced solution for baffle. The last property ensure accuracy of homogenization technique.



Picture 1. Transformation of heterogeneous baffle/reflector cell into equivalent "homogenized" cell.

Here, d = thickness of baffle; $d \to 0$; ρ^w=density of water; h=width of fuel assembly $\rho^w = \rho_0^w (h - d_0)^2 / (h-d)^2$ for L-shaped reflector; $\rho^w = \rho_0^w (h - d_0) / (h-d)$ for the slab reflector; the subscript o labels initial values (before baffle compression).

III. *Derivation of interface matrix from 2D spectral calculations.*

Suppose, the total partial currents through the external and internal boundary of baffle are known from 2D spectral calculations. For the simplicity, let us neglect slowing down in the baffle. Hence, we can treat thermal and fast group separately. Using the simple relation of diffusion theory, derive neutron flux and net current: $J = J^+ - J^-$; $\varphi = 2(J^+ + J^-)$. Consider the baffle of volume V, whose boundary $S = \partial V$ consists of two components $S = S_1 \cup S_2$, $S_1 \cap S_2 = \varnothing$ where S is L-shaped or slab interface between active core and baffle. Define:

$$J^l = \int_{S_1} \vec{J} d\vec{S} \Big/ \int_{S_1} dS; \quad J^r = \int_{S_2} \vec{J} d\vec{S} \Big/ \int_{S_1} dS; \quad \varphi^l = \int_{S_1} \varphi dS \Big/ \int_{S_1} dS; \quad \varphi^r = \int_{S_2} \varphi dS \Big/ \int_{S_1} dS \qquad (1)$$

The integrals are known from HELIOS or from MCNP calculation. Consider the thin layer, which has net currents J^l, J^r at the left and right side respectively and absorption rate $S_1(J^l - J^r)$. Noting that,

$$\textit{ABSORPTION RATE OF BAFFLE} = \int_V \Sigma_a \varphi dr = \oint_S \vec{J} d\vec{S} = \int_{S_1} \vec{J} d\vec{S} + \int_{S_2} \vec{J} d\vec{S} = S_1(J^l - J^r);$$

conclude: if (1) is true then the thin layer conserves absorption rate of baffle. Write the simple relation of 1D diffusion theory for the baffle of thickness d.

$$\begin{cases} J(x) = -D d\varphi / dx \\ dJ / dx = -\Sigma_a \varphi(x) \\ J(0) = J^r, \varphi(0) = \varphi^r \end{cases} \Rightarrow \quad \begin{pmatrix} \varphi^l \\ J^l \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} \varphi^r \\ J^r \end{pmatrix}; \qquad \mathbf{R} = \begin{pmatrix} \alpha & \gamma\beta \\ \gamma/\beta & \alpha \end{pmatrix} \qquad (2)$$

$$L^2 = D/\Sigma_a, \ \beta = -L/D; \ \alpha = \cosh(d/L); \ \gamma = \sinh(d/L) = \sqrt{\alpha^2 - 1}; \qquad (3)$$

As far as the values φ^l J^l φ^r J^r are known, then we can derive $\alpha, \beta, \mathbf{R}$. Combining (2), (3), write the final expression:

$$\alpha = \left(\varphi^l J^l + \varphi^r J^r\right) \Big/ \left(\varphi^r J^l + \varphi^l J^r\right); \ \gamma = \sqrt{\alpha^2 - 1}; \ \beta = \left(\alpha\varphi^r - \varphi^l\right) \Big/ \left(J^r \gamma\right); \qquad (4)$$

Note, if the coarse-mesh solution conserves referenced surface-averaged flux φ^l and surface-averaged current J^l at the core-baffle interface, then interface matrix R simulates accurately absorption rate of referenced solution (HELIOS,MCNP) for baffle. The interface matrix technique looks attractive for simulation of UO2/MOX interface. Reasons: (a) IM simulates the region of steep flux gradients more accurately than polynomials; (b) diffusion theory does not

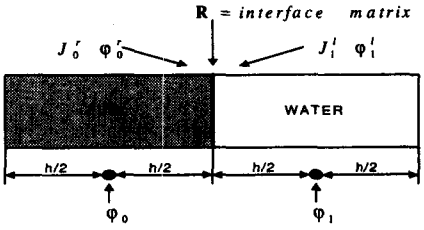
work well at UO2/MOX interface, while IM conserves functionals of multigroup non-diffusional referenced solution. The reasons (a), (b) are also true for core-reflector interface.

IV. *Incorporation of interface matrix in finite difference scheme.*

The starting point is the conventional approximation of currents:

$$J_0^r = -D_0\left(\varphi_0^r - \varphi_0\right)/(h/2); \quad J_1^l = -D_1\left(\varphi_1^l - \varphi_1\right)/(h/2); \tag{5}$$

In contrast to the standard FD scheme, let us suppose the following interface boundary conditions:



$$\begin{pmatrix} \varphi_1^l \\ J_1^l \end{pmatrix} = \mathbf{R} \cdot \begin{pmatrix} \varphi_0^r \\ J_0^r \end{pmatrix}$$

Picture 2. Finite difference approximation.

Substitution (5) in (2) leads to the linear system, which has standard structure of FD system but new sense of mesh coupling coefficients. Finally, the 5-point equation for 2D rectangular geometry is:

$$P_0\varphi_0 - \sum_{k=1}^{4} A_k\varphi_k = Q_0; \tag{6}$$

where

$$P_0 = \Sigma_a h^2 + \sum_{k=1}^{4} B_k; \quad \Sigma_a \equiv \text{absorption cross section}; \quad Q_0 = q_0 h^2; \tag{7}$$

$$A_k = 2d_0\Big/\left(g_k\beta^{-1}d_k^{-1} + a_k + a_k d_k^{-1} d_0 + g_k\beta d_0\right); \qquad d_k = D_k/(h/2) \tag{8}$$

$$B_k = 2d_0\left(g_k\beta^{-1}d_k^{-1} + a_k\right)\Big/\left(g_k\beta^{-1}d_k^{-1} + a_k + a_k d_k^{-1} d_0 + g_k\beta d_0\right); \tag{9}$$

$$\begin{cases} a_k = \alpha, \quad g_k = \gamma, & \text{if absorbing thin layer (baffle) divides two nodes;} \quad (10) \\ a_k = 1, \quad g_k = 0, & \text{otherwise (ordinary case)} \quad (11) \end{cases}$$

Note, in case of $a_k = 1$, $g_k = 0$ we have standard finite difference formula for coupling coefficients:

$$A_k = B_k = 2d_0 d_k/(d_0 + d_k).$$

V. *Incorporation of interface matrix in the* PANBOX *code.*

To incorporate new concept of homogenized cell in PANBOX, we modify standard full-core model:

$$\vec{J}^{in} = \mathbf{R}'\vec{J}^{out} + \mathbf{P}\vec{q}; \qquad \vec{J}^{out} = \mathbf{H}\vec{J}^{in}$$

where the first linear system gives response of nodes and the second system defines topographical relation between incoming and outgoing currents. The standard approach supposes that the matrix **H** is the so-called permutation matrix, having the following properties: (a) the matrix consists of 0 and 1; (b) the **H** contains only one non-zero element per row and per column; (c) $\mathbf{H} \times \mathbf{H} =$ *unit matrix*. The offered concept of equivalent reflector supposes modification of **H**. The concept employs the new coupling of partial currents at the core-reflector interface:

$$\begin{pmatrix} J_w^{in} \\ J_w^{out} \end{pmatrix} = \mathbf{A} \begin{pmatrix} J_f^{in} \\ J_f^{out} \end{pmatrix}; \quad \mathbf{A} = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \alpha & -\gamma\beta \\ -\gamma/\beta & \alpha \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}$$

instead of standard coupling $J_f^{in} = J_w^{out}$, $J_w^{in} = J_f^{out}$. Here, the subscripts f,w label fuel assembly and water respectively. Also, the new formula for multiplication factor $K_{eff} = G/(A + L + A_b)$ is used to take into account absorption within layer, where G=generation rate, A=absorption rate, L=leakage, A_b=absorption rate of layer. The modified PANBOX uses the following algorithm for core-reflector interface:

(a) for given $J_f^{in,k}$ calculate $J_f^{out,k+1}$ by solving standard 2D nodal equation for fuel assembly;

(b) put $J_w^{in,k+1} = \left(J_f^{out,k+1} - \mathbf{A}_{22} J_w^{out,k}\right) / \mathbf{A}_{21}$;

(c) for given $J_w^{in,k+1}$ calculate $J_w^{out,k+1}$ by solving standard 2D nodal equation for reflector cell;

(d) put $J_f^{in,k+1} = \left(J_w^{out,k+1} - \mathbf{A}_{22} J_f^{out,k+1}\right) / \mathbf{A}_{21}$;

(e) $k=k+1$ go to (a).

V. *NUMERICAL RESULTS.*

The accuracy of interface matrix technique was tested on ZION-1 benchmark problem[7].The following calculations were made: (1) standard finite difference scheme with explicit baffle (FDE); (2) finite difference with homogeneous reflector and interface matrix (FD-IM) at core-reflector interface; (3) PANBOX with interface matrix (PANBOX-IM) at core-reflector interface. The FDE, FD-IM used 16*16 meshes per assembly, while PANBOX-IM used 4*4 meshes per assembly. In order to provide proper comparison, IM was derived from FDE rather than from HELIOS. The partial currents from FDE and equations (1)-(4) were used to calculate IM. The fig. 1 shows power distribution and multiplication factor for FDE, FD-IM, PANBOX-IM. The fig. 2 shows independency of model from in-core conditions. The data for FDE, FD-IM, PANBOX-IM were: (a) ZION-1 benchmark data, with the exception for enrichment of one fuel assembly facing at L-shaped reflector; (b) cross sections and interface matrix from "unperturbated" state of core. The fig. 3 compares FDE and FD-IM pin-by-pin powers at the region of L-shaped reflector.

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FDE : finite difference, explicit baffle simulation
FD-IM : finite difference, interface matrix boundary conditions
PANBOX-IM: PANBOX, interface matrix boundary conditions.

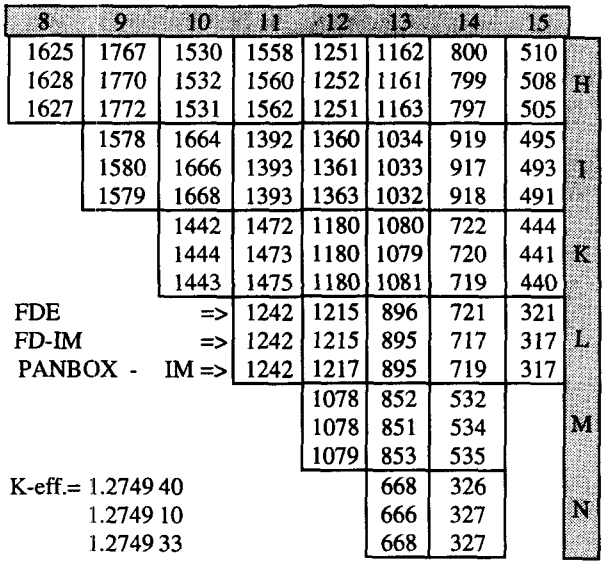

Fig. 1. Power distribution for ZION-1 benchmark problem.

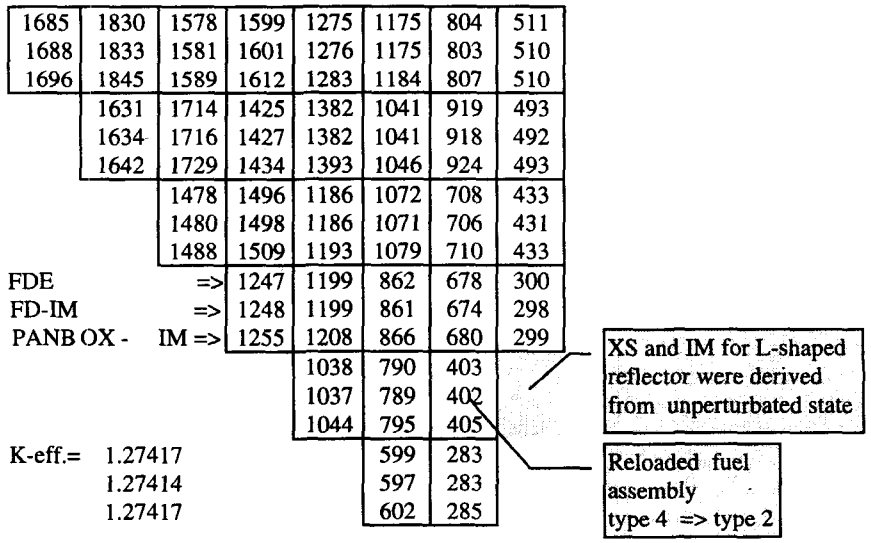

Fig. 2. Independency of homogenized parameters from in-core conditions.

RELATIVE ERROR OF PIN-BY-PIN CALCULATION

X=10000*(B-A)/A
where A=pin power for FDE
B=pin power for FD-IM

Results for cell L-15

-12	**-10**	**-7**	**-4**	**-2**	**0**	**3**	**6**	**8**	**10**	**12**	**13**	**14**	**14**	**14**
-14	-11	-8	-5	-1	1	4	8	10	13	15	17	18	19	19	18
-16	-13	-9	-5	-1	2	6	10	14	17	19	21	23	24	24	23
-19	-15	-10	-5	-1	3	8	13	17	21	25	27	29	30	30	30
-22	-17	-12	-6	0	5	11	17	22	27	31	34	36	38	38	38
-26	-20	-14	-7	0	7	15	22	28	34	39	43	45	47	47	47
-30	-24	-16	-7	1	10	19	28	36	43	49	53	56	58	59	58
-37	-29	-19	-8	2	13	25	35	45	54	61	66	70	72	72	71
-45	-35	-23	-10	3	18	32	45	58	68	77	83	88	89	89	87
-55	-43	-29	-12	5	23	41	58	73	86	97	105	110	111	110	106
-70	-55	-36	-15	7	31	54	75	94	111	124	133	138	139	136	130
-90	-71	-47	-18	11	41	71	98	122	142	158	170	175	175	170	159
-119	-94	-61	-23	16	56	94	128	159	185	205	219	225	224	214	196
-161	-126	-80	-27	26	78	127	171	209	242	268	285	293	290	275	245
-226	-174	-105	-30	42	110	173	230	279	320	353	376	388	385	362	312
-337	-244	-133	-26	72	162	242	314	376	429	472	503	522	523	498	424

Results for cell L-14

-20	**-21**	**-22**	**-22**	**-22**	**-23**	**-23**	**-23**	**-23**	**-22**	**-22**	**-21**	**-20**	**-18**	**-17**	**-15**
-22	-23	-24	-24	-25	-25	-26	-26	-26	-25	-25	-24	-23	-21	-19	-17
-25	-25	-26	-27	-28	-28	-29	-29	-29	-29	-28	-27	-26	-24	-22	-19
-27	-28	-29	-30	-31	-31	-32	-32	-32	-32	-32	-31	-30	-28	-25	-22
-29	-30	-32	-33	-34	-35	-36	-36	-37	-37	-36	-35	-34	-32	-29	-26
-32	-33	-35	-36	-37	-38	-40	-40	-41	-41	-41	-41	-39	-37	-34	-30
-34	-36	-38	-39	-41	-43	-44	-45	-46	-47	-47	-47	-46	-44	-40	-36
-37	-39	-41	-43	-45	-47	-49	-51	-52	-54	-54	-54	-53	-51	-48	-43
-40	-42	-45	-47	-49	-52	-54	-57	-59	-61	-62	-63	-63	-61	-58	-52
-43	-46	-48	-51	-54	-57	-60	-63	-67	-69	-72	-74	-74	-73	-70	-64
-46	-49	-52	-55	-59	-62	-66	-71	-75	-79	-83	-86	-89	-89	-87	-80
-49	-52	-56	-60	-64	-68	-73	-78	-84	-90	-96	-102	-106	-109	-108	-102
-52	-56	-60	-64	-69	-74	-80	-87	-94	-102	-110	-119	-127	-134	-137	-133
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-58	-62	-67	-72	-78	-85	-94	-103	-114	-127	-143	-161	-183	-207	-230	-244
-61	-65	-70	-76	-83	-91	-100	-111	-124	-139	-159	-183	-215	-254	-304	-354

Results for cell M-14

-63	**-68**	**-73**	**-79**	**-87**	**-95**	**-105**	**-117**	**-132**	**-150**	**-173**	**-203**	**-243**	**-301**	**-390**	**-555**
-65	-70	-76	-82	-90	-99	-109	-122	-138	-158	-183	-216	-261	-325	-421	-573
-67	-72	-78	-84	-92	-101	-112	-125	-142	-162	-188	-221	-266	-327	-412	-527
-68	-73	-79	-86	-94	-103	-113	-127	-142	-162	-187	-218	-259	-312	-379	-462
-70	-74	-80	-87	-94	-103	-113	-126	-141	-159	-181	-209	-243	-285	-334	-390
-70	-75	-80	-87	-94	-102	-112	-123	-136	-152	-171	-193	-220	-250	-283	-316
-71	-75	-80	-86	-93	-100	-108	-118	-129	-142	-157	-174	-192	-211	-229	-242
-71	-75	-80	-85	-91	-97	-104	-112	-121	-130	-141	-152	-162	-170	-174	-169
-71	-75	-79	-83	-88	-93	-99	-105	-111	-117	-123	-127	-130	-128	-119	-97
-71	-74	-77	-81	-85	-88	-92	-96	-99	-102	-103	-102	-96	-85	-64	-27
-70	-73	-76	-78	-81	-83	-86	-87	-87	-86	-83	-75	-62	-42	-9	40
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-70	-70	-70	-70	-69	-67	-63	-58	-49	-37	-20	4	39	89	159	254
-70	-69	-68	-67	-65	-61	-56	-48	-37	-22	0	30	72	133	219	340
-70	-69	-67	-64	-60	-55	-48	-38	-25	-6	18	54	103	175	280	439