A Study on Orbit Maintenance and Required Fuel Consumption

정도희*, 이성복(한라중공업 우주항공연구소)

1. Introduction

For low Earth orbit, the atmosphere causes orbit altitude to decrease. If this decrease is not corrected by the satellite propulsive unit, then the orbit decays continuously until reaches the dense atmosphere and satellite life ends. If active orbit maintenance is made by satellite propulsive unit then fuel consumption is necessary, which must be considered in the satellite design. Especially interesting is the method for evaluating the fuel consumption rate for maintenance of elliptical orbit developed in this paper.

2. Difference Equations of orbit correction

The differential equation of orbit perturbation can be written in general from as follows

$$\frac{dx_j}{dt} = \sum_{i=1}^{m} R_i f_i \qquad (j = 1,2,\cdots,n) \tag{1}$$

where x_j is the j-th orbit element, f_i is the i-th component of control perturbing acceleration, R_i is the associated variable coefficient. Eq.(1) is rewritten as

$$dx_j = (\sum_{i=1}^{m} R_i f_i) dt$$

For a finite and small time Δt, the velocity impulse generated by the rocket engine is

$$\Delta v_i = f_i \Delta t$$

and the differential equation [Eq. (1)] becomes difference equation

$$\Delta x_i = \sum_{i=1}^{m} R_i \Delta v_i \tag{2}$$

Because the action of orbit maintenance or orbit correction is always moderate, it is possible to transform the differential equations of orbit perturbation into difference equation.

For the first group of equations of orbit perturbation from Basic Equation, the velocity impulses in r, u, h directions can be expressed as product of control acceleration(i.e., control force per unit mass) and time interval :

$$\Delta v_r = f_r \Delta t , \qquad \Delta v_u = f_u \Delta t , \qquad \Delta v_h = f_h \Delta t \tag{3}$$

thus the first group of difference equations of orbit correction is derived :

$$\begin{aligned}
\Delta a &= \frac{2a^2}{\sqrt{\mu p}}[e\sin\theta\Delta V_r + (1+e\cos\theta)\Delta v_u \\
\Delta e &= \sqrt{\frac{p}{\mu}}\{\sin\theta\,\Delta V_r + [(1+\frac{r}{p})\cos\theta + \frac{er}{p}]\Delta v_u\} \\
\Delta p &= 2\sqrt{\frac{p}{\mu}} r\Delta v_u \\
\Delta\Omega &= \frac{1}{\sqrt{\mu p}\sin i} r\sin(\omega+\theta)\Delta v_h \\
\Delta i &= \frac{1}{\sqrt{\mu p}} r\cos\theta(\omega+\theta)\Delta v_h \\
\Delta\omega &= \frac{1}{e}\sqrt{\frac{p}{\mu}}[-\cos\theta\,\Delta v_r + (1+\frac{r}{p})\sin\theta\,\Delta v_h - \frac{er}{p}\sin(\omega+\theta)\cot i\Delta v_h]
\end{aligned} \tag{4}$$

For the second group of equations of orbit perturbation from another Basic Equation, the velocity impulses in t, n, h directions are

$$\Delta v_t = f_t \Delta t , \qquad \Delta v_n = f_n \Delta t , \qquad \Delta v_h = f_h \Delta t \tag{5}$$

therefore the second group of difference equations of orbit correction is obtained :

$$\begin{aligned}
\Delta a &= \frac{2a^2 v}{\mu}\Delta v_t \\
\Delta e &= \frac{1}{v}[2(e+\cos\theta)\Delta V_t - \frac{r}{a}\sin\theta\Delta v_n \\
\Delta p &= \frac{2p}{v}\Delta v_t + \frac{2r}{v}\sin\theta\Delta v_n \\
\Delta\Omega &= \frac{1}{\sqrt{\mu p}\sin i} r\sin(\omega+\theta)\Delta v_h \\
\Delta i &= \frac{1}{\sqrt{\mu p}} r\cos(\omega+\theta)\Delta v_h \\
\Delta\omega &= \frac{2}{ve}\sin\theta\Delta v_t + \frac{a(1+e^2)-r}{ave^2}\Delta v_n - \frac{\cot i}{\sqrt{\mu p}} r\sin(w+\theta)\,\Delta v_h
\end{aligned} \tag{6}$$

When the required orbit element corrections $\Delta a, \Delta e, \Delta p, \Delta\Omega, \Delta i, \Delta\omega$ are given ,the necessary velocity impulses $\Delta v_r, \Delta v_u, \Delta v_h, or\, \Delta v_t, \Delta v_n, \Delta v_h$ can be found from the above equations. It is noticed that the velocity impulses are developed by the rocket engine(or generally by the propulsive unit) of the satellite at the expenditure of fuel consumption.

The amount of fuel needed for orbit correction must be taken into consideration in the satellite design.

3. Orbit Decay Caused by Atmosphere and Estimation of Lifetime

For circular orbit. from $\Delta r_{turn} = -4\pi\sigma\rho r^2$ and formula of orbit period P we have

$$dh/dt = \Delta r_{turn} / P = -2\sqrt{\mu}\sigma\rho\sqrt{r}$$

Making integration leads to

$$t_f - t_0 = \frac{1}{2\sigma\sqrt{\mu}} \int_{h_f}^{h_0} \frac{dh}{\rho(h)\sqrt{r}} \tag{7}$$

where 0 denotes 'initial', f denotes 'final'. This expression gives relation of t_f and h_f, therefore can be used to estimate the satellite lifetime in terms of orbit decay.

For elliptical orbit the expressions for change per revolution (turn), i.e., eq. (3-25)) for Δe_{turn} are used. Dividing them by orbit period $P = 2\pi\sqrt{a^3/\mu}$ and using relation $p = a(1-e^2)$ leads to differential equations

$$\begin{aligned} \frac{da}{dt} &= -\frac{\sigma}{\pi\mu\sqrt{1-e^2}} \int_0^{2\pi} r^2 v^3 \rho du \\ \frac{de}{dt} &= -\frac{\sigma}{\pi a^2 \sqrt{1-e^2}} \int_0^{2\pi} r^2 v (3+\cos\theta) \rho du \end{aligned} \tag{8}$$

These differential equations describe the variation of semimajor axis a and eccentricity e with time t, i.e., describe the decay of elliptical orbit due to atmosphere. They are solved numerically, and the results are used to estimate the satellite lifetime.

Eq. (8) is a special kind of system of differential equations the right side of which contains not only variables but also definite integrals.

4. Fuel consumption Rate Needed for Orbit Maintenance

For case of circular orbit, although the time interval between two successive maintenance maneuvers in form of Hohmann transfer depends on the altitude tolerance, but the average fuel consumption rate(FCR) can be approximately calculated from the condition of equilibrium:

T(thrust)=D(drag).

Thrust is

T=FCR*w

where w is equivalent speed of exhaust jet from the rocket nozzle, it is equal to g multiplied by specific impulse [when specific impulse is given in (sec)], or equal to specific impulse [when specific impulse is given in (m/s)].

Drag is

$$D = 0.5\rho v^2 SC_D$$
$$= 0.5\mu C_D S\rho / r$$

because $v^2 = \mu / r$ for circular obit. From the condition T=D we have

$$\text{FCR}=\frac{SC_D}{2w}\frac{\mu\rho}{r} \tag{9}$$

For case of elliptical orbit the average fuel consumption rate is derived in the following.

Previously the expressions for change of semimajer axis a and eccentricity e caused by atmospheric drag in one revolution are given [$\Delta a_{turn} = -\frac{2\sigma a^2}{\sqrt{\mu^3 p}}\int_0^{2\pi} r^2 v^3 \rho du \quad < 0$, $\Delta e_{turn} = -\frac{2\sigma}{\sqrt{\mu p}}\int_0^{2\pi} r^2 v(e+\cos\theta)\rho du \quad < 0$]. For more clarity subscript "atm" is used to replace "turn", i.e.,

$$\Delta a_{atm} = -\frac{2\sigma a^2}{\sqrt{\mu^3 p}} S_a$$
$$\Delta e_{atm} = -\frac{2\sigma}{\sqrt{\mu p}} S_e \tag{10}$$

with

$$S_a = \int_0^{2\pi} r^2 v^3 \rho du$$
$$S_e = \int_0^{2\pi} r^2 v(e+\cos\theta)\rho du \tag{11}$$

On the other hand , if every time when the satellite passes perigee P and apogee A the propulsive unit exerts correction velocity impulse Δv_p and Δv_a (Fig. 1) they will cause correction a and e as follows

$$\Delta \alpha_{cor} = \frac{2a^2}{\mu}(v_p \Delta v_p + v_a \Delta v_a)$$
$$= \frac{2a^2}{\sqrt{\mu p}}[(1+e)\Delta v_p + (1-e)\Delta v_a]$$
$$\Delta e_{cor} = \frac{2}{V_p}(e+1)\Delta v_p + \frac{2}{v_a}(e-1)\Delta v_a = \sqrt{\frac{p}{\mu}}[2\frac{e+1}{1+e}\Delta v_p + 2\frac{e-1}{1-e}\Delta v_a]$$
$$= 2\sqrt{\frac{p}{\mu}}(\Delta v_p - \Delta v_a)$$

(12)

These expressions are derived from the second group of difference equations of orbit correction and by use of

$$v_p = \sqrt{\mu/p}(1+e)$$
$$v_a = \sqrt{\mu/p}(1-e)$$

For complete correction it is required that

$$\Delta a_{cor} + \Delta a_{atm} = 0$$
$$\Delta e_{cor} + \Delta e_{atm} = 0 \qquad (13)$$

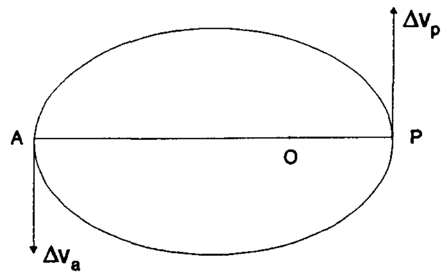


Fig. 1

Combination of Eq. (3), Eq. (10), Eq. (12) leads to

$$\frac{2a^2}{\sqrt{\mu p}}[(1+e)\Delta v_p + (1-e)\Delta v_a] = \frac{2\sigma a^2}{\sqrt{\mu^3 p}} S_a$$

$$2\sqrt{\frac{p}{\mu}}(\Delta v_p - \Delta v_a) = \frac{2\sigma}{\sqrt{\mu p}} S_e$$

or

$$(1+e)\Delta v_p + (1-e)\Delta v_a = \frac{\sigma}{\mu} S_a$$
$$\Delta v_p - \Delta v_a = \frac{\sigma}{p} S_e \qquad (14)$$

The solution of these linear algebraic equations are

$$\Delta v_p = \sigma[-\frac{S_a}{\mu} - (1-e)\frac{S_e}{p}$$
$$\Delta v_a = \sigma[-\frac{S_a}{\mu} + (1+e)\frac{S_e}{p} \qquad (15)$$

The sum of the two velocity impulse is

$$\Delta v_{sum} = \Delta v_p + \Delta v_a = 2\sigma[-\frac{S_a}{\mu} + e\frac{S_e}{p}] \qquad (16)$$

The simplified relation of mass of fuel consumption Δm_p and velocity is

$$\Delta m_p = \frac{m \Delta v_{sum}}{\omega} \qquad (17)$$

where ω is equivalent speed of exhaust jet from nozzle.

The average fuel consumption rate is got by diving Δm_p by orbit period P

$$FCR = \frac{\Delta m_p}{P} = \frac{\Delta m_p}{2\pi\sqrt{a^3/\mu}} \qquad (18)$$

Putting eqs(4-g) and(4-h) into (4-i) leads to the final result

$$FCR = \frac{C_D S}{2\omega} \cdot \frac{1}{\pi\sqrt{a^3/\mu}}\left(-\frac{S_a}{\mu} + e\frac{S_e}{p}\right) \qquad (19)$$

Although in practice the correction of orbit is carried out not so frequently(twice per revolution), but the formula for FCR[Eq. (18)] still give suitable estimation of fuel consumption.

5. Conclusion

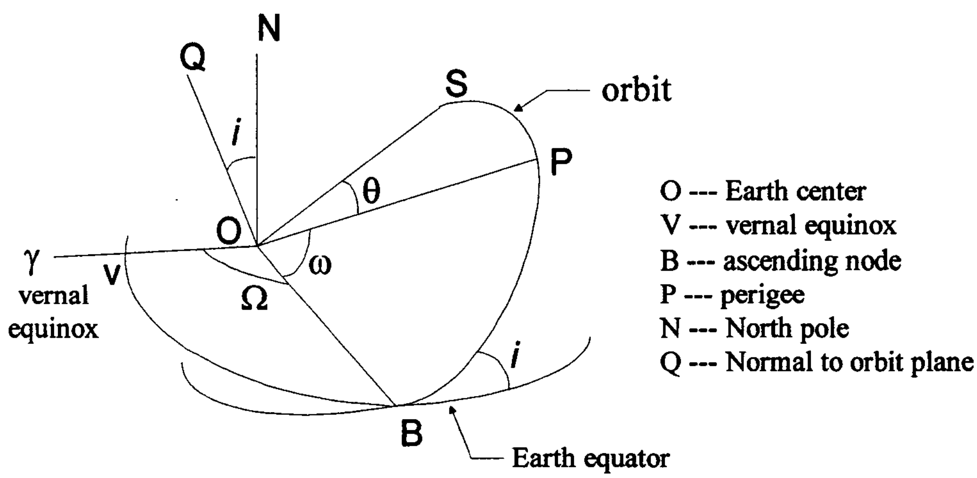
The satellite preliminary design includes selection of initial orbit elements, selection of launch window, specification of major parameters of the electric power subsystem, propulsion subsystem with amount of propellant needed, attitude stabilization and control subsystem, telecommunication subsystem. All of this relies on the results of satellite orbit dynamics analysis and calculation. Therefore it is important for the satellite designer to have some knowledge and tools of orbit dynamics. The purpose of this dissertation has been to be build a bridge between the theory of orbit dynamics and the practice of satellite design.

For this purpose this paper summarizes the major methods including relevant formulas and equations that are useful for the satellite preliminary design.

Although the "classical" parts are quoted from books and reports available, with some work of systemization, some parts are developed or improved by author, they are listed in the following.

1) transformation from the differential equations for variation of orbit elements into difference equations for finite changes in orbit elements;

2) estimation of fuel consumption rate required for orbit maintenance in case of elliptical orbit;



References

[1] Griffin M. D., French J. R., Space Vehicle Design, AIAA Education Series, AIAA Inc., Washington DC, 1991

[2] Xiao Yelun, Principles of Spacecraft Flight Dynamics, Press Astronautics, Beijing, 1995 (in Chinese)

[3] Werts J. R. (ed), Spacecraft Attitude Determination and Control, Publ. D. Reidel, Boston MA, 1985

[4] Suggs R., Space Station Freedom Natural Environment Design Models, AIAA 93-0492, 1993

[5] Chobotov V.A. (ed), Orbital Mechanics, AIAA Education Series, AIAA Inc. , Washinton DC , 1991

[6] Bate R.R. et al. , Fundamentals of Astrodynamics, Dover Publications, New York, 1971

[7] Cornelisse J. W. et al. , Rocket propulsion and Spaceflight Dynamics, Pitman Publ. Ltd. , London, 1979

[8] Werts J.R., Larson W.J., Spacecraft Mission Analysis and Design, Kluwer Academic Publishers, 1994

[9] M. H. Kaplan, Modern Spacecraft Dynamics and control. Wiley & Son, New York, 1976

[10] Rimrott F.P.J., Introductory Orbit Dynamics, Vieweg, 1989