인수분해된 광각 빔 전파기법 및 방향전환 거울 해석에의 응용

A Factored Wide Angle Propagation Technique Applied to Turning Mirror Simulation

정 영 철
광운대학교 전자통신공학과

Youngchul Chung
Department of Electronic Communications Engineering
Kwangwoon University

Abstract

A wide angle propagation techinque is formulated through an expansion of the Holmholtz operator followed by a Pade expansion and factorization of the resulting polynomials. Its accuracy is checked through the successful modeling of integrated waveguide turning mirrors, indicating that 6-th order polynomial can handle as large as 55° tilt angle very accurately.

Various forms of wide angle beam propagation algorithms have been developed to overcome the limitation of the paraxial propagation technique and to meet the need for designing ever evoving photonic integrated circuits(PICs) which might include integrated waveguide turning mirrors or wide angle branching waveguides. The previous approaches include a higher order expansion of a Helmholtz propagation operator[1] with the operator splitting or using a recursive Pade formula[2], or the explicit finite difference algorithm based on the Taylor series expansion of the amplitude Helmholtz wave equation[3].

The purpose of this letter is to introduce a fomulation of a wide angle BPM resulting from factoring of the denominator and numerator polynomials which are derived from the Taylor series expansion of the Helmholtz operator in combination with the Crank-Nicholson Scheme.

Then we applied the wide angle BPM to the simulation of integrated waveguide turning mirrors and check the accuracy of various orders of series expansion of the Helmholtz operator.

The propagation of scalar optical wave in the guiding structure can be described by the amplitude Helmholtz wave equation

$$\left(\frac{\partial^2}{\partial z^2} - j2k_0 n_r \frac{\partial}{\partial z} + \nabla_t^2 + k_0^2\left(n^2(x,y,z) - n_r^2\right)\right) \cdot E(x,y,z) = 0 \quad \text{.........(1)}$$

where n_r is the reference refractive index, and $k_0 = 2\pi/\lambda$, and E is the slowly varying amplitude of the electric field. This amplitude Helmholtz equation can be written as a factored form[4]

$$\left(\frac{\partial}{\partial z} - jk_0 n_r + jk_0 n_r (1+L)^{1/2}\right) \cdot$$

$$\left(\frac{\partial}{\partial z} - jk_0 n_r - jk_0 n_r (1+L)^{1/2}\right) E(x,y,z) = 0 \quad \text{....(2)}$$

where

$$L = \frac{\nabla_t^2 + k_0^2 (n^2(x,y,z) - n_r^2)}{(k_0 n_r)^2} \quad \text{....(3)}$$

The propagation of the forward going(positive z direction) wave is described by the following wave equation:

$$\frac{\partial E}{\partial z} = j k_0 n_r (1 - (1+L)^{1/2}) E =$$

$$jk_0 n_r \left(-\frac{1}{2} L + \frac{1}{8} L^2 - \frac{1}{16} L^3 + \frac{5}{128} L^4 \cdots\right) E$$

....(4)

The formal operator solution of Eq. (4) is of form

$$E(z+\Delta z) = \exp(j k_0 n_r \Delta z (1 - (1+L)^{1/2}) E(z) \quad \text{...(5)}$$

which can be further approximated through the Pade approximation as follows:

$$E(z+\Delta z) = \frac{1 + j\frac{1}{2} k_0 n_r \Delta z (1 - (1+L)^{1/2})}{1 - j\frac{1}{2} k_0 n_r \Delta z (1 - (1+L)^{1/2})} E(z)$$

.....(6)

The accuracy of the operator solution given by eq. (6) under various orders of Taylor series expansion can be checked by looking at the angled plane wave propagation in the homogeneous medium. The refractive index of the medium and the reference index are taken to be the same. Then the operator L corresponds to $\sin^2(\theta)$ where θ is the angle of plane wave propagation relative to z axis. The phase variation of the plane wave along the direction of z for the propagation distance of Δz is given as

$$\phi(\theta) = 2\arctan(k_0 n_r \Delta z)(1 - (1 + \sin^2\theta)^{1/2})$$

$$= 2\arctan(k_0 n_r \Delta z) \cdot$$

$$\left(-\frac{1}{2}\sin^2\theta + \frac{1}{8}\sin^4\theta - \frac{1}{16}\sin^6\theta + \frac{5}{128}\sin^8\theta \cdots\right) \quad \text{..(7)}$$

The ralative phase error $\frac{\Delta\phi}{k_0 n_r \Delta z}$ for various orders of Taylor series expansion is plotted in Fig. 1 as a function of the propagation angle. In the calculation $k_0 n_r \Delta z$ is taken to be 0.4. A similar behavior is observed when the value of $k_0 n_r \Delta z$ is 0.8 and 1.6. It is seen that the propagation angle larger than 50° can be treated very accurately when the Taylor series expansion up to order of five or more is used.

If we take the Taylor series expansion of the Helmholtz operator upto order N and performing factorization, the Eq. (6) can be written as

$$E(z+\Delta z) \approx \frac{\sum_{n=1}^{N} a_n^* L^n}{\sum_{n=1}^{N} a_n L^n} E(z) = \frac{\prod_{n=1}^{N} (L - x_n^*)}{\prod_{n=1}^{N} (L - x_n)} E(z) \quad \text{...(8)}$$

Then propagating a beam through an interval Δz can be performed by applying the N operators $(L - x_n^*)/(L - x_n)$ successively. Note that in the case of a two-dimensional structure each operation results in a tridiagonal linear matrix equation which can be solved very easily. This approach requires N times more computing time than the paraxial propagation. A similar approach has been adopted by Hadley, in which the starting polynomial was found by Pade approximants from the recursion formula[2]. It is expected that the accuracy of the wide angle propagation can be increased by including the higher order terms in the Taylor series expansion of Helmholtz operator.

The accuracy and efficiency of the wide angle propagation algorithm can be checked by simulating the optical wave propagation through the integrated waveguide turning mirrors as shown in Fig. 2. In this kind of turning mirrors the optical wave around the turning mirror is a mixture of the z-directed wave and the wave propagating in the direction angled by θ relative to z-axis, which can not be properly modeled using the paraxial wave propagation. In the calculation the computational window size is 50 μm, the number of mesh points 4096, the reference refractive index n_r 3.377, the wavelength 1.3 μm, and the propagation step size Δz 0.05 μm. The length of the waveguide between two mirrors is maintained to be 40 μm for all the angles θ. The overlap intergral of the transmitted wave with the eigenmode of the output waveguide is calculated for various tilt angles θ which is π minus the turning angle. The results are plotted in Fig. 3 for different orders of polynomial in the expansion of Helmholtz operator. The paraxial approximation can handle the tilt angles upto about 15^0, and the expansion upto second order can give accurate results for the tilt angles upto 30^0. Obviously increasing the order of polynomial can handle the wider angle propagation, indicating upto 55^0 tilt angle can be handled very accurately by using the sixth-order polynomial, which is in good agreement with the results shown in Fig. 1.

In summary, the formulation of the wide angle propagation technique is presented and the accuracy is demonstrated through the successful simulation of the integrated waveguide turning mirrors of various turning angles.

References

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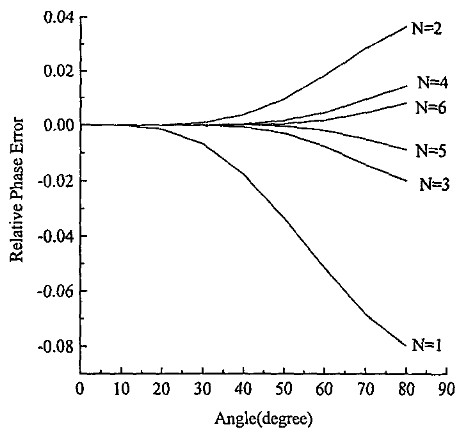

Fig. 1 The ralative phase error of the plane wave as a function of propagation angle for various orders of polynomials.

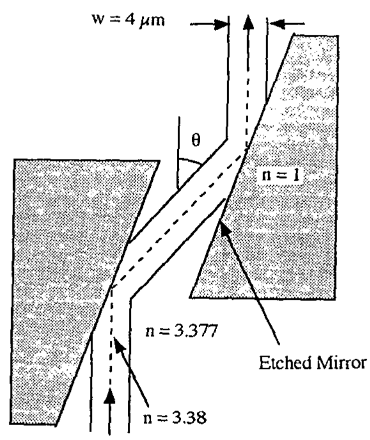

Fig. 2 An integrated waveguide turning mirror structure.The width of the waveguide is 4 μm, the core refractive index 3.38, and the clad index 3.377. The length of the tilt waveguide is 40 μm.

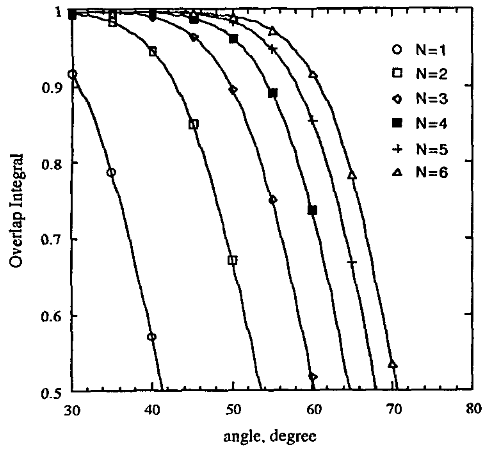

Fig. 3 The amplitude transmission as a function of the tilt angles for various orders of polynomial for the Helmholtz operator expansion.The amplitude transmission is measured through the overlap integral between the eigenmode and the optical field at the output of the turning mirror structure shown in Fig. 2.