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첨가질량이 작용하는 적층복합판의 고유진동수에 대한 자중 무시효과
The Effect of Neglecting Own Weight on The Natural Frequency of Vibration of Laminated Composite Plates with Attached Mass/Masses

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ABSTRACT

In this paper, the effect of neglecting the own weight of the composite plate on the natural frequency of vibration of the laminated plates is presented. The method used has been developed by the author since 1974. This method is very effective for the plates with arbitrary boundary conditions and irregular sections. When the attached mass is equal to the weight of the plate, the effect of neglecting the plate weight is 9.26 percent.

1. INTRODUCTION

Several structural elements such as the floor slabs of a factory or a builing and others may be subject to point mass/masses in addition to its own masses. Design engineers need to calculate the natural frequencies of such elements but obtaining exact solution to such problems is very much difficult. Pretlove(1987) reported a method of analysis of beams with attached masses using the concept of effective mass. This method, however, is useful only for certain simple types of beams. In case of a laminated composite plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution. A method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures was developed and reported by the author in 1974. In this report, the effect of neglecting the weight of beams on the natural frequency is given for several beam support types.

Recently, this method was extended to the first mode vibration analysis of two dimensional problems including composite laminates, and was reported at the first Japan International Society for the Advancement of Materials and Process Engineering Symposium and Exhibition(JISSE Ⅰ) in 1989. Further extension of this method to the second mode vibration of such two dimensional problems was reported at the Eighth Structures Congress

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of American Society of Civil Engineers in 1990. This method, applied to thick laminated plates, was reported at EASEC III, April, 1991., ICCM 8, July, 1991. and JISSE II, 1991.

This paper presents the method of application of this method to laminated plates with point mass/masses, in addition to its own weight. This procedure can easily be applied to any type of laminates with arbitrary boundary counditions and non-uniform sections. In order to illustrate this method, some details already reported by the author(1,2,3,4) are repeated in this paper.

2. METHOD OF ANALYSIS

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections(maximum) of both the converged and the previous one should remain unchanged under the inertia force related with this natural frequency. Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function. Considering only the first mode as a start, the deflection shape of a structural member can be expressed as

$$w = W(x, y)F(t) = W(x, y)\sin \omega t \tag{1}$$

where

W : the maximum amplitude

ω : the circular frequency of vibration

t : time.

By Newton's Law, the dynamic force of the vibrating mass,m, is

$$F = m\frac{\partial^2 w}{\partial t^2}. \tag{2}$$

Substituting Equation (1) into this,

$$F = -m(\omega)^2 W\sin\omega t. \tag{3}$$

In this expression, ω and W are unknowns. In order to obtain the natural circular frequency, ω, the following process is taken. The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w(i,j)(1) = W(i,j)(1) \tag{4}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accelerating convergence. The dynamic force corresponding to this(maximum) amplitude is

$$F(i,j)(1) = m(i,j)[\omega(i,j)(1)]^2 w(i,j)(1) \tag{5}$$

The "new" deflection caused by this force is a function of F and can be expressed as

$$\begin{aligned} w(i,j)(2) &= f\{ m(k,l)[\omega(i,j)(1)]^2 w(k,l)(1) \} \\ &= \sum^{k,l} \Delta(i,j,k,l) \{ m(k,l)[\omega(i,j)(1)]^2 w(k,l)(1) \} \end{aligned} \tag{6}$$

where Δ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, w(i,j)(1) and w(i,j)(2), have to remain unchanged and the following condition has to be held :

$$w(i,j)(1)/w(i,j)(2) = 1. \tag{7}$$

From this equation, ω(i,j)(1) at each point of (i,j) can be obtained, but they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e., ω(i,j) should be equal for all (i,j), this step is repeated until sufficient equal magnitude of ω(i,j) is obtained at all (i,j) points.

However, in most cases, the defference between the maximum and the minimum values of ω(i,j) obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of ω(i,j) where the deflection is the maximum. For the second cycle, w(i,j)(2) in

$$w(i,j)(3) = f\{ m(i,j)[\omega(i,j)(2)]^2 w(i,j)(2) \} \tag{8}$$

the absolute numerics of w(i,j)(2) can be used for convenience.

In case of a structural member with irregular section including composite one, and non-uniformly distributed mass, regardless of the boundary conditions, it is convenient to consider the member as divided by finite number of elements[1]. The accuracy of the result is proportional to the accuracy of the deflection calculation.

For practical design purposes, it is desirable to simplify the vibration analysis procedure. One of the methods is to neglect the weight of the structural element. The effect of neglecting the weight (thus mass) of the plate is studied as follows. If a weightless plate is acted upon by a concentrated load Fig(1-a), $P=N \cdot q \cdot a \cdot b$, the critical circular frequency of this plate is

$$\omega_n=\sqrt{\frac{g}{\delta_{st}}} \tag{9}$$

where δ_{st} is the static deflection.
Similar result can be obtained by the use of Eqns(6) and (7).

$$[\omega(i,j)]^2=\frac{1}{\left\{\Delta(i,j,i,j)\cdot\frac{P(i,j)}{g}\right\}} \tag{10}$$

where $P(i,j)=N \cdot q \cdot a \cdot b$ (11)

In case of the plate with more than one concentrated loads,

$$[\omega(i,j)]^2=\frac{1}{\left\{\sum^{k,l}\Delta(i,j,k,l)\cdot\frac{P(k,l)}{g}\right\}} \tag{12}$$

If we consider the mass of the plate as well as the concentrated loads, Fig (1-b),

$$w(i,j)(1)=w(i,j)(2)=\left\{\sum^{k,l}\Delta(i,j,k,l)\cdot m(k,l)\cdot w(k,l)(1) + \sum^{m,n}\Delta(i,j,m,n)\cdot\frac{P(m,n)}{g}\cdot w(m,n)(1)\right\}\cdot[\omega(i,j)(1)]^2 \tag{13}$$

where (m,n) is the location of the concentrated loads. The effect of neglecting the weight of the plate can be found by simply comparing Eqns(12) and (13).

3. EXAMPLE

As a numerical example, the special orthotropic laminate given in Reference [7] is considered, Fig. 2. This example illustrates the method of analysis.

The material properties are :

Matrix Modulus,	$E_m=3.4$ *GPa*	Fiber Modulus,	$E_f=110$ *GPa*
Matrix Poisson's Ratio,	$\nu_m=0.35$	Fiber Poisson's Ratio,	$\nu_f=0.22$
Matrix Volume Ratio,	$V_m=0.4$	Fiber Volume Ratio,	$V_f=0.6$.

By the use of rule of mixture, in simpler forms,

$$E_1 = E_f V_f + E_m V_m = 67.36\ GPa\ , \quad (14) \qquad E_2 = \frac{E_f E_m}{E_f V_m + E_m V_f} = 8.12\ GPa\ , \quad (15)$$

$$G_{12} = \frac{G_m G_f}{V_m G_f + V_f G_m}\ , \quad (16) \qquad G_m = \frac{3.4\ GPa}{2(1+0.35)} = 1.2593\ GPa\ ,$$

$$G_f = \frac{110\ GPa}{2(1+0.22)} = 45.0820\ GPa\ , \qquad G_{12} = 3.0217\ GPa\ ,$$

$$\nu_{12} = V_m \nu_m + V_f \nu_f = 0.2720\ , \quad (17) \qquad \nu_{21} = \nu_{12} \cdot \frac{E_2}{E_1} = 0.0328\ .$$

The stiffnesses are

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q}_{ij})_k \cdot (h_k - h_{k-1}), \quad \text{in N/m}\ , \tag{18}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (\overline{Q}_{ij})_k \cdot (h^2_k - h^2_{k-1}), \quad \text{in N}\ , \tag{19}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (\overline{Q}_{ij})_k \cdot (h^3_k - h^3_{k-1})\ , \quad \text{in Nm.} \tag{20}$$

and obtained as

$$\underset{(MN/m)}{A(i,j)} = \begin{vmatrix} 720.67 & 33.43 & 0 \\ 33.43 & 421.79 & 0 \\ 0 & 0 & 45.33 \end{vmatrix}\ , \qquad \underset{(N-m)}{D(i,j)} = \begin{vmatrix} 18492 & 627 & 0 \\ 627 & 2927 & 0 \\ 0 & 0 & 849 \end{vmatrix}\ .$$

B(i,j) = 0 from symmetry,
The influence surfaces are calculated by

$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi x}{b}\ , \tag{21}$$

where

$$w_{mn} = \frac{P_{mn}/\pi^4}{D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4} \tag{22}$$

in which

$$P_{mn} = \frac{4(1)}{ab} \cdot \sin\frac{m\pi\xi}{a} \cdot \sin\frac{n\pi\eta}{b}\ , \tag{23}$$

From Eqn (4),

$$w(i,j)(1) = W(i,j)(1)$$

where W is the maximum amplitude, (i,j) or (x,y) is the point under consideration, and (1) after (i,j) indicates the first assumed mode shape. The first mode shape is assumed as

$$w(i,j)(1) = \begin{vmatrix} 10 & 20 & 30 & 20 & 10 \\ 20 & 30 & 40 & 30 & 20 \\ 30 & 40 & 50 & 40 & 30 \\ 20 & 30 & 40 & 30 & 20 \\ 10 & 20 & 30 & 20 & 10 \end{vmatrix}$$

By Eqn (5) which is

$$F(i,j)(1) = +m(i,j)[\omega(i,j)(1)]^2 w(i,j)(1)$$

where

m(i,j) = the mass at (i,j) point = $\rho h(i,j)\Delta x \Delta y$,

where Δx and Δy are the mesh sizes in the x- and y- directions, respectively, and ρ is the mass density at (i,j), h is the thickness of the plate at (i,j),

ω(i,j)(1) = the "first" natural cicular frequency at (i,j) point.

F(i,j)(1) is obtained in terms of ω(i,j)(1).

Substituting F(i,j)(1) into Eqn (6)

$$\begin{aligned} w(i,j)(2) &= \sum^{k,l} \Delta(i,j,k,l) \cdot F(i,j)(1) \\ &= \sum^{k,l} \Delta(i,j,k,l) \cdot \{+m(k,l) \cdot [\omega(i,j)(1)]^2 \cdot w(k,l)(1)\} \end{aligned}$$

where Δ (i,j,k,l) is the influence surface, i.e., the deflection at (i,j) point caused by a unit load at all of (k,l) points, w(i,j)(2) can be obtained.

The numerical calculation is carried out for several cases of the amount of the attached mass, at the center of the plate. For simplicity, only one mass is considered for this example.

4. CONCLUSION

The method of obtaining the effect of neglecting the own weight of the laminated plates on the natural frequency of vibration is presented. This method is very simple but extremely accurate to solve such problems for any type of beams and plates. In case of square special orthotropic laminate, the effect is only 9.26 percent when the attached mass is equal to the weight of the plate. When it is twice of the plate weight, the result is 4.7 percent off. It can be concluded that an engineer can neglect the own weight of the plate if the concentrated mass is equal to the weight of the plate.

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Table 1. Effect of Neglecting Weight of Plates

$\omega_n / \sqrt{q/g}$

N \ CASE	A	B	DIFFERENCE (%)
1	741.2701	678.4405	9.2608
2	524.1571	500.6899	4.6869
3	427.9725	414.9556	3.1369
5	331.5061	325.3633	1.8879
7	280.1737	276.4409	1.3503
10	234.4120	232.2131	0.9469

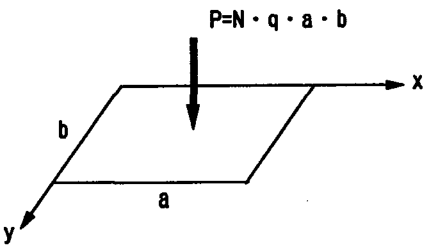


Fig. 1-a. CASE-A

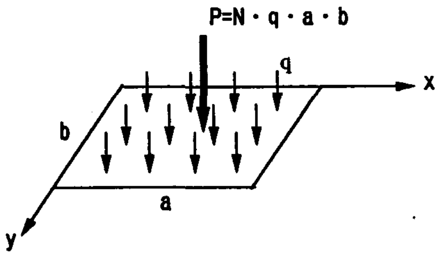
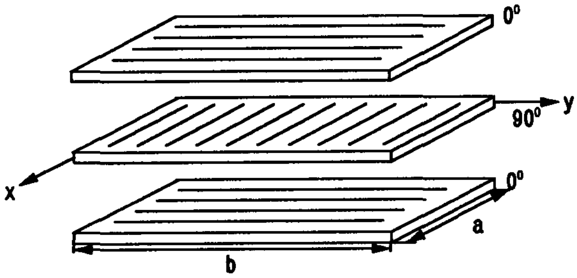


Fig. 1-b. CASE-B

Figure 1. Plates With and Without Own Weight



$t_1 = t_2 = t_3 = 0.005$ m

$a = b = 1$m

Fiber Orientation : 0°/90°/ 0°

Figure 2. Specially Orthotropic Laminate