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# Reactor Condition Monitoring via Wavelet Transform De-noising

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## Abstract

*Wavelets are localized in space and in frequency. This localization properties result from the multiresolution analysis of wavelets. The wavelet transform can be used to detect singularity of dynamic systems after the signal is de-noised. We applied the wavelet transform decomposition and de-noising procedures to the Hanaro dynamics consisting of 39 nonlinear differential equations plus Gaussian noise. The numerical tests demonstrate that the wavelet transform de-noising is effective for detection of the abrupt reactivity change and computationally efficient. Thus this wavelet theory could be profitably utilized in a real-time system for automatic event recognition (e.g., reactor condition monitoring).*

## I. Introduction

Wavelets are useful in many different fields of science and engineering, e.g., sound analysis and reconstruction, decomposition of visual data, detection of edges and singularities.[1,2] Also, several people have tried to solve differential equations using orthonormal wavelet functions,[3] including the neutron diffusion equation.[4]

Here, we concentrate on the detection of edge and singularities for process signal and image. The de-noising capability of wavelet transform is exploited for making plant signal smoother and more readable. We study through the wavelet theory the method of detecting abrupt changes in signals and thus in dynamic systems.

In our algorithm, we use the orthonormal basis of compactly supported wavelets constructed by Daubechies.[1] The wavelet functions are generated by dilation and translation operations such as

$$\psi_{n,k}(x) = 2^{n/2}\psi(2^n x - k) \tag{1}$$

for some $\psi \in L^2(\mathbf{R})$.

A *multiresolution·analysis(MRA)* of $L^2(\mathbf{R})$ is defined as a set of closed subspaces $V_j$ with $j \in \mathbf{Z}$ that exhibit the following properties :

1) $V_j \subset V_{j+1}$,
2) $v(x) \in V_j \Leftrightarrow v(2x) \in V_{j+1}$ and $v(x) \in V_0 \Leftrightarrow v(x+1) \in V_0$ ,
3) $\bigcup_{j=-\infty}^{+\infty} V_j$ is dense in $L^2(\mathbf{R})$ and $\bigcap_{j=-\infty}^{+\infty} V_j = \{0\}$,
4) A scaling function $\phi(x) \in V_0$ exists such that the set $\{\phi(x-l) | l \in \mathbf{Z}\}$ is a basis of $V_0$.

Consequently, a sequence $(h_k) \in l^2(\mathbf{Z})$ exists such that the scaling function satisfies *a two-scale difference equation*

$$\phi(x) = \sum_k h_k \phi(2x-k). \tag{2}$$

The set of functions $\{ \phi_{j,l}(x) \mid l \in \mathbf{Z} \}$ with $\phi_{j,l}(x) = 2^{j/2}\phi(2^j x - l)$ is an orthonormal basis of $V_j$ . A complement space of $V_j$ in $V_{j+1}$ is denoted by $W_j$ , so $V_{j+1} = V_j \oplus W_j$, and consequently,

$$\bigoplus_{j=-\infty}^{+\infty} W_j = L^2(\mathbf{R}). \tag{3}$$

The complementary spaces are chosen such that

$$w(x) \in W_j \Leftrightarrow w(2x) \in W_{j+1} \quad \text{and} \quad w(x) \in W_0 \Leftrightarrow w(x+1) \in W_0. \tag{4}$$

A function $\psi(x)$ is a mother wavelet if the set of functions $\{ \psi(x-l) \mid l \in \mathbf{Z} \}$ is an orthonormal basis of $W_0$. Since the mother wavelet is also an element of $V_1$, a sequence $(g_k) \in l^2(\mathbf{Z})$ exists such that

$$\psi(x) = \sum_k g_k \phi(2x-k). \tag{5}$$

The set of functions $\{ \psi_{j,l}(x) \mid l \in \mathbf{Z} \}$ with $\psi_{j,l}(x) = 2^{j/2}\psi(2^j x - l)$ is an orthonormal basis of $V_j$ . The coefficients $h_k$ and $g_k$ are related by

$$g_k = (-1)^k h_{2N-1-k}, \tag{6}$$

where $h_k$ and $g_k$ are the coefficients of Daubechies' wavelets.

Suppose that a finite sequence $s_k^0$, $k = 1, 2, \cdots, K$ is given. The fast wavelet transform (FWT)[5] can be written as:

$$\begin{aligned} s_k^j &= \sum_{n=0}^{2N-1} h_n s_{n+2k-1}^{j-1}, \\ d_k^j &= \sum_{n=0}^{2N-1} g_n s_{n+2k-1}^{j-1}, \end{aligned} \tag{7}$$

and the inverse fast wavelet transform can be written as:

$$\begin{aligned} s_{2n}^{j-1} &= \sum_{k=1}^{N} h_{2k-1} s_{n-k+1}^j + \sum_{k=1}^{N} g_{2k-1} d_{n-k+1}^j, \\ s_{2n-1}^{j-1} &= \sum_{k=1}^{N} h_{2k-2} s_{n-k+1}^j + \sum_{k=1}^{N} g_{2k-2} d_{n-k+1}^j, \end{aligned} \tag{8}$$

where $N$ is the Daubechies' order.

Recently, we developed a wavelet transform method for the flux reconstruction problem in reactor analysis.[6]

## II. De-noising with Thresholding Algorithms

Besides the advantages due to its multiresolution properties, the wavelet transform shows another important characteristic which can be exploited in the signal domain, i.e., the capability of discriminating a typical "noise" process from a typical "signal" one.

The de-noising capability is based on the different evolution across scales of its modulus ( $d^j$ ) maxima, depending on the signal regularity degree, defined in terms of the Lipschitz exponent.[7] The wavelet transform modulus maxima of a signal that can be described in terms of a negative Lipschitz exponent tend to vanish as the scale increases, whereas this does not happen if the signal is characterized by a positive exponent. The wavelet modulus maxima are related to the sharp variation points in the signal; therefore, they carry very useful information about the signal.

Given a wavelet coefficient $\{s\}$, the threshold (t) is $\sqrt{2\log(n)}\sigma$ ( $\sigma$ : noise level). The *hard-thresholded* value is given by $T_{hard}(s;t) = sI \ (|s| > t)$, (keep-or-kill) and the *soft-thresholded* value by $T_{soft}(s;t) = sgn(s)(|s| - t)I \ (|s| > t)$, (shrink-or-kill) where I is the usual indicator function.

The difference between soft and hard thresholding is that in the soft case, the finer scale terms that are included are also shrunk. This shrinking has the effect of reducing variance, but at some cost in increased bias.

An attractively simple choice for the threshold value is based on the idea of "de-noising", which attempts to eliminate pure noise terms. For white noise, $\sigma$ can be estimated very well, say by a robust scale estimate applied to the finest scale wavelet coefficients, so we will assume that $\sigma$ is known. Assuming the errors are Gaussian allows use of the interesting property that for $X_1, \cdots, X_n$ $i.i.d. N(0, \sigma^2)$,

$$P(\max_{i=1,\cdots,n}|X_i| \leq \sigma\sqrt{2\log(n)}) \rightarrow 1, \tag{9}$$

Hence the threshold $t=\sqrt{2\log(n)}\sigma$ will zero out every term that has all noise and no signal.

## III. Singularity Detection

Many problems in signal processing are concerned with the detection of edges and singularities. The conventional method of edge and singularity detection is signal segmentation algorithms which will lead to event detection, stationarity hypothesis testing, for example, for applications in fault detection, non-destructive testing, bio-medical, or space signal processing. Segmentation algorithms for signal or images generally are based on one of the two basic properties of level values: discontinuity and similarity.

Basically, the idea underlying the most edge-detection techniques is the computation of a local derivative operator. The wavelet based method is based on fast wavelet transform.

The presence of noise makes the wavelet transform decomposition useless for the signal detection. But the wavelet transform coefficients are able to effectively localize the sharp variation points of a signal (and in particular the onset of anomaly), provided that the signal is de-noised, as has been proven by applying the technique on synthetic data.[7]

We call a smoothing function $\theta(x)$ whose integral is equal to 1 and that converges to 0 at infinity. If we have a wavelet $\psi(x)$ equal to the first-derivative of a smoothing function $\theta(x)$, the wavelet transform is computed by convolving the signal with a dilated wavelet. The wavelet transform of $f(x)$ at the scale $s$ and position $x$, computed with respect to the wavelet $\psi(x)$, is defined by

$$W_s f(x) = f * \psi_s(x) = \int_{-\infty}^{+\infty} f(u)\psi_s(x-u)\,du. \tag{10}$$

We can easily derive that

$$W_s^1 f(x) = f*(s\frac{d\theta_s}{dx})(x) = s\frac{d}{dx}(f*\theta_s)(x). \tag{11}$$

The local extrema of $W_s^1 f(x)$ correspond to the inflection points of $f*\theta_s(x)$. For edge and singularity detection, we are only interested in the local maxima of $|W_s^1 f(x)|$. When detecting the local maxima of $|W_s^1 f(x)|$, we can also keep the value of the wavelet transform at the corresponding location. We note that the regularity of Daubechies wavelets increases linearly with $N$ (=Daubechies' order).

$\phi_{n,k}, \psi_{n,k} \in C^{\lambda(N)}$ = space of Holder continuous functions with exponent $\lambda(N)$.

$$\lambda(2)=0.55000, \quad \lambda(3)=1.08783, \quad \lambda(4)=1.67179, \quad \lambda(N)=0.3485N, \quad \text{for large } N. \tag{12}$$

Therefore, Daubechies' wavelet $\psi(x)$ could be equal to the first-derivative of a smoothing function $\theta(x)$. One signal sharp variation produces modulus maxima at different scales $2^j$.

The value of a modulus maximum at scale $2^j$ measures the derivative of the signal smoothed at scale $2^j$.

## IV. Numerical Tests and Conclusions

The first test is a step signal which has the Gaussian noise, that is $N(0, \sigma^2)$, $\sigma=0.5, 1$. Total 128 data are given. Fig. 1 shows the de-noised step signal and its modulus( $d^1$ ) for D4 wavelet. The behavior of root mean square error with various Daubechies' order is given in Fig. 2.

Our wavelet thresholding method is applied to the dynamics of Korea Multipurpose Research Reactor (KMRR, or Hanaro), and Runge-Kutta method is used as integrator. We use 39 nonlinear differential equations to describe the KMRR model.[8] The reactivity is a series of step functions and the time step is $\Delta t$=0.01sec. Total 128 data are given with the Gaussian noise $N(0,0.5^2)$. The results are presented in Figs. 3 and 4. From the wavelet modulus, we can ascertain the abrupt change in the reactivity of the reactor.

From the comparison of root mean square errors, the shorter D4 wavelet is shown to be more effective for the step signal but the longer D6 wavelet is more effective for the reactor power signal. This can be explained easily considering that the high-pass filtered signal obtained at the first octave wavelet transform retains the information about the high frequency components of the original signal which carry information about the abrupt variations in the signal itself. In transformed signal, the high frequency is related to the transient behavior and the low frequency to the stationary behavior.

In conclusion, the wavelet transform can be used to detect singularity of dynamic systems after the signal is de-noised. The wavelet transform decomposition and de-noising procedures have proved to be computationally efficient. So this wavelet theory could be profitably utilized in a real-time system for automatic event recognition (e.g., reactor condition monitoring).

## References


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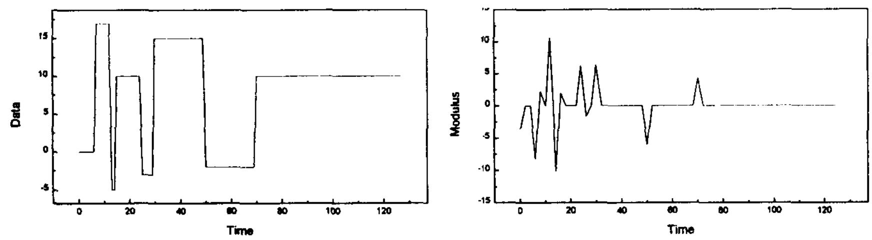
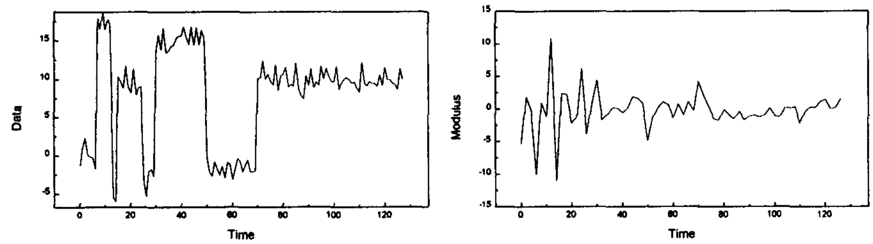
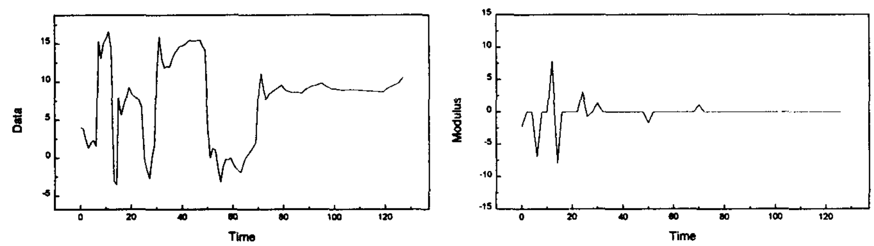


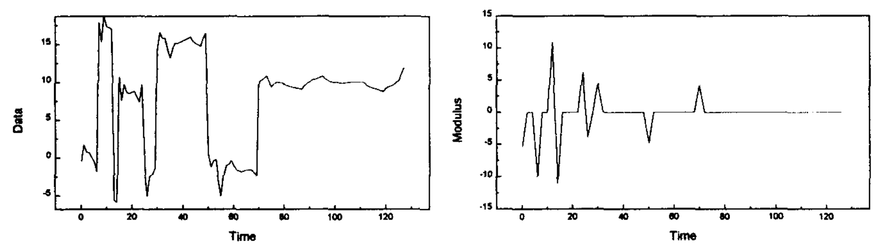

(a) Input signal and its wavelet modulus




(b) Noisy signal and its wavelet modulus




(c) Soft threshold de-noised signal and its wavelet modulus




*(d) Hard threshold de-noised signal and its wavelet modulus*

Fig. 1. De-noised step signal and its modulus for D4 wavelet and noise level (σ) 1.

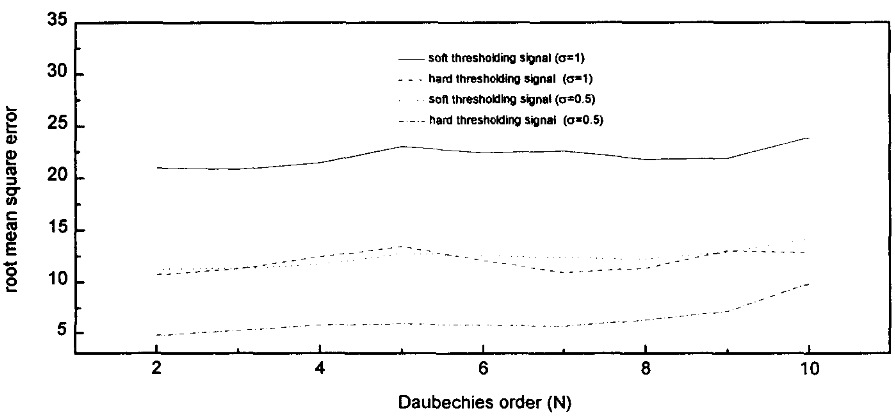
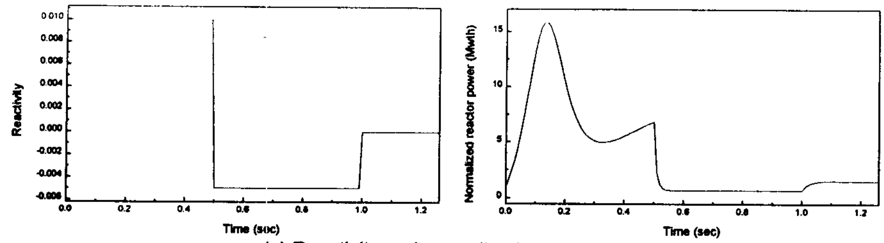
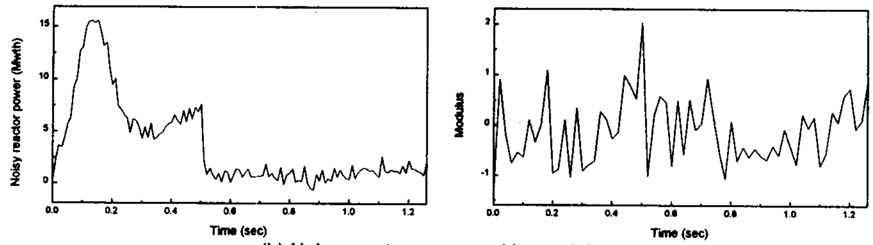


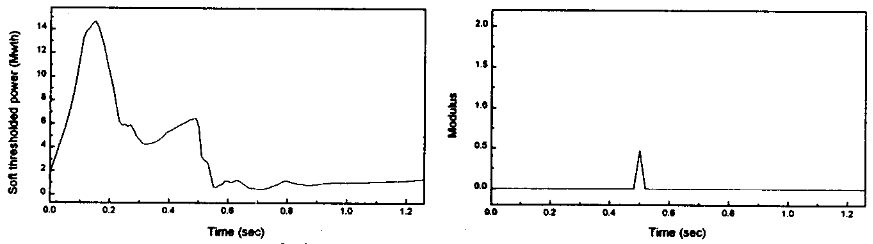

Fig.2. Root mean square errors of de-noised step signal with soft and hard thresholding (σ= 0.5, 1).



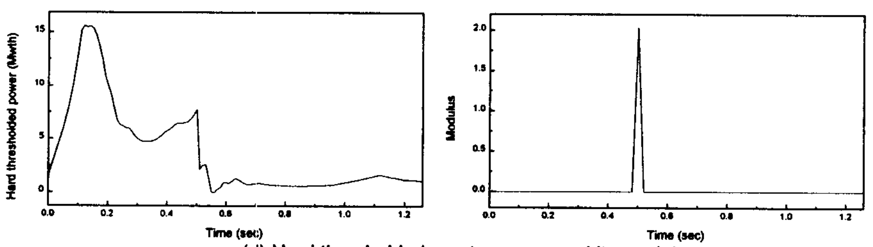

(a) Reactivity and normalized reactor power




(b) Noisy reactor power and its modulus ($\sigma$=0.5)




(c) Soft thresholded reactor power and its modulus




(d) Hard thresholded reactor power and its modulus

Fig.3. De-noised reactor power and its wavelet modulus for D6 wavelet with soft and hard thresholding.

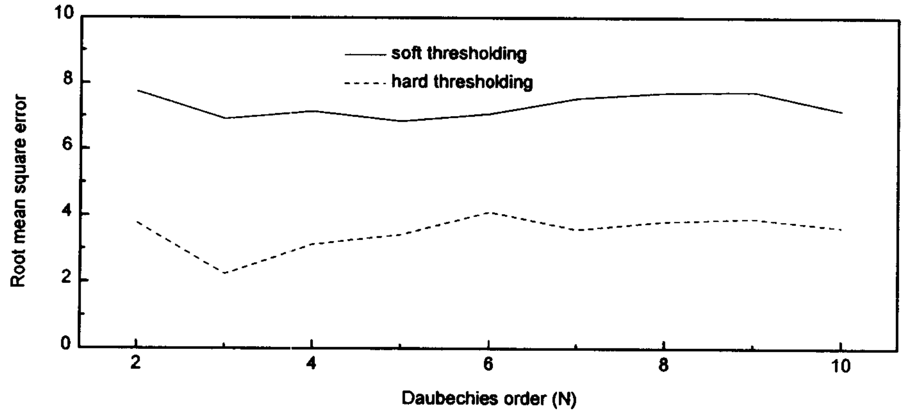



Fig. 4. Root mean square error of de-noised reactor power with soft and hard thresholding ($\sigma$=0.5).