스트럿-타이 모델의 스트럿과 노드의 강도 결정방법에 관한 연구

A Method for Determination of Strengths of Struts and Nodes in Strut-Tie Models

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국문요약

본 논문에서는 스트럿-타이 모델을 이용한 콘크리트 구조물의 설계 및 해석과정에서 필요로 하는 스트럿 유효강도의 결정 및 절점영역 지지력의 검토를 일반적이고 일관성 있게 수행할 수 있는 방법을 제안하였다. 콘크리트 스트럿의 유효강도는 스트럿-타이 모델의 스트럿 영역에 해당되는 유한요소들의 주응력비를 고려하여 결정하였으며, 스트럿의 기하학적 형상을 이용하여 형성된 절점영역의 지지력은 조합응력을 받는 콘크리트의 파괴기준을 고려하는 비선형 유한요소 해석을 이용하여 검토하였다. 제안한 방법을 예증하기 위해 실험된 철근콘크리트 보의 해석을 스트럿-타이 모델 방법을 이용하여 실시하였다.

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1. INTRODUCTION

In general, strength design procedures for structural concrete members consist of three major steps. First, the designer chooses the initial concrete dimensions based on practical considerations and/or architectural constraints. In the second step, the concrete dimensions and the amount and detailing of the reinforcement are determined using equilibrium and ultimate strength considerations. An estimate of the deformations under service conditions can be considered as the last step. The strut-tie models are conceptual tools employed in the second stage to investigate the equilibrium between the loads, the reactions and the internal forces in concrete and reinforcement. Strut-tie models are discrete representations of the actual stress fields

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resulting from applied load and support conditions and provide static lower bound solutions. These models represent the load carrying mechanism of a structural member by approximating the flow of internal forces by means of struts representing the flow of compressive stresses and ties representing the flow of tensile stresses.

An accepted design philosophy in structural concrete is to produce members in which the critical section will exhibit ductile behavior under extreme overload. This is done by ensuring that the reinforcement yields before the concrete fails and that flexure controls the mode of failure. To ensure ductile behavior, it is necessary to place a limit on the stress levels in the concrete. Since the compressive strength of concrete is affected by several factors such as disturbance caused by cracks and reinforcement, confinement and multiaxial state of stress, it is not possible to have all the concrete components of the load carrying mechanism reaching simultaneously a unique peak value of compressive strength. Various values of the effective stress levels of concrete struts are suggested based on either experimental observations or analytical studies. In this paper, a general and consistent approach for determining the effective stress levels of concrete struts is proposed by implementing the principal stress ratios obtained from the finite element analysis of the uncracked plain concrete structure. This represents an improvement of previous works since it provides a more general evaluation process of effective stress levels of concrete struts.

The struts and ties represent one-dimensional stress fields, whose intensity should not exceed the compressive strength of the concrete and yield strength of the steel, respectively.

The intersections of three or more struts or ties form nodal zones. To allow safe transfer of strut and tie forces through nodal zones, concrete stress levels in nodal zones must be controlled as well. Herein, a technique for verifying the bearing capacity of nodal zones is also proposed. To illustrate the proposed approaches, an analysis example of a reinforced concrete beam tested to failure is conducted using the strut-tie model approach.

2. MATERIAL STRENGTHS IN STRUT-TIE MODEL

The effective stress of concrete strut, f_c, has been the subject of much debate. In general, it is chosen as some fraction of the uniaxial concrete cylinder compressive strength f_c', i.e., $f_c = \nu f_c'$, where ν is an effectiveness factor which is less than unity. Since the strut-tie model is associated with the ultimate limit state, substantial cracking may be expected to reduce the concrete compressive strength. Hence, the effectiveness factor is introduced to reflect the reduction of the usable concrete strength due to cracking of the struts and tensile strains transverse to the struts.

Based on test results, Nielsen et al.(10), Ramirez and Breen(11), Marti(9), Schlaich et al.(12), MacGregor(8) and Bergmeister et al.(3) proposed values and equations for effective stresses of concrete struts. Detail literature review on this is conducted by Yun and Ramirez(15).

More recently, Alshegeir(2) evaluated the effective stress levels of concrete struts based on the analysis of experimental results from four continuous reinforced deep beams subjected to two concentrated loads, three prestressed concrete deep beams subjected to high shear stresses and four simply supported reinforced concrete beams with different stirrup detailing.

In this study, the effective stress levels of concrete struts are more generally determined form Fig. 1 which is made based on the experimental test results of two-dimensional plain concrete under biaxial stresses by Kupfer et al.(6). In the figure, the maximum and minimum effective stress level of concrete strut is limited by $1.0f'$ and $0.1f'$, respectively. To obtain the effective stress level for a given concrete strut, the principal stress ratio determined by taking the average values of the principal stress ratios of the finite elements representing the concrete strut must be evaluated. The effective stress level determined from Fig. 1 is only valid when the strut is placed within 10 degrees of compressive principal stress flow. If the strut deviates more than 10 degrees from the compressive principal stress flow, the value obtained from Fig.1 is multiplied by $\cos\alpha$, where α is the deviation angle between the compressive principal stress flow and strut. The effective stress level may be increased up to 5 to 20 percent depending on the confinement provided by reinforcement or any anchorage or bearing plate(14). Though the tensile resistance of concrete is not utilized, either the concrete modulus of rupture or the tension failure stress of two-dimensional plain concrete under biaxial compression-tension loading, whichever smaller, is taken as the effective tensile stress level of concrete ties. This is sometimes the only alternative for load carrying in some locations where steel reinforcement can not be provided due to practical considerations. The yield strength of steel reinforcement is taken as effective stress of tension steel tie.

Table 1 Principal Stress Ratios, Effective Strength and Corresponding Peak Strains of Concrete Struts in the Strut-Tie Model for the Test Specimen

Member No.	P. Stress ratio (σ_2/σ_1)	Eff. Strength (f_c/f_c')	Peak Strain ($\times 10^{-3}$)
8	-1.5	0.30	-0.446
10	-2.2	0.30	-0.446
12	-4.1	0.40	-0.567
14	-4.0	0.40	-0.567
22	-5.0	0.50	-0.749
23	-10.0	0.70	-0.977
24 & 25	-35.0	0.85	-1.289
29	-3.0	0.35	-0.505

- σ_1 & σ_2 : Max. and Min. Principal Stresses
- Symmetric Member Only
- See Figure 4 for Member Location

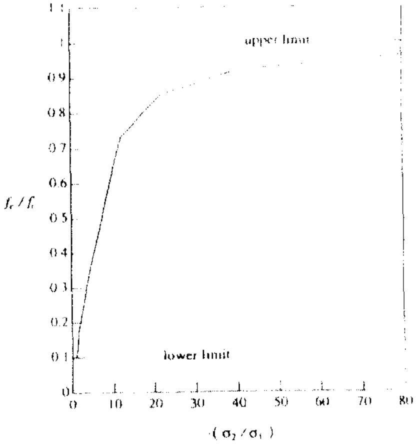

Fig.1 Relationship between Concrete Stress Level and Principal Stress Ratio

The strength of concrete in the nodal zones depends on a number of factors such as (a)the confinement of the zones by reactions, compression struts, anchorage plates for prestressing, reinforcement from the adjoining members and hoop reinforcement; (b)the effects of strain discontinuities within the nodal zone ; (c)the splitting stresses and hook bearing stresses resulting from the anchorage of the reinforcing bars of a tension tie in or immediately behind the nodal zone. Equations or values of effective strengths of nodal zones were proposed by the Canadian Code(4), Marti(9), Schlaich et al.(12), M. Schlaich and Anagnostou(13), Jirsa et al.(5), Bergmeister et al.(3) and Adebar and Zhou(1). Detail literature review on this is conducted by Yun and Ramirez(15).

In this study, the bearing capacity of nodal zone is verified using a finite element nonlinear analysis with the failure criteria which incorporate the different state of stresses. For plane structures, the nodal areas are considered as two-dimensional stress fields whose boundaries are determined by the intersection of the stress fields framing into the node. Tension ties are considered as equivalent struts acting on the node in opposite direction to the ties. Tension ties can also be incorporated as forces/unit of length when developed through the node. The failure criteria employed in this study is determined from the experimental test results of two-dimensional plain concrete by Kupfer and Gerstle(7). Test results show that the compressive limit stress of two-dimensional plain concrete under biaxial compression and tensile or compressive limit stress of two-dimensional plain concrete under biaxial tension-compression are, respectively, larger and smaller than the compressive limit stress of uniaxial compression. Hence, the interpolated values of the maximum compressive and tensile stresses under different loading (stress) ratios are employed as the failure criteria in the nodal zone analysis. In the finite element nonlinear analysis, the tangent modulus of elasticity of concrete in the critical principal direction is set to be very small when the maximum or minimum principal stress of a finite element with certain principal stress ratio reaches the compressive or tensile stress. The finite element nonlinear analysis will continue until nodal zone model becomes unstable due to the development of a failure mechanism. A failure mechanism is assumed to occur when cracks propagate from one side to the opposite side of the nodal zone and/or when all boundary elements along any nodal zone face crush.

3. EXAMPLE APPLICATION OF PROPOSED APPROACHES

To illustrate the proposed approaches for the evaluation of the effective of the stress levels of concrete struts and for the verification of bearing capacity of nodal zone, an example of a rectangular reinforced concrete beam tested to failure is presented. Detail procedures and explanation on the application of the proposed approaches are given in Ref..(15). The information on the test specimen and analysis results are briefly shown in Table 1 and Figures 2-7.

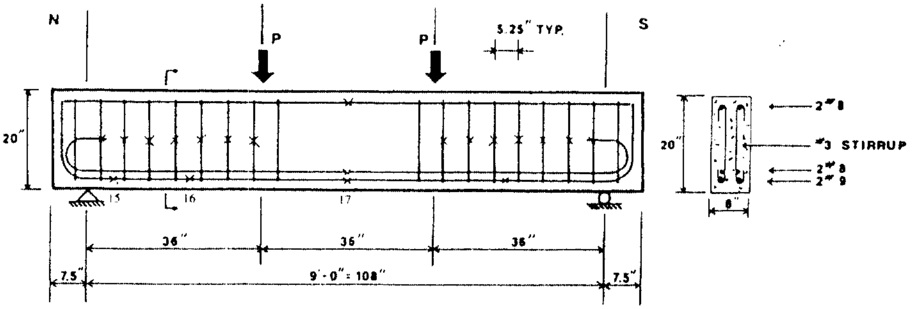

Fig.2 Test Specimen of the Reinforced Concrete Beam

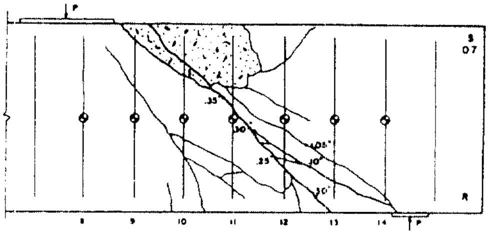

Fig.3 Detailed Crack Plot

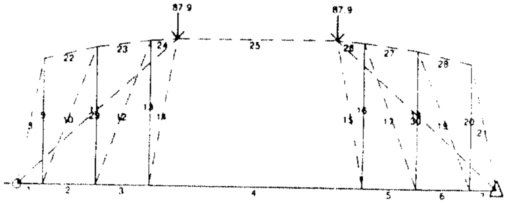

Fig.4 Strut-Tie Model

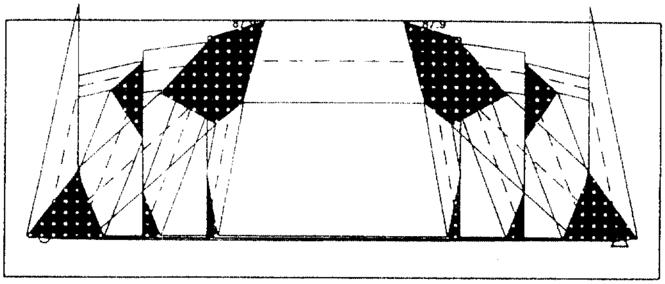

Fig.5 Dimensioned Strut-Tie Model

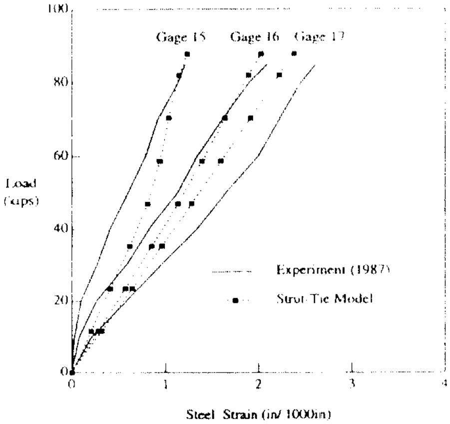

Fig.6 Longitudinal Strain Behavior

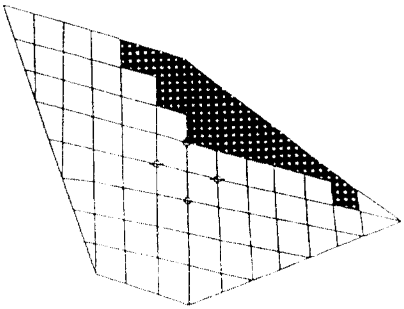

Fig.7 Crushed Shape of Critical Nodal Zone

4. SUMMARY AND CONCLUSIONS

General approaches for determining the effective stress levels of concrete struts by implementing the principal stress ratios of the finite elements of the corresponding strut regions in the uncracked plain concrete structure, and for verifying the bearing capacity of nodal zones are proposed. The values and equations for effective stress levels of concrete struts and nodal zones proposed elsewhere are suitable for specific design situations. The approaches proposed in this paper provide more general evaluation and verification processes of effective stress levels for concrete struts and bearing capacity of nodal zones in the case of complex design situations. To illustrate the proposed approaches, an analysis of a reinforced concrete beam tested to failure was conducted using the strut-tie model approach with the aid of an interactive computer graphics program.

5. REFERENCES

1. Adeber, P. and Zhou, Z. (1993). "Bearing Strength of Compressive Struts Confined by Plain Concrete." ACI Structural Journal, 90, 534-541.
2. Alshegeir, A. (1992). "Analysis and Design of Disturbed Regions with Strut-Tie Models." Ph.D. Thesis, School of Civil Engineering, Purdue University, West Lafayette, Indiana, 274.
3. Bergmeister, K., Breen, J.E. and Jirsa, J.O. (1991). "Dimensioning of the Nodes and Development of Reinforcement." Report, IABSE Colloquium Structural Concrete, Stuttgart, 555-564.
4. Canadian Standards Association (1984). "Design of Concrete Structures for Buildings." CAN3-A23.3-M84, Rexdale, Ontario, 288.
5. Jirsa, J.O., Breen, J.E., Bergmeister, K., Barton, D., Anderson, R. and Bouadi, H. (1991). "Experimental Studies of Nodes in Strut-and-Tie Models." Report, IABSE Colloquium Structural Concrete, Stuttgart, 551-564.
6. Kupfer, H., Hilsdorf, H.K. and Rusch, H. (1969). "Behavior of Concrete under Biaxial Stresses." ACI Journal, 66, 656-666.
7. Kupfer, H. and Gerstle, K. (1973). "Behavior of Concrete under Biaxial Stresses." Journal of Engineering Mechanics, ASCE, 99, No. EM4, 853-866.
8. MacGregor, J.G. (1988). "Reinforced Concrete Mechanics and Design." Prentice Hall, Englewood Cliffs, New Jersey.
9. Marti, P. (1985). "Basic Tools of Reinforced Concrete Beam Design." ACI Structural Journal, 82, 46-56.
10. Nielsen, M.P., Braestrup, M.W., Jensen, B.C., and Bach, F. (1978). "Concrete Plasticity, Beam Shear-Shear in Joints-Punching Shear." Special Publication, Danish Society

of Structural Science and Engineering, Technical University of Denmark, Lyngby.

11. Ramirez, J.A. and Breen, J.E. (1983). "Proposed Design Procedure for Shear and Torsion in Reinforced and Prestressed Concrete." Research Report 248-4F, Center for Transportion Research, University of Texas at Austin, 254.
12. Schlaich, J., Schaefer, K. and Jennewein, M. (1987). "Towards a Consistent Design of Structural Concrete." Journal of the Prestressed Concrete Institute, 32, 4-150.
13. Schlaich, M. and Anagnostou, G. (1990). "Stress Fields for Nodes of Strut-and-Tie Models." Journal of Structrual Engineering, ASCE, 116, 13-23.
14. Yun, Y.M. (1994). "Design and Analysis of 2-D Structural Concrete with Strut-Tie Model." Ph.D. Thesis, School of Civil Engineering, Purdue University, West Lafayette, Indiana, 247.
15. Yun, Y.M. and Ramirez, J.A. (1996). "Strength of Struts and Nodes in Strut-Tie Model", Journal of Structural Engineering, ASCE, 112.