한국전산구조공학회
1996년도 봄학술발표회 논문집

Ductility Demand Estimation Methods at Structural System Level for Seismic Design of Structures

Dong-Guen Lee[1), Chung-Bang Yun[2) and Jong-Keol Song[3)

ABSTRACT

The ductility demand for seismic design of a single degree of structure or an individual structural member can be determined easily. However, there is no established method to determine the ductility demand for a structural system. The object of this paper is to develop a method for the estimation of the ductility demand for structural systems, in which the inelastic behavior can be taken into account properly. The validity of the proposed method has been examined for several cases with different structures and different earthquake excitations. The method is also compared with two alternative methods.

1. INTRODUCTION

The current state of practice for earthquake resistant design of building structures may be well represented by the Uniform Building Code(UBC).[4] In the UBC method, the design earthquake load is computed based on the specified elastic response spectrum. Then, it is reduced by applying the response modification factor *R*, which is related to the ductility capacity of structure. Proper earthquake resistant design requires that the ductility capacity of the structure exceeds the ductility demand imposed by the design earthquake load. The ductility capacity may be determined experimentally. However, the ductility demand may be determined from the analysis of the inelastic behavior of the structure. Although the ductility

---

1) *Professor, Department of Architectural Engineering, Sung Kyun Kwan Univ.*
2) *Professor, Department of Civil Engineering, KAIST*
3) *Graduate Student, Department of Civil Engineering, KAIST*

demands for a single degree of freedom(SDOF) system and an individual structural member can be determined easily, it is not quite clean how to determine the ductility demand for a whole structural system.

The existing methods for the ductility estimation of multistory structures such as Q-model[3] are based on an equivalent SDOF equation derived from the multi-degree of freedom (MDOF) structure.[1],[2],[6] In those methods, at first, the linear and nonlinear characteristics of the SDOF system are evaluated by the inelastic static analysis of the MDOF system. Then, the inelastic behavior of the MDOF structure subjected to an earthquake excitation is approximately obtained by the inelastic dynamic analysis of the SDOF system. Since the inelastic behavior of the MDOF structure obtained by the above methods is hardly accurate, the estimation of the ductility demand can not be expected to be reasonable. Owing to the fast development of computer technology, the accurate inelastic behavior of the MDOF structure can be easily computed. Hence, the ductility demand may be estimated more properly, if the inelastic behavior obtained by the inelastic dynamic analysis of the MDOF structure is utilized. From this background, two methods for ductility demand estimation at structural system level are proposed in this study. The proposed methods are compared with the existing method such as Q-model.

2. DEFINITIONS OF DUCTILITY

In general, two definitions of ductility demand are accepted for the estimation of inelastic deformations. They are displacement ductility μ for a SDOF system and rotation ductility μ_θ for a bilinear element of a multistory frame. Displacement ductility is evaluated from the ratio of the maximum displacement $U_{\max}$ to the yield displacement U_y as ;

$$\mu = \frac{U_{\max}}{U_y} \tag{1}$$

Rotation ductility is evaluated from the ratio of the maximum rotation at the end of a member to the yield rotation. Because the inelastic flexural deformation of a beam is assumed to be concentrated at the ends of the beam, plastic rotations occur at the plastic hinges that form at both ends of a beam. Therefore, rotation ductility can be written as ;

$$\mu_\theta = \frac{\theta_{\max}}{\theta_y} \tag{2}$$

in which, $\theta_{\max}$ and θ_y represent the maximum and the yield rotations at the end of the member.

3. ESTIMATION METHODS FOR DUCTILITY DEMAND

In earthquake resistance design procedures, the ductility demand is customarily defined based on a SDOF equation as

$$m\ddot{u}(t)+c\dot{u}(t)+R(u(t))=-m\ddot{x}_g(t) \tag{3}$$

Hence, the ductility demand for seismic design of a multistory frame can be effectively estimated, if an equivalent SDOF equation is derived for the MDOF system. The equation of motion of a multistory frame subjected to earthquake excitation can be obtained as

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+\{R(x(t))\}=-[M]\{1\}\ddot{x}_g(t) \tag{4}$$

where $\{x(t)\}$ is the relative story displacement vector to the ground, $[M]$ is the diagonal mass matrix, $[C]$ is the damping matrix, and $\{R(x(t))\}$ is the restoring force vector. Assuming the structure behaves approximately as

$$\{x(t)\}=\{\Psi\}u(t) \tag{5}$$

where $\{\Psi\}$ is a vector that describes the deflected shape of the multistory frame, and $u(t)$ is the displacement of the equivalent SDOF system, an equivalent SDOF equation can be derived from Eq. (4) as

$$m\ddot{u}(t)+c\dot{u}(t)+R(u(t))=-L\ddot{x}_g(t) \tag{6}$$

where $m=\{\Psi\}^T[M]\{\Psi\}$, $c=\{\Psi\}^T[C]\{\Psi\}$, $R(u(t))=\{\Psi\}^T\{R(\{\Psi\}u(t))\}$, and $L=\{\Psi\}^T[M]\{1\}$.

By normalizing $\{\Psi\}$ as

$$\{\Psi\}^T[M]\{\Psi\}=\{\Psi\}^T[M]\{1\} \tag{7}$$

the earthquake load coefficient L in Eq. (6) becomes equal to m. Then, Eq. (6) becomes identical to Eq. (3), which is for a SDOF system subjected to an earthquake excitation.

By premultiplying $\{\Psi\}^T[M]$ on Eq. (5) and using Eq. (7), the equivalent SDOF $u(t)$ of Eq. (6) can be evaluated as

$$u(t)=\frac{\{\Psi\}^T[M]\{x(t)\}}{\{\Psi\}^T[M]\{1\}} \tag{8}$$

For a MDOF structure subjected to inelastic deformation, an equivalent SDOF $u(t)$ can be evaluated using the instantaneous inelastic structural response $\{x(t)\}$ as the shape vector $\{\Psi\}$ in Eq. (8) as

$$u(t)=\frac{\{x(t)\}^T[M]\{x(t)\}}{\{x(t)\}^T[M]\{1\}} \tag{9}$$

Then, the ductility demand can be estimated from the relationship between *u(t)* and the base shear $V_o(t)$ of the MDOF structure. In this study, the method of the ductility demand estimation using Eq. (9) is refered as **Method I.**

Alternatively, the ductility demand may be estimated by using an existing Q-model,[3] which is based on the inelastic response obtained using an equivalent SDOF equation. In this method, the linear characteristics of the SDOF system, i.e., mass, damping coefficient and stiffness, are obtained using the elastic deflection of the MDOF system under a monotonically increasing static load with a triangular profile and using Eq. (5) and Eq. (7). Then, the nonlinear characteristics, i.e., the strain hardening ratio and the yield force, are determined from the relationship between *u(t)* and the base shear $V_o(t)$ obtained by the inelastic static analysis of the MDOF structure. The ductility demand for the SDOF system subjected to earthquake excitation is evaluated by the inelastic dynamic analysis of the SDOF system. In this study, the method using an inelastic dynamic response of the equivalent SDOF system is refered as **Method II**.

Finally, the third method is proposed as follows. At first, a floor is determined, which can reasonably represent the inelastic behavior of the structural system. Then, the ductility demand may be estimated from the relationship between the floor displacement and the dynamic base shear. In this study, the representative floor is determined from the elastic deflections due to three kinds of lateral static load profiles frequently used in seismic design of structures as shown Fig. 1a. If the deflected shapes are normalized using Eq. (7), it is very interesting to notice that three deflection shapes become very alike as in Fig. 1. Based on the results, the representative floor is selected as the one corresponding to the story displacement being equal to 1. Then, the height of the floor is approximately obtained as 70 % of the structure height. The method based on the floor displacement at 70 % of the structure height is refered as **Method III**.

4. EXAMPLE ANALYSIS

Three example structures are constructed based on the UBC standards(1988). They are 3, 8, and 20-story frames, and each structure has 3 bays as illustrated in Fig. 2. The story height is 18 ft (5.49 m) for the bottom story and 12 ft (3.66 m) for all others. The seismic design loads are computed for UBC Seismic Zone 4 with a soil factor of 1.2 and an importance factor of 1.0. The member sizes of the strong-column weak-beam(SCWB) and weak-column strong-beam(WCSB) frames are provided in Table 1 with the member identities given in Fig. 2. The fundamental periods are computed as 0.48 and 0.47 sec for 3-story SCWB and WCSB frames. They are 1.17 and 1.16 sec for 8-story frames, and 2.77 and 2.97 sec for 20-story frames.

Three earthquake records are selected for this study: EW record measured in Mexico City on September 19, 1985; N10E record in Llolleo, Chile, on March 3, 1985; NS record in El Centro, California, on May 18, 1940. The maximum rotational ductility demands at story levels for 8-story frames subjected to Mexico City earthquake are shown in Fig. 3. Force-displacement relationships obtained for 8-story frames by three ductility demand estimation methods are shown in Figs. 4 and 5. The ductility demands at structural system level are shown in Table 2. From Fig. 3, it can be observed that the ductility demand at story level has a broad range. Hence, it is required to determine the ductility demand representing the structural system, which will be used to determine the response modification factor and the earthquake load for structural design. From the Table 2, it can be observed that results of Methods I and III are nearly the same. Also, the results by Method II are slightly different from the above two.

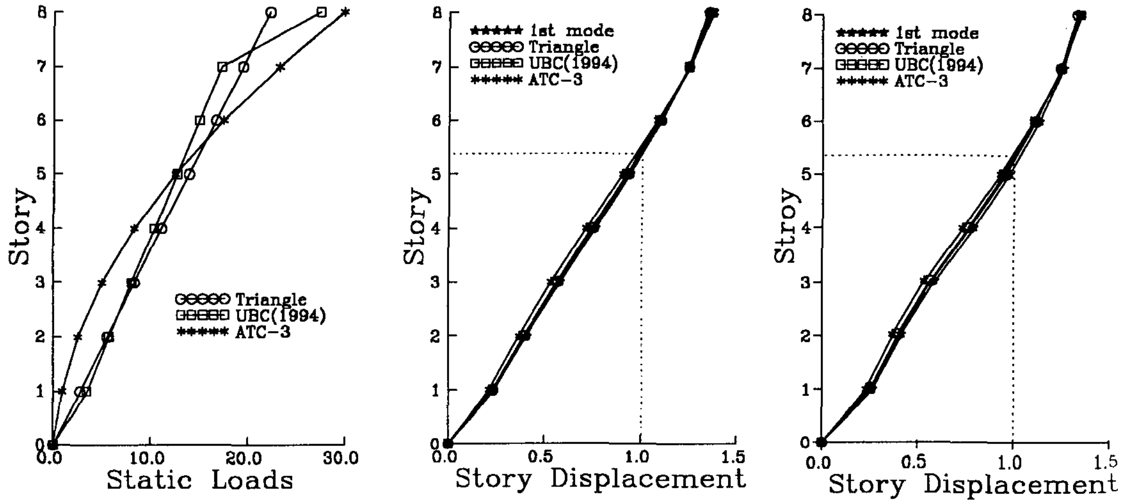
5. CONCLUDING REMARKS

A method for the estimation of the ductility demand at structural system level is proposed. By this method the inelastic behavior of the structure can be more properly considered than the conventional method using dynamic analysis on an equivalent SDOF system. Numerical example studies on 3 different multi-story frames indicate that the estimated ductility demands by two methods based on the inelastic response of the MDOF systems are very similar. Also, those based on the inelastic response of the equivalent SDOF systems are slightly different. For better estimation, Method I which is based on the inelastic response of the MDOF systems can be used.

6. REFERENCES

1. Baik, S. W., Lee, D. G. and Krawinkler, H., "A Simplified Model for Seismic Response Prediction of Steel Frame Structures," *Proceedings of 9WCEE*, Vol. V, 1988, pp. V375-V380
2. Lee, D. G., "Accurate and Simplified Models for Seismic Response Prediction of Steel Frame Structures," Ph. D. Dissertation, Department of Civil Engineering , Stanford University, Stanford, California, February 1984
3. Saiidi, M. and Sozen M. A., "Simple Nonlinear Seismic Analysis of R/C Structures," *Journal of the Structural Division*, ASCE, Vol. 107, No. ST5, 1981, pp. 937-952
4. *Uniform Building Code (UBC)* 1994. International Conference of Building Officials, 1994.
5. Tsai, K.C. and Li, J. W. "DRAIN2D+ - A General Purpose Computer Program for Static and Dynamic Analyses of Inelastic 2D Structures Supplemented with a graphic processor," *Report No. CEER/R83-03*, National Taiwan Univ., 1994.

6. Newmark, N.M. and Hall, W.J., *Earthquake Spectra and Design*, Earthquake Engineering Research Institute
7. Manual of Steel Construction, 8th Edition, *American Institute of Steel Construction*, New York, NY, 1980
8. Roeder, C. W., Schneider, S. P. and Carpenter, J. E., "Seismic Behavior of Moment-Resisting Steel Frames: Analytical Study," *Journal of the Structural Engineering*, ASCE, Vol. 119, No. 6, 1993, pp. 1866-1884
9. Lee, Seok-Yong and Lee, Dong-Guen, "Effects of Gravity Load on Seismic Response of Multistorey Buildings," *Engng Struct.*, 1994, Vol. 16, Number 6, pp. 445-454
10. Bertero, Vitelmo V. "Design Guidelines for Ductility and Drift Limits," *Report EERC 91-15*, Univ. of California, Berkeley, Calif., 1991.
11. Uang, Chia-Ming "Establishing R (or R_w) and C_d Factors for Building Seismic Provisions," *Journal of the Structural Engineering*, ASCE, Vol. 117, No. 1, January, 1991, pp. 19-28
12. Uang, Chia-Ming "Comparison of Seismic Force Reduction Factors Used in U.S.A. and Japan," *Earthquake Engineering and Structural Dynamics*, Vol. 20, 1991, pp 389-397
13. Uang, Chia-Ming and Bertero, Vitelmo V. "UBC Seismic Serviceability Regulations: Critical Review," *Journal of the Structural Engineering*, ASCE, Vol. 117, No. 7, July, 1991, pp. 2055-2068
14. Uang, Chia-Ming "An Evaluation of Two-Level Seismic Design Procedure," *Earthquake Spectra*, Vol.9, No.1, 1993, pp. 121-135
15. Mahin, S. A. and Lin, J., "Construction of inelastic Response Spectra for Single Degree of Freedom Systems," *Report No. UCB/EERC-83/17*, Earthquake Engineering Research Center, University of California, Berkeley, CA, 1983.



(a) Distribution of lateral static loads

(b) Normalized deformed shape for SCWB8 frame

(c) Normalized deformed shape for WCSB8 frame

Fig. 1. Distribution of lateral static loads and the normalized deformed shape

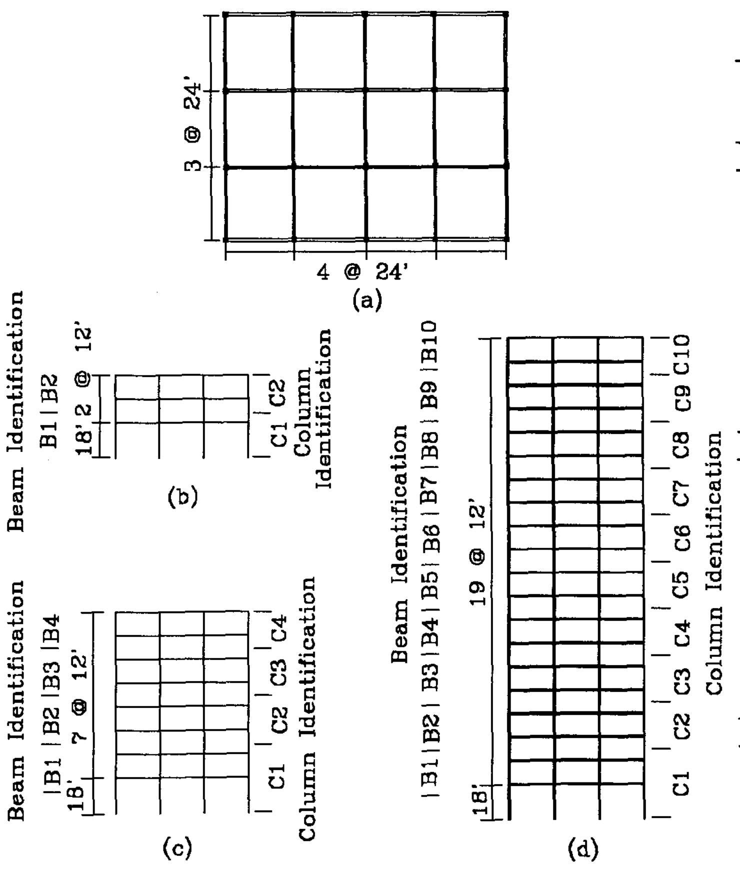
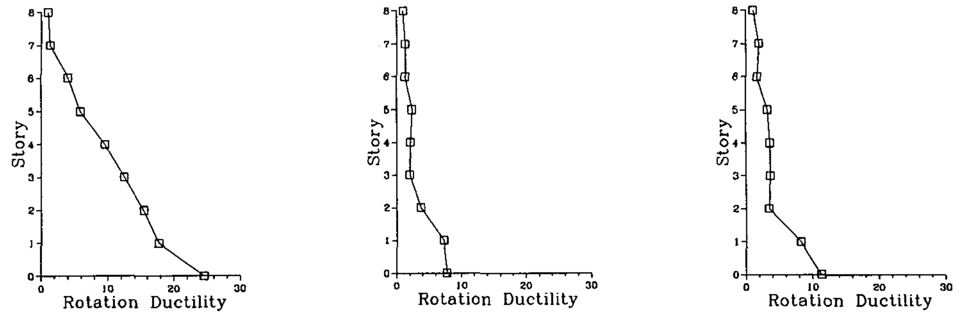


Fig. 2. Geometries of Example Structure : (a) Plan,
(b) Three-Story Elevation; (c) Eight-story Elevation;
(d) 20-Story Elevation

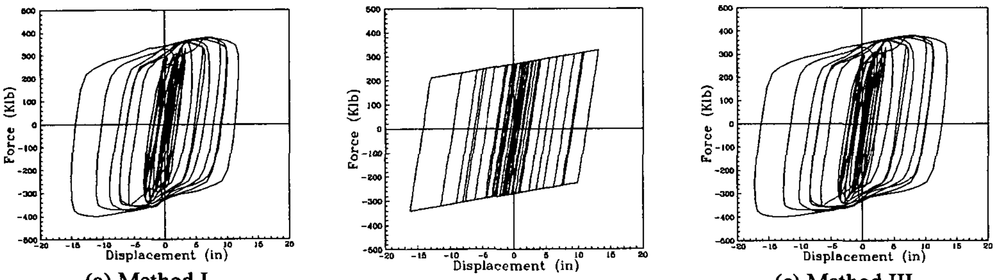
Table 1. Member Sizes

Member Identifier (1)	Three-story Frames		Eight-Story Frames		20-Story Frames	
	WCSB (2)	SCWB (3)	WCSB (4)	SCWB (5)	WCSB (6)	SCWB (7)
(a) Interior Columns						
C10	-	-	-	-	W14X74	W27X84
C9	-	-	-	-	W14X90	W27X102
C8	-	-	-	-	W14X109	W30X116
C7	-	-	-	-	W14X132	W30X132
C6	-	-	-	-	W14X145	W36X135
C5	-	-	-	-	W14X159	W36X150
C4	-	-	W18X40	W24X55	W14X176	W36X160
C3	-	-	W18X60	W24X84	W14X193	W36X170
C2	W14X43	W14X68	W18X76	W24X94	W14X211	W36X182
C1	W14X68	W14X99	W21X122	W24X131	W14X283	W36X232
(b) Exterior Columns						
C10	-	-	-	-	W14X53	W14X74
C9	-	-	-	-	W14X68	W14X90
C8	-	-	-	-	W14X82	W14X109
C7	-	-	-	-	W14X99	W14X120
C6	-	-	-	-	W14X109	W14X132
C5	-	-	-	-	W14X120	W14X145
C4	-	-	W14X43	W14X48	W14X132	W14X159
C3	-	-	W14X48	W14X74	W14X145	W14X176
C2	W14X30	W14X48	W14X61	W14X90	W14X176	W14X193
C1	W14X53	W14X68	W14X82	W14X99	W14X193	W14X211
(c) Girders						
B10	-	-	-	-	W24X76	W24X62
B9	-	-	-	-	W27X94	W24X68
B8	-	-	-	-	W27X94	W24X76
B7	-	-	-	-	W30X108	W24X76
B6	-	-	-	-	W30X124	W27X84
B5	-	-	-	-	W30X124	W27X84
B4	-	-	W24X68	W18X40	W30X132	W27X94
B3	-	-	W24X76	W21X50	W30X132	W27X94
B2	W18X50	W18X40	W24X68	W24X55	W30X173	W27X102
B1	W27X84	W18X55	W24X103	W24X68	W30X173	W27X102



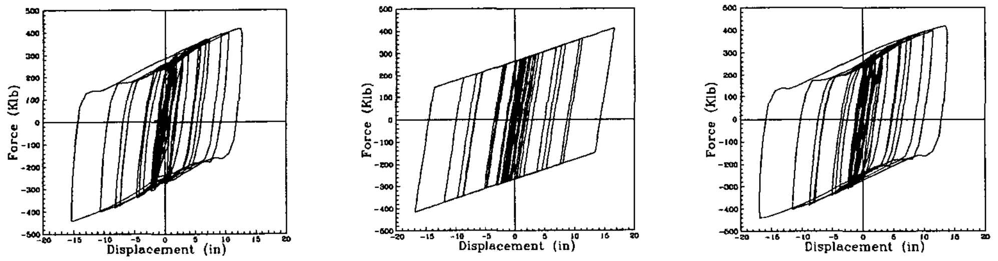
(a) Beams of SCWB8 (b) Exterior columns of WCSB8 (c) Interior columns of WCSB8

Fig. 3. Maximum Rotational Ductility Demands at Story Levels for 8-Story Frames Subjected to Mexico City Earthquake.



(a) Method I (b) Method II (c) Method III

Fig. 4. Force-Displacement Relationships for SCWB8 Frame Subjected to Mexico City Earthquake.



(a) Method I (b) Method II (c) Method III

Fig. 5. Force-Displacement Relationships for WCSB8 Frame Subjected to Mexico City Earthquake.

Table 2. Ductility Demands at Structural System Level

Frame ID	**El Centro**			**Chile**			**Mexico**		
	Method I	Method II	Method III	Method I	Method II	Method III	Method I	Method II	Method III
SCWB3	5.8	4.9	6.8	6.3	6.5	6.8	9.7	8.9	10.4
WCSB3	9.5	9.6	9.8	7.2	9.1	8.2	32.8	36.2	35.8
SCWB8	3.7	4.9	3.7	1.9	2.5	1.9	9.3	10.1	10.5
WCSB8	3.8	4.5	4.2	1.9	2.5	1.9	9.8	10.6	10.7
SCWB20	2.3	2.2	2.5	1.4	1.2	1.5	4.5	7.0	4.8
WCSB20	2.2	2.2	2.2	1.6	1.6	1.6	5.0	5.7	5.5