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Singular Residual Stresses at Interface of Compound Cylinders

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Abstract

This paper concerns the cladding residual stresses in a reactor vessel induced during cooling from the manufacturing temperature down to room temperature. Finite element results show that very large stress gradients are present at the interface corner and such stress singularity might lead to local yielding or cladding-base metal debonding.

1. Introduction

Engineering components often feature dissimilar materials bonded along planar surfaces. Elastic analyses of such configurations generally exhibit stress singularities of various types depending on the loading, geometry, and materials[1-3]. The singular behavior of the stresses at the point of intersection of the boundary and the bonded orthogonal wedge is given as $r^{-2+(\alpha+i\beta)}$, where $i=\sqrt{-1}$, the complex quantity $\alpha+i\beta$ is a simple root of a certain entire function of the elastic constants of the two materials and $-2<\alpha<-1$. The imaginary part $i\beta$ presents the oscillatory behavior of stress. Bimaterial interfaces are common in composite technology where the mismatch of thermal and mechanical properties can cause severe manufacturing, service and durability problems[4]. Parallels can also be drawn in reactor vessel technology. It is well known that the residual thermal stresses are developed in the reactor vessel upon cool down from manufacturing temperature[5]. Such stresses are due to the mismatch in thermoelastic properties between the cladding layer and the base metal.

The present study examines the cladding residual stresses at the interface of a hollow cylinder by finite element method using the ANSYS[6]. The interface region between the cladding layer and the base metal may suffer from highly localized stresses in the vicinity of the ends. Axisymmetric problem of welded cylinder of different materials are investigated in this paper. A comprehensive study of this type, to our knowledge, is not available in the literature.

2. Plane-Strain Solution

An elastic stress analysis of cooled compound cylinder is performed, in which a plane-strain axisymmetric approach is used to predict the stress distribution of such a cylinder. Consider a compound cylinder subjected to uniform temperature change DT(t), in which the inner cylinder has an inner radius a and an outer radius b while the outer cylinder has an inner radius b and an outer radius c (see Figure 1). In this calculation, it is assumed that the inner cylinder and the outer cylinder are bonded perfectly, with no defects or cracks. Then the radial displacement u_r , radial and circumferential stresses σ_r , σ_θ are obtained by applying the displacement and traction continuity conditions over the common interface of two cylinders as follows:

(i) $a \leq r \leq b$ (inner cylinder)

$$(\sigma_r)_1 = -\frac{P^*b^2}{(b^2 - a^2)}\left[1 - \frac{a^2}{r^2}\right]$$

$$(\sigma_\theta)_1 = -\frac{P^*b^2}{(b^2 - a^2)}\left[1 + \frac{a^2}{r^2}\right] \tag{1}$$

$$(u_r)_1 = -\frac{P^*b^2}{(b^2 - a^2)}\left[\frac{1 - 2\nu_1}{G_1} r + \frac{a^2}{G_1 r}\right] + (1 + \nu_1)\alpha_1 \, \Delta T(t)\, r$$

(ii) $b \leq r \leq c$ (outer cylinder)

$$(\sigma_r)_2 = \frac{P^*b^2}{(c^2 - b^2)}\left[1 - \frac{c^2}{r^2}\right]$$

$$(\sigma_\theta)_2 = \frac{P^*b^2}{(c^2 - b^2)}\left[1 + \frac{c^2}{r^2}\right] \tag{2}$$

$$(u_r)_2 = -\frac{P^*b^2}{(c^2 - b^2)}\left[\frac{1 - 2\nu_2}{G_2} r + \frac{c^2}{G_2 r}\right] + (1 + \nu_2)\alpha_2 \, \Delta T(t)\, r$$

where subscripts 1 and 2 indicate the inner cylinder and outer cylinder, α_i , ν_i , and G_i represent the coefficient of thermal expansion, Poisson's ratio, and shear modulus, respectively and P* is

$$P^* = \frac{\left[(1 + \nu_1)\,\alpha_1 - (1 + \nu_2)\,\alpha_2\right]\Delta T(t)}{\left[\frac{b^2}{2\,(c^2 - b^2)}\left(\frac{1 - 2\nu_2}{G_2}\right) + \frac{c^2}{2\,(c^2 - b^2)}\left(\frac{1}{G_2}\right) + \frac{b^2}{2\,(b^2 - a^2)}\left(\frac{1 - 2\nu_1}{G_1}\right) + \frac{a^2}{2\,(b^2 - a^2)}\left(\frac{1}{G_1}\right)\right]} \tag{3}$$

The above two-dimensional solutions may be applicable to the mid-zone of the compound cylinder, but not very close to the ends. This plane-strain solutions could not account for the zero axial stress conditions at the ends of the compound cylinder. The shear stresses at the interface near the ends might be significant.

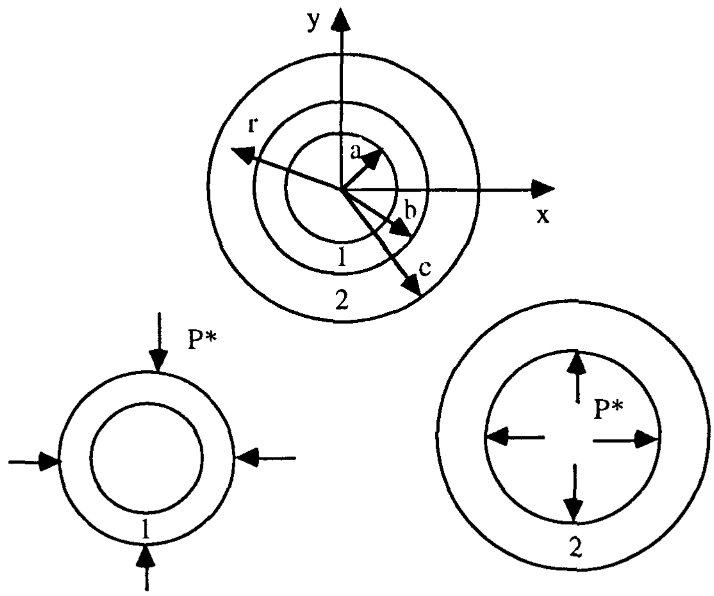

Figure 1. Plane-strain model of compound cylinder subjected to uniform temperature change $\Delta T(t)$

3. Finite Element Model

The two-dimensional axisymmetric finite element program ANSYS was used to predict thermal stress distribution in the compound cylinder. Figure 2 displays a cylinder with a length of L = 87.15 in, an inner radius of 69.25 in, an outer radius of 77.3 in, and cladded inside by a 0.25 in thick austenitic deposit. The interface radius between the cladding layer and the base metal was 69.25 in. Due to the symmetry of the problem only one quarter of the cylinder needs to be discretized. The two-dimensional axisymmetric model was analyzed using eight-node axisymmetric element. The two-dimensional axisymmetric finite element model is attractive since it can be performed with modest computational resources even for very refined meshes. Figure 3 shows the finite element grids used for the analysis. Constant material properties (see Table 1) and uniform temperature distribution were assumed. Very refined meshes were used to capture very large stress gradients within the region near the end.

	Cladding Layer	Base Metal
Young's Modulus, E (psi)	27 x 10^6	28.5 x 10^6
Poisson's Ratio, ν	0.3	0.3
Coefficient of Thermal Expansion, α (°F^{-1})	9.1 x 10^{-6}	7.5 x 10^{-6}

Table 1. Material properities.

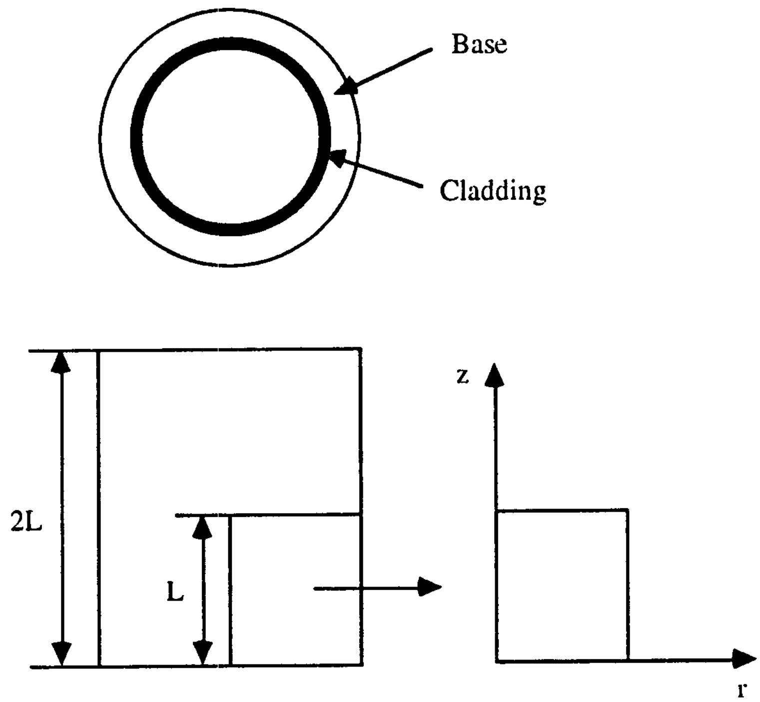

Figure 2. Compound cylinder used in the analysis.

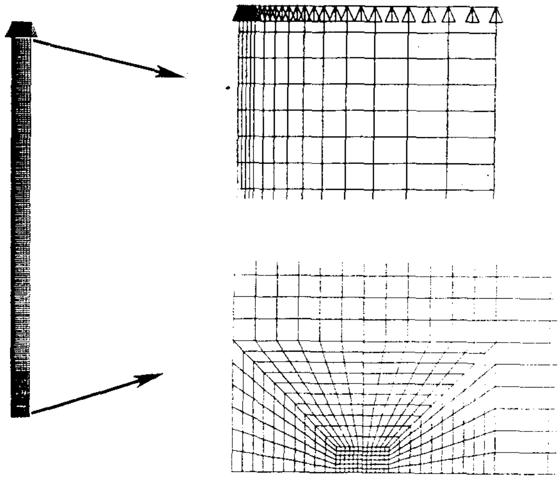

Figure 3. Finite element model of compound cylinder

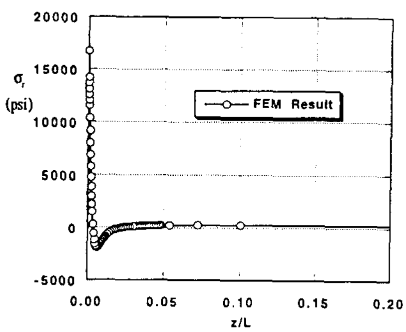

Figure 4. Variation of normal stress along interface

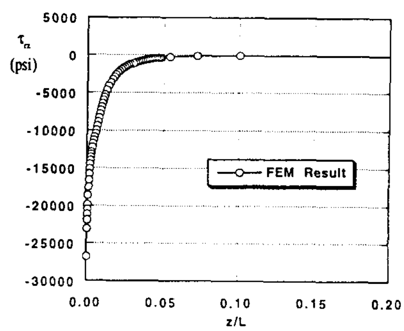

Figure 5. Variation of shear stress along interface

4. Numerical Results

The starting point for this calculation is the final heat treatment at 1238 °F (670 °C). It is assumed that all stress components are zero at this temperature. Then, the temperature is lowered from 1238° F to 68 °F (20 °C). In this calculation, it is assumed that the temperature changes throughout the body are uniform, i.e. the cylinder is subjected to uniform temperature fluctuation. Elastic stress profiles were plotted along interface to investigate the nature of stresses. Figure 4 shows the distribution of the radial interface stresses σ_r. At the mid-zone of the cylinder, this stress is in agreement with the plane-strain solutions of eqns (1) and (2). A large gradient in σ_r is observed in the vicinity of the end. Figure 5 is a plot of shear interface stress τ_{rz} vs. z/L. This profile shows a stress singularity at the interface corner. Figures 4 and 5 suggest that the thermal stress singularity on the interface does dominate a large region relative to cladding layer thickness for the case examined.

5. Conclusions

The FEM analysis has been performed to investigate the singular thermal stresses at the interface corner between the cladding layer and the base metal of a hollow cylinder induced during cooling from the manufacturing temperature down to room temperature. Numerical results show that very large stress gradients are present at the interface corner and such stress singularity dominates a large region relative to cladding layer thickness. Since the exceedingly large stresses at the interface corner can not be borne by cladding and base metals, local yielding or cladding-base metal debonding may occur in the vicinity of the end. To investigate the response in such a region in detail, a suitable non-linear model must be employed. Experimental evidence is required to assess the impact that this phenomenon has on the onset of failure in crack-free structures.

6. References

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