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ROBUST CONTROL OF POSITIONING SYSTEMS WITH A BANG-BANG ACTUATOR
(뱅-뱅 액츄에이터를 가진 위치 제어계의 강인제어)

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Abstract

A nonlinear control scheme for preventing limit cycle due to the nonlinearity of the multi-step bang-bang actuator in mechanical position control systems is proposed. A linearized model, SIDF, for a multi-step bang-bang actuator is introduced to compensate the nonlinearity of the multi-step bang-bang actuator. Using that model, a H_∞ robust controller for position control systems with a bang-bang actuator is proposed by loop shaping techniques with normalized coprime factorization stabilization to address the robustness. The proposed scheme needs a smaller deadband as a result of compensating the nonlinearity of the bang-bang actuator. A single-axis servo system is served in order to verify the proposed control scheme experimentally. Experimental results show that the controller can satisfy the special interests, silent contact switching of the actuator.

1. INTRODUCTION

High-power motors actuate the big reclaimers used to reclaim ores in the mines and raws yards of the steelworks, and wall cranes to transfer heavy materials in the shops. Most of the motors in these machines are of multi-step bang-bang type, because workers can easily operate them by several speed control levers, and the motors are more inexpensive than linear motors. One of the difficulties encountered in automating reclaimers is to overcome the position errors and stable oscillation by bang-bang actuators. The hard nonlinearities of the bang-bang actuator entail the limit cycle in the feedback control system. Limit cycle tends to cause poor position accuracy. The constant oscillation associated with the limit cycle can bring about increasing wear, undesirable chattering and mechanical failures of the systems. In general, a deadband is used to avoid the problem of the persistent switching due to limit cycle, but it necessarily brings about the steady-state position error. The quasi-linearization methods serve as viable tools in the analysis and synthesis of nonlinear systems with hard nonlinearities. In the previous research, Taylor and Strobel[1] designed a fully nonlinear PID compensator for hard nonlinear systems via a sinusoidal input describing function. Beaman[2] proposed the quasi-linear quadratic Guassian(QLQG) control, and Kim[3] proposed the quasi-linear quadratic Gaussian control with loop transfer recovery (QLQG/LTR).

In this paper, for hard nonlinear systems, a nonlinear robust control scheme, which can address the stability robustness problem and design controllers with relative ease is proposed. The designed control method is applied to prevent limit cycle due to the nonlinearity of the multi-step bang-bang actuator in mechanical position control systems. Using a linearized model, SIDF, for a multi-step bang-bang actuator, a H_∞ controller for position control systems with a bang-bang actuator is designed. The position controller with the compensator of the bang-bang actuator is synthesized by loop shaping techniques. The controller attenuates the nonlinear effects of the bang-bang actuator by introducing the describing function inverse, and simultaneously suppresses the persistent contact switching of the actuator by limit cycle. The designed control system has been implemented in a positioning system with a 2-step bang-bang amplifier. The phase portraits for step references verify that the designed control system is effective to suppress limit cycle by the bang-bang actuator in the vicinity of the reference position. Experimental results show that the controller can satisfy the special interests, silent contact switching of the actuator.

2. DESCRIBING FUNCTION FOR THE BANG-BANG ACTUATOR

The describing function or sinusoidal input describing functions of a nonlinear element is defined to be the complex ratio of the fundamental harmonic component of the output to the input[4]. Consider the sinusoidal input $x(t)=M\sin\omega t$ to the multi-step bang-bang actuator with dead band. The output is supposed to be $g(t)=g\sin\omega t$ by considering only the fundamental component of the output. The input and the output functions are plotted in Fig. 1. The output is seen to be symmetric over the four quarters of a period. In the first quarter, it can be expressed as

$$g(t)=\begin{cases} g_0 & 0\le\omega t\le\alpha_1 \\ g_1 & \alpha_1\le\omega t\le\alpha_2 \\ g_2 & \alpha_2\le\omega t\le\alpha_3 \\ & \vdots \\ g_n & \alpha_n\le\omega t\le\pi/2 \end{cases} \tag{1}$$

where $\alpha_1 = \sin^{-1}(x_1 / M), \ldots, \alpha_n = \sin^{-1}(x_n / M)$ and $g_0 = 0$.

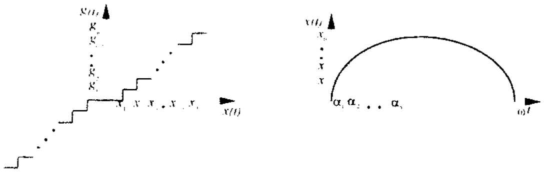


Fig.1 Multi-step nonlinearity

Using Fourier series, the output function is

$$g(t) = b_1 \sin(\omega t) \tag{2}$$

,where $$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} g(t)\sin(\omega t)d(\omega t) = \frac{4}{\pi}\left[\sum_{i=1}^{n}\cos\alpha_i(g_i - g_{i-1}\right] \tag{3}$$

The actuator output, $g(t)$, may be expressed as follows:

$$g(t) = \frac{4}{\pi}\left[\sum_{i=1}^{n}\cos\alpha_i(g_i - g_{i-1}\right]\sin(\omega t) \tag{4}$$

The describing function N is then given by

$$N = \frac{g(t)}{x(t)} = \frac{4}{\pi M}\left[\sum_{i=1}^{n}\cos\alpha_i(g_i - g_{i-1})\right] \tag{5}$$

From the relation of the trigonometric function,

$$\cos\alpha_i = \sqrt{1-(x_i / M)^2}\ . \tag{6}$$

Applying Eq. (6) to Eq. (5), the describing function for the multi-step bang-bang actuator is obtained as follows.

$$N = \frac{4}{\pi M}\left[\sum_{i=1}^{n}(g_i - g_{i-1})\sqrt{1-(x_i / M)^2}\right] \tag{7}$$

3. CONTROLLER DESIGN USING LOOP SHAPING TECHNIQUES

Any other perturbed system of the same input/output dimensions by normalized right coprime factorization(NRCF) can be written in the form

$$G_\Delta = (N + \Delta_N)(M + \Delta_M)^{-1} \tag{8}$$

where $\Delta_N, \Delta_M \in H_\infty$ are stable transfer functions.
The robust stabilization problem is to stabilize the nominal system, G, with NRCF(N,M) and the family of systems G_ε defined

$$G_\varepsilon = \{(N + \Delta_N)(M + \Delta_M)^{-1} : \left\|\begin{matrix}\Delta_N \\ \Delta_M\end{matrix}\right\|_\infty < \varepsilon\} \tag{9}$$

using a proper feedback control K[5].

Definition (NRCF Robust Stabilization): Let (N,M) be a NRCF of G, then, 1) Find the largest positive number ε_{max} called the maximum stability margin, such that (N, M, ε_{max}) is robustly stable. 2) For a particular value $\varepsilon \le \varepsilon_{max}$, synthesize a feedback controller, K, such that (N,M,K,ε) is robustly stable.
The maximum stability margin is written as

$$\varepsilon_{max} = (1 + \lambda_{max}(XY))^{-\frac{1}{2}}. \tag{10}$$

Given $0 < \varepsilon < \varepsilon_{max}$, then a state-space realization of the central controller satisfying the stability margin is

$$K = \left[\begin{array}{c|c} A + BB^T + \varepsilon^{-2}W_1^{-1}YC^TC & \varepsilon^{-2}W_1^{-1}YC^T \\ \hline B^TX & 0 \end{array}\right] \tag{11}$$

where $W_1 = (1-\varepsilon^{-2})I + YX$.

The Loop-Shaping Design Procedure

Loop Shaping : using a loop compensator, W, the magnitude of the nominal system G, is shaped to give a desired target loop which determines the open-loop shape the closed-loop system. The selection of target loop uses familiar SISO loop-shaping guidelines. The nominal system and the loop compensator, W are combined form the shaped system, G_S, where $G_S = GW$.

Robust Stabilization : a feedback controller, K_∞, is synthesized using the NRCF stabilization procedure which robustly stabilizes the NRCF of G_S, with stability margin ε[6].

The final controller, K, is then constructed by combining the H_∞ controller, K_∞, with the loop compensator W such that $K = WK_\infty$.

4. DESIGN OF THE PROPOSED CONTROLLER

The actuator treated in this paper is of the 2-step bang-bang type without dead band, and its describing function is obtained from Eq. (7).

$$N(M) = \frac{4}{\pi M}\left[g_1 + (g_2 - g_1)\sqrt{1-(x_2 / M)^2}\right] \tag{12}$$

Since the electrical time constant is much smaller than the mechanical time constant in the servomotor-driven position control system, the armature inductance effects can be neglected. Therefore, the dynamic model of the system, is, in general, characterized by the 2nd order transfer function, and has no finite zeros. The transfer function for the index table not including the bang-bang actuator is experimentally obtained using frequency response and approximately given as follows:

$$G(s)=\frac{k}{s(Ts+1)} \qquad (13)$$

where k is 1.52 and T is 0.009 sec. Thus, the positioning system including the linearized model of the bang-bang actuator is

$$G(s,M)=\frac{kN(M)}{s(Ts+1)} \qquad (14)$$

The open loop shape of $G(s,M)$ moves upward and downward according to the magnitude of the control input M. Therefore, the crossover frequency varies, and the bandwidth of the system is greatly dependent on the control input. The varying bandwidth means that the position control system may have bad servo performance when the M is large, because the nonlinearity dominates the response dynamics. The plant dynamics is loop-shifted to $k/s(Ts+1)$ by dividing $N(M)$ to simplify the solution of a H_∞ controller. Next, we shape the $k/s(Ts+1)$ to the desired form with linear loop compensator to meet our control specifications.

Inserting a finite zero with an appropriate proportional gain as a loop compensator, which guarantees a good phase margin is necessary for the target loop to have -20db/dec near the crossover frequency. The roll-off rate in the crossover frequency is -20 db/dec. But if a large proportional gain is used in the feedback controller, the shape goes up necessarily and the roll-off rate in the crossover frequency will be -40 db/dec. So the insertion of a finite zero near the crossover frequency guarantees for the target loop to have -20db/dec. The shaped plant $G_s(s)$ is obtained by inserting another loop compensator, $k_p(s/\omega_z+1)$, where we assign 105 rad/sec to ω_z. A ω_z much larger than the natural frequency, 1/0.009, can't transform the roll-off rate near the crossover frequency, and a ω_z smaller than that natural frequency reduces the bandwidth of the closed-loop system.

The shaped transfer function is

$$G_s(s)=\frac{kk_p(s/\omega_z+1)}{s(Ts+1)} \qquad (15)$$

Since poles and zeros in the H_∞ controller is mostly located near or beyond the crossover frequency, and the controller is strictly proper as shown in the state-space model of the controller, it has a higher roll-off rate in the high frequency and good noise immunity.

The stability margin, ε=0.5, appears to give the good loop recovery, because the designed loop has slightly lower gain than the target loop. This result from the selection of ε smaller than ε_{max}. ε near to ε_{max} gives the good recovery, but has a disadvantage to introduce fast poles. The designed loop shows that it gently crosses the 0 db with -20db/dec, and it is thought that it will have a nice transient response. The maximum stability margin calculated from Eq. (10) is 0.72. Here, the H_∞ controller, K_∞, is designed with stability margin, ε =0.5 to prevent the fast poles from being introduced to the H_∞ controller. The final feedback controller K is obtained by combining the loop compensator and K_∞.

$$K(s)=\frac{140.06(s+\omega_z)(s+109.54)}{(s+327.24)(s+104.9)}N(M)^{-1} \qquad (16)$$

The final controller $K(s)$ will be a kind of an adaptive controller with varying gain which is dependent on the magnitude of the control input to cancel the nonlinear effects of the bang-bang actuator. Fig. 2 shows the magnitude Bode plot for the target loop, the designed loop shape and the controller K_∞. The designed loop shape shows higher roll-off rate in the high frequency region and therefore robustness to the measurement noise. The magnitude of the H_∞ controller is slightly below 0 db in the low frequency region because of a smaller ε than ε_{max}.

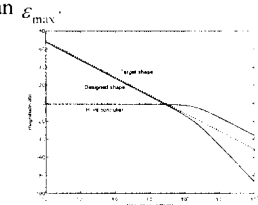


Fig. 2 Magnitude plot of the designed system

The final controller consists of the H_∞ controller for linear plant dynamics, $\boldsymbol{K}$ and the compensator for the bang-bang actuator as shown in Fig. 3. Therefore, we have to generate the amplitude, $\boldsymbol{M}$, of the input signal to the actuator to implement N^{-1} into the controller. Colgren[7] took the absolute value of the control input to the actuator, m, and filtered it through a low pass filter. An amplitude estimator in this paper is also accomplished by taking the absolute value of the actuator input signal.

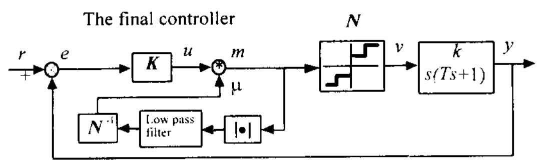


Fig. 3 H_∞ control system with the conpensator

The usefulness of the nonlinear compensator for the actuator to reduce the persistent switching is discussed according to the magnitude of the linearized actuator output. Consider the input to the actuator $m = M\sin\omega t$. The actuator output is simply the product of the controller output m and the describing function N.

$$v=N(M)m=\frac{4[g_1+(g_2-g_1)\sqrt{1-(x_2/M)^2}\,]}{\pi M}m \qquad (17)$$

The v has a very large magnitude when the $\boldsymbol{M}$ is close to zero. As the large proportional gain in the linear system causes overshoots, such a large actuator output gives rise to limit cycle by the similar effect. Therefore, if the control input to the bang-bang actuator gets smaller, limit cycle can be avoided. The deadband of the actuator bounds the maximum magnitude of the linearized output of the bang-bang actuator as shown in Fig. 4. It has severe effects on the switching behaviors by bounding the magnitude of controller out, as well as it extinguishes the actuation.

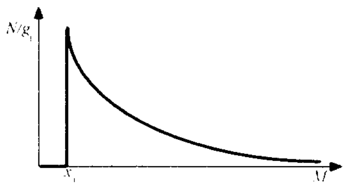


Fig. 4 Describing function of the bang-bang actuator

The describing function inverse N^{-1} for compensating the nonlinearity in this controller is considered to have no deadband term, because the deadband term makes the magnitude of the describing function inverse infinite when the magnitude of the input signal to the N^{-1} is within the deadband. Therefore, the deadband is inserted into the output terminal of the controller to constraint the control output.

5. EXPERIMENTAL RESULTS

Fig. 5 shows a testbed for the position control system with a 2-step bang-bang actuator. The system consists of two main elements: the servomotor-driven index table and VME computer with interface. The used position sensor is an incremental encoder. The motor used is DC servo motor, and the controller is implemented using MVME143 computer that has VME bus and 32 bit microprocessor. The computer is interfaced to a power amp via a DAC board and a encoder through a counter board. The H_∞ controller designed in the continuous-time domain is transformed to the discrete- time domain controller through bilinear transformation. The digital codes written in C are executed every 2 msec under assistance of real time operating system, VMEexec.

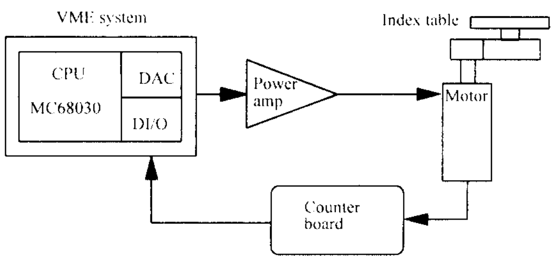
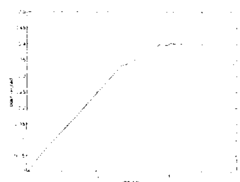


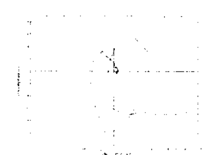
Fig.5 Experimental setup of a servomechanism

Step response experiments were carried out to demonstrate the effectiveness of the proposed controller. Step responses are useful information to examine the existence of the limit cycle in the vicinity of the target position. The index table was given 0.4 radian step command. The proposed controller has the deadband equivalent to 0.0003, while the input range of the power amp is ±0.7. The value is appropriately determined to be the minimum that doesn't bring about persistent switching. The position response shows good performance despite of the hard nonlinearity of the actuator as shown in Fig. 6a. The actuator output for the above step response is also given in Fig. 6b. The actuator switches only once near the reference position, which is encouraging to implement the controller into the real mechanical systems. The phase portrait informs the nature of the system response and corresponding actuator switching behaviors from the velocities and positions of the position control system.
The phase portraits in the vicinity of the reference position as shown in Fig. 6c represent the stable convergence and gentle switching.

The proposed nonlinear controller is compared with the linear H_∞ controller not including the nonlinear compensator for the bang-bang actuator. The wider deadband whose value is 0.015 is set to this linear controller. The phase portraits of the step reference for the two controllers are plotted in Fig. 7.



(a) position response



(c) phase portrait

Fig. 6 Time responses for the step reference

The linear controller yields larger steady-state error than the proposed controller as a result of the wider deadband. It also produces more switchings though it has wider deadband. The linear controller shows worse switching behaviors as seen in the phase portrait. The deadband of the proposed controller is much smaller than that of the linear controller.
Robustness for the load uncertainty which causes a parametric modeling error is demonstrated with a additional 6 kg mass which is corresponding to 4/5 of the allowable maximum load attachable to the index table.

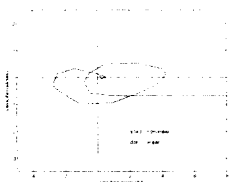
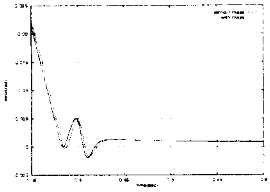
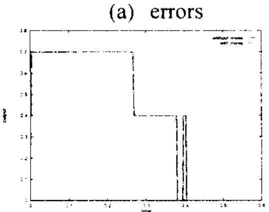


Fig.7 Comparison of the phase portraits

Fig.8 shows the error response with the load. The angle errors which show the error responses for an angle command of 0.4 radian after 0.35 second illustrate a robust performance to the load perturbation. The steady-state errors approach to the nearly same value despite the additional load. The actuator outputs also demonstrates the silent switching.



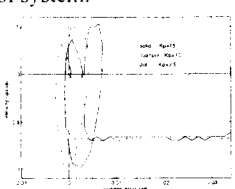
(a) errors



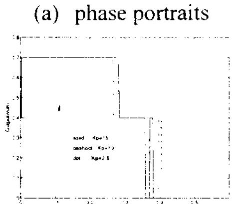
(b) actuator outputs

Fig.8 Robustness for the load variations

The deadband in the linear controller without the nonlinearity compensation, in general, has approximately linear proportional relationship to the controller gain. Since the bang-bang actuator is highly dependent on the magnitude of the input signal, the controller gain variations will change the bandwidth of the control loop, and therefore tracking ability will be deteriorated. Therefore, it is meaningful to examine the effects of gain changes on the performances for real system application. To ascertain the insensitivity to the controller gain variations, three controllers designed from the different loop compensator gains are tested. Three designed nonlinear controllers that have the three proportional gains, 2.5, 10 and 15 give the similar switching characteristics indifferent from the gain changes as shown in Fig. 9. Note that he proposed controller has switching robustness for the wide variation of the controller gain. But the steady- state errors is dependent on the gain as the position error is, in general, inversely proportional to the magnitude of the gain in the linear control system.



(a) phase portraits



(b) actuator responses

Fig. 9 Robustness for the gain variation

It is shown that the persistent switchings of the actuator are sufficiently suppressed by the compensator of the bang-bang actuator as expected. It is believed that the designed controller has the insensitivity to the perturbations of the load and controller gain through the above experimental results.

6. CONCLUSIONS

A nonlinear control scheme is proposed to prevent persistent actuator switching due to the nonlinearity of the multi-step bang-bang actuator in mechanical position control systems. A robust H_∞ controller for the position control system with the bang-bang actuator is designed by loop shaping techniques with normalized coprime factorization stabilization. The nonlinearity of the actuator was compensated with the describing function inverse for a multi-step bang-bang actuator. The proposed controller was applied to the index table to ensure applicability for the real mechanical systems. Experimental results showed that the designed H_∞ controller is effective to suppress undesirable persistent switching. Despite of the dependence of the bang-bang actuator on the input amplitude, the compensated system illustrated the insensitivity to the variations of the controller gain.

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