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Delayed Hopfield-like Neural Network for Solving Inverse Radiation Transport Problem

Sang Hoon Lee and Nam Zin Cho

Korea Advanced Institute of Science and Technology
Department of Nuclear Engineering
373-1 Kusong-dong, Yusong-gu
Taejon, Korea 305-701

Abstract

The identification of radioactive source in a medium with a limited number of external detectors is introduced as an inverse radiation transport problem. This kind of inverse problem is usually ill-posed and severely under-determined, however, its applications are very useful in many fields including medical diagnosis and nondestructive assay of nuclear materials. Therefore, it is desired to develop efficient and robust solution algorithms. As an approach to solving inverse problems, an artificial neural network is proposed. We develop a modified version of the conventional Hopfield neural network and demonstrate its efficiency and robustness.

1. Introduction

The phenomena of radiation transport within a given medium are described by *Boltzmann transport equation* and using this equation we can calculate the boundary values. On the contrary, consider another problem that if we only know the boundary values(usually detected values) then how to estimate where the sources are?

A problem involving the estimation of sources' spatial distribution within a medium of known properties and boundary conditions based on the measured external activities is a kind of **inverse radiation transport problem.** Some methods for solving inverse radiation transport problems are usually formulated in terms of constrained least squares and their corresponding objective function. An artificial neural network, more precisely, Hopfield Neural Network(HNN) was proposed as a new approach for these problems[1]. In some optimization problems, the HNN which has one-layered and fully interconnected neurons with feed-back topology showed that it worked well with acceptable fault tolerance[2]. As proven by *Takeda et al.*, when diagonal elements of the interconnection matrix are not zero, the HNN becomes unstable. However, most problems including this identification problem require non-zero diagonal elements when programmed on neural networks[3].

According to *Soulie* and *Weisbuch*, discrete random iteration could produce stable minimum state of associative memory[4]. In this study, we modify the conventional HNN into a new HNN which has random delayed updating intervals in order to alleviate the above unstable phenomenon. So we call it as "**Delayed Hopfield-like Neural Network(DHNN)**".

2. Radioactive Source Identification with DHNN

Consider a uniform, isotropic medium which is divided into N uniform meshes and M external detectors deployed around the medium. Under these assumptions, the measured activity at i^{th} detector is determined by

$$y_i = \sum_{j=1}^{N} a_{ij} x_j, \qquad i = 1, \cdots, M, \tag{1}$$

where x_j is the source strength in mesh j and the coefficient a_{ij} which describes the attenuation along the distance from mesh j to detector i is given by

$$a_{ij} = \frac{1}{4\pi r_{ij}^2} \exp(-\mu r_{ij}). \tag{2}$$

In this study, the measurement vector $\mathbf{y}$ is expressed as a linear transformation $\mathbf{A}$ of the desired solution vector $\mathbf{x}$ plus additive noise vector $\mathbf{n}$

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{3}$$

where $\mathbf{A}$ is the attenuation matrix whose elements are given by Eq. (2). What is common to all these problems is that $\mathbf{A}$ is usually ill-conditioned, in other words, unstable to small perturbations in input data. Although we know the inverse of $\mathbf{A}$, its direct use often amplifies noise and thereby prohibits a high quality recovery of $\mathbf{x}$. All such problems are often named "inverse problems".

This kind of inverse problem can be converted into least squares optimization problem and solved with HNN. However as stated in Section 1, we apply some algorithms such as randomized updating intervals, simulated annealing to guarantee and to improve stability of our DHNN. Even if some modified strategies are applied, the stable performance is not expected under additive noise environment. To overcome this undesired phenomenon, regularization theory is adopted. The estimated solution $\hat{\mathbf{x}}$ to this problem is formulated in terms of least squares with a regularization term. In this approach, the objective function E is defined as follows:

$$E = \frac{1}{2}\|\mathbf{y} - \mathbf{A}\hat{\mathbf{x}}\|^2 + \frac{1}{2}\lambda\|\mathbf{D}\hat{\mathbf{x}}\|^2, \tag{4}$$

where λ is a Lagrangian multiplier and the last term is the regularization term; its purpose is to force a distinctive constraint on the estimated solution $\hat{\mathbf{x}}$. The constant λ determines the relative importance of this term. The matrix $\mathbf{D}$ is a second-order differential operator and its element is described as d_{ij}.

A discretized objective function which should be minimized for the best estimated solution is given as follows:

$$E = \frac{1}{2}\sum_{i=1}^{M}[(y_i - \sum_{j=1}^{N} a_{ij}x_j)^2 + \lambda(\sum_{j=1}^{N} d_{ij}x_j)^2]. \tag{5}$$

The neural parameters that correspond to this objective function are interconnection weight term

$$w_{jl} = -\sum_{i=1}^{M}(a_{ij}a_{il} + \lambda d_{ij}d_{il}), \tag{6}$$

and threshold term

$$\theta_j = \sum_{i=1}^{M} y_i a_{ij}. \tag{7}$$

According to *Hopfield*, the updating of HNN is done as follows:

1. calculate the weighted sum u_j for j^{th} neuron at k^{th} iteration,

$$u_j^k = \sum_{l=1}^{N} w_{jl}x_l^k + \theta_j. \tag{8}$$

2. update sequentially(asynchronously),

$$x_j^{k+1} = \begin{cases} 1 & u_j^k > 0, \\ x_j^k & u_j^k = 0, \\ 0 & u_j^k < 0. \end{cases} \tag{9}$$

If diagonal elements of $\mathbf{W}$, i.e., w_{jj} are negative(in this problem) then a modified(randomized) updating strategy will be better. For this purpose, we adopt random updating intervals as delayed updating.

There is another sensitive point in this kind of optimization problem, that is, how to escape from local minima? Therefore, we adopt a hill climbing strategy by simulated annealing using *Boltzmann transition rule*. At each iteration, we calculate the energy difference of DHNN by comparing present state energy with the previous one

$$\Delta E = E(x_k) - E(x_{k-1}). \tag{10}$$

We accept the updated state if and only if $\Delta E \leq 0$, otherwise, accept it by *Boltzmann transition rule*

$$Prob(x \to x') = \frac{1}{1 + \exp(-\Delta E/T)}. \tag{11}$$

During this procedure, the parameter T known as system temperature continuously decreases(simulated annealing) from an initial high value(hot state) to a given low value(cold state)[5].

3. Results and Discussion

To evaluate DHNN, we consider a 10×10cm size medium which has 64 uniform meshes with 3 distributed constant sources and 12 detectors as depicted in Fig. 1. The material in the 2-D plane is assumed that its attenuation coefficient μ is 0.002 cm^{-1} and tested with various Gaussian noise corrupted data. The noise level is given by signal-to-noise ratio(SNR) and the lower SNR means the higher additive noise.

We consider that DHNN is converged when the energy of DHNN is less than the given limit value or the neuron states are not changed during a certain iteration length. The DHNN dynamics with respect to iterations shows the hill climbing search feature in Fig 2.

In the view of noise robustness, DHNN is converged to its stable state, i.e., identified source locations with noise-free data quickly(typically a few seconds) and could identify perfectly. Although the probability of successful identification decreases as the noise level increases, the success rate is still above 80% until SNR is higher than about 23dB (Fig. 3). However, the number of iterations shows not specific trend but oscillatory behavior mostly due to two convergence criteria. For low level noise corrupted data, energy-limit criterion is dominant, on the contrary, for high level noise, stability(fixed iteration) criterion is dominant.

As a concluding remark, DHNN can identify the source location efficiently and it also works with reasonable robustness under certain noise environment.

References

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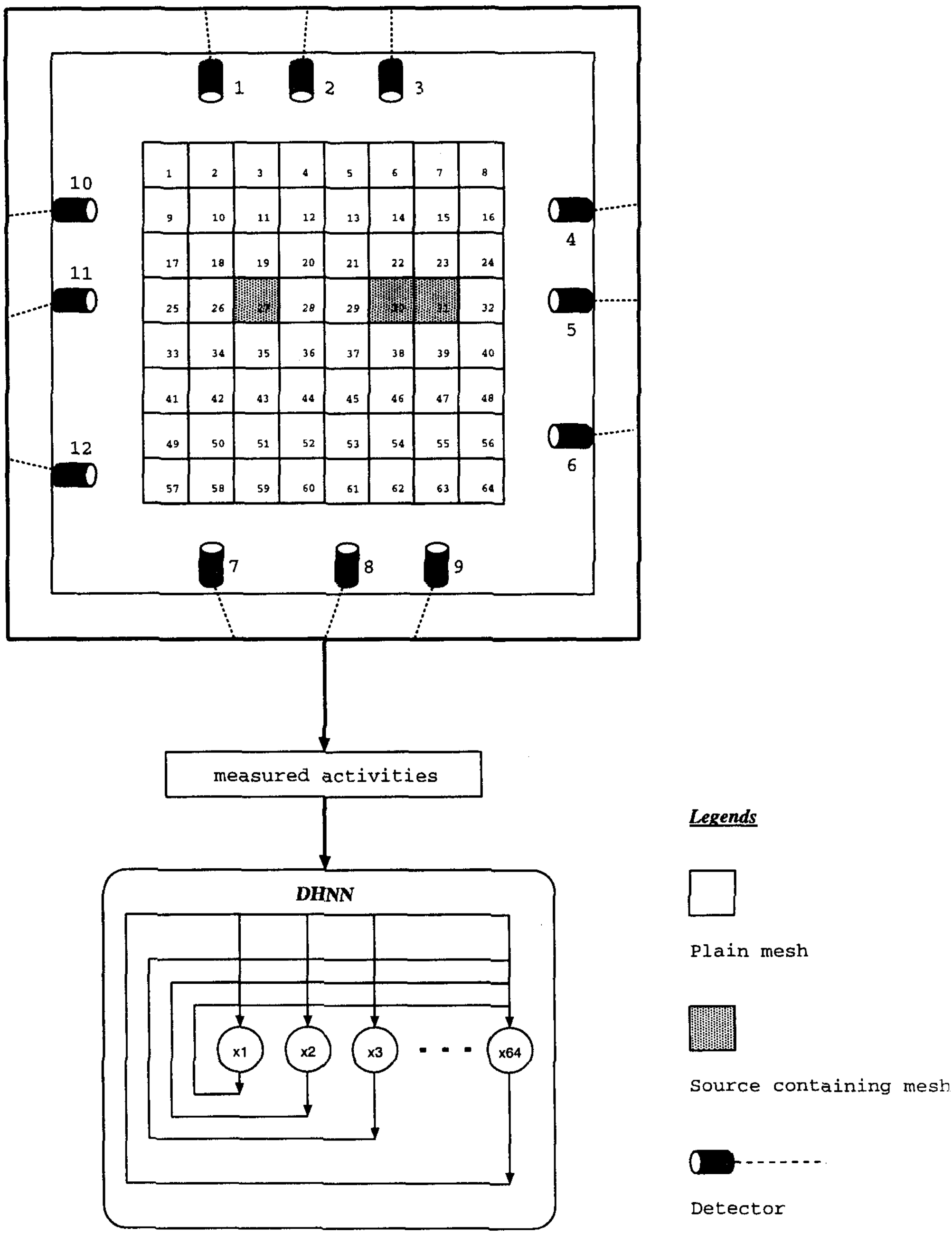

Figure 1: Description of Inverse Radiation Transport Problem

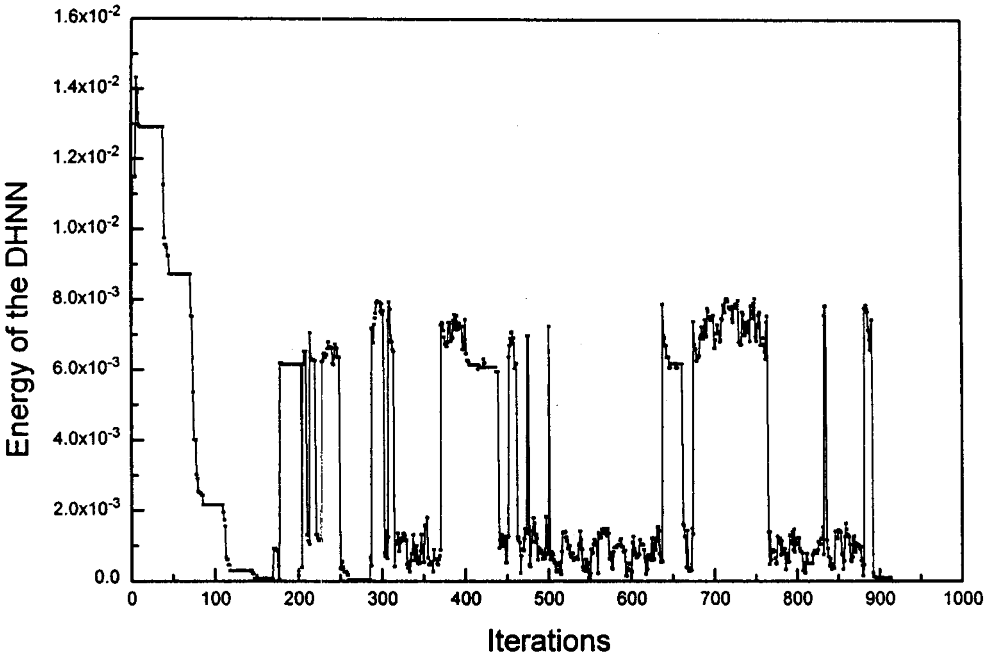

Figure 2: Dynamics of the DHNN

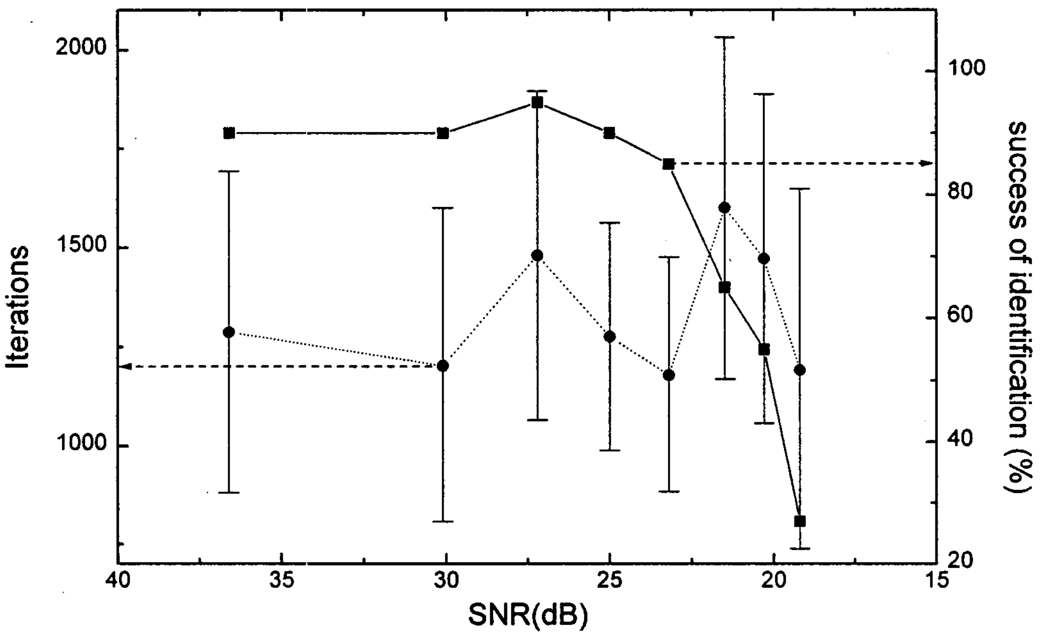

Figure 3: Robustness and Iterations with Respect to Noise