

Orbit Estimation of the Satellite using GPS

GPS 를 이용한 위성궤도추정

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abstract

Orbit Determination is process of obtaining values of those parameter which completely specify the motion of an orbiting body through space, based on a set of observation of the body. For the narrow land of Korea, the ground tracking system has very limited time of operation. In this connection the use of GPS for orbit determination has advantage of full autonomy on the ground station. It would be more powerful economical method for near-earth satellites.

Therefore we have better to pay attention to the research of satellites of orbit determination by use of GPS. So in this research, we studied the motion of the satellites with estimation using GPS. As a result, the result of computer simulation show that good convergence and indicated effective for real operation.

1. Introduction

An earth satellite collecting GPS(Global Positioning System) data with onboard receiver can compute its state(position and velocity) in a diversity of ways, the choice depending in part on the type of orbit and mission requirements. Tracking and navigation requirements can include real-time state knowledge and active control during launch and orbit insertion and during reentry and landing; real-time relative navigation between vehicles during rendezvous, autonomous station-keeping and near-real time orbit knowledge for operation and orbit maintenance; rapid post-maneuver orbit recovery; and after the fact precise orbit determination for scientific analysis.

Orbit Determination is process of obtaining values of those parameter which completely specify the motion of an orbiting body a satellites through space, based on a set of observation of the body. Orbit accuracy requirements can range from hundreds of meters or more for routine operations to a few centimeters for precise remote sensing. Among existing tracking systems, GPS can meet the most stringent of these needs for the most dynamically unpredictable vehicles.

The GPS signal beamwidths extend roughly 3000km beyond the earth's limb, enabling an earth orbiter below that altitude to receive continuous three dimensional coverage. This study focuses on orbit estimation for satellites in low circular orbits, below a few thousand kilometers, with emphasis on the high accuracy that GPS so ably provides. Real-time techniques fall under what we call direct GPS orbit determination, in which only the GPS data collected by the orbiter can used in the solution. [Ref.1]

The potential of GPS to provide accurate and autonomous satellite orbit determination was noted early in its development. Early studies of direct GPS-based tracking include those in [Ref.2], which surveyed applications from near earth to beyond geosynchronous altitudes; [Ref.3] which examined GPS tracking of the space shuttle; [Ref.4], which focused on a autonomous near earth navigation. In Korea, there are a little study about direct GPS-based tracking but a ground tracking system. [Ref.5]

Especially, for the narrow land of Korea, the ground tracking system has very limited time of operation. In this connection the use of GPS for orbit determination

has advantage of full autonomy on the ground station. It would be more powerful and economical method for near earth satellites. Therefore we have better to pay attention to the research of satellites of orbit determination by use of GPS. So in this research, we studied the motion of the satellites with estimation using GPS. The user satellite is near earth satellite which has sunsynchronous orbit.

2. Modelling Dynamics of the Satellite

When developing the two body equation of motion, we can obtain an equation for acceleration vector of the satellite. [Ref.6]

$$\frac{d^2 \mathbf{r}}{dt^2} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{a} \quad (1)$$

r is the distance from the center of the earth to the satellite

\mathbf{r} is position vector from the center of the earth to the satellite

μ is the earth's gravitational parameter

\mathbf{a} is the perturbation acceleration effect

In this study, we consider the nonspherical effect and the drag acceleration by earth's atmosphere as the perturbation acceleration effect.

When considering the nonspherical gravitation perturbation effect, the equation for acceleration is

$$\begin{aligned} \ddot{x} &= -\mu \frac{x}{r^3} \left[1 + \frac{3}{2} J_2 \frac{r_e^2}{r^2} \left(1 - 5 \frac{z^2}{r^2} \right) \right] \\ \ddot{y} &= -\mu \frac{y}{r^3} \left[1 + \frac{3}{2} J_2 \frac{r_e^2}{r^2} \left(1 - 5 \frac{z^2}{r^2} \right) \right] \\ \ddot{z} &= -\mu \frac{z}{r^3} \left[1 + \frac{3}{2} J_2 \frac{r_e^2}{r^2} \left(3 - 5 \frac{z^2}{r^2} \right) \right] \end{aligned} \quad (2)$$

And the drag acceleration by earth's atmosphere is

$$\dot{\mathbf{r}} = -\frac{C_D \Lambda}{2m} \rho(H) \mathbf{V}_b \mathbf{V}_b \quad (3)$$

Relative velocity of the satellite is

$$\mathbf{V}_b = (V_x + y\omega)\mathbf{i} + (V_y - x\omega)\mathbf{j} + V_z\mathbf{k} \quad (4)$$

where $\rho(H)$ is the atmospheric density, C_D is the drag coefficient, Λ is the cross-sectional area perpendicular to \mathbf{V}_b , m is the satellite mass, \mathbf{V}_b is the speed relative to the atmosphere.

We can get observation data using [ref.8]

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = [c(\Delta T - e_T)]^2 \quad (5)$$

x, y, z : the used satellite's position component

x_i, y_i, z_i : the GPS satellite's position component

c : speed of light

e_T : clock error

The quantities (x, y, z, e_T) are the unknown which can be solved for using four simultaneous equation. So we need at the least four GPS satellite's observed data.

3. Kalman Filter Formulation

A spaceborne GPS user may require a continuous real-time state solution more accurate than point positioning can provide. Although filtering is needed to achieve this, a batch solution is generally inappropriate because it may require a long accumulation of measurements and a large amount of computation at once. In such cases, a sequential estimator is called for, a popular example of which is the Kalman filter. A sequential filter continually updates the current state estimate with each new measurement. The computation needed for each update is small compared with that for a full batch solution; hence, an onboard processor can maintain the solution real time. It should be noted that the sequential current state employs only data from the past up to the present, whereas a batch filter may estimate a state with data from both before and after an epoch.

The Kalman filter is formulated in discrete time recursion relation. Suppose the filter has produced a state estimate \hat{x}_i at time t_i (using data up through time t_i) and the estimated covariance matrix for \hat{x}_i is

\hat{P}_i . The state solution \hat{x}_{i+1} at time t_{i+1} is derived in two step. Step 1 is the time updated, in which a predicted or a priori solution \hat{x}_{i+1} and covariance matrix \hat{P}_{i+1} are generated from their estimated values at time t_i , with no new data yet added. Step 2 is the measurement updated, in which the new estimates \hat{x}_{i+1} and \hat{P}_{i+1} are generated from the

data at time t_{i+1} , as corrections to the predicted values.

Also observe unmodeled motion, we model the time-varying satellite force as the sum of a deterministic component (our standard dynamic model) and a stochastic component. The latter is often called a process noise model. Augmenting a Kalman filter with process noise model is away of telling the filter that the state transition information in transition matrix is incomplete - that there is another component that the filter can't predict, but there it can try to observe in the data and estimate at each time step. in the context of orbit determination, this means that at each time step, in addition to applying the standard dynamic updates, the filter will examine the discrepancy between the dynamic state estimate and the apparent state as indicated geometrically by the measurements. From that discrepancy, it will estimate a local correction to dynamic model, valid only over the update time interval. When added to dynamic model, that correction will reduce the disagreement between the observations and the solution trajectory at time.

As it proceeds through the data, the filter will generate a sequence of local force model corrections, one at each updated time, bringing solution trajectory into better agreement with the observation. [Ref.1,7]

4. Select Observed Data of the User Satellite using GPS

The accuracy of the navigation solution for the user's satellite position is usually specified by a quantity known as GDOP (geometric dilution of precision)

$$GDOP = \sqrt{\text{trace}[A^T \cdot A]^{-1}} \quad (6)$$

$$[a_i \ \beta_i \ \gamma_i \ 1], \quad i = 1, 2, 3, 4$$

$(a_i \ \beta_i \ \gamma_i)$ is direction cosine in user satellite and GPS satellites. [Ref 8]

For a unique solution of eqn(6), data from only four satellites are needed.

At a given instant, if there are more than four GPS satellite in view, the user satellite has options for choosing a group of four among the visible satellite.

So, we can choose the option like as this step.

- 1) choose the GPS satellite that visible angle is over 10 degree. (Fig.1)
- 2) choose the four GPS satellite that satisfied minimum GDOP quantity

5. Simulation

The user satellite in the simulation which is near earth satellite in the sunsynchronous orbit, and its data is like as table 1, its initial classic orbit element is like as table 2.

We select four GPS satellite using GDOP per 5 minute in the computer simulation.

So we can choose the optimal four GPS satellite that was applied to orbit determination.

Fig.2 is the result of the case 1, it shows that position and velocity error is converge within error bound. Position error is that average value is about 4m and maximum value is about 10m, and velocity error is that average value is about 0.04m/s and maximum value is about 0.1m/s.

Fig.3 is the result of the case 2, it also shows that position and velocity error is converge within error bound. Position error is that average value is about 10m and maximum value is about 30m, and velocity error is that average value is about 0.04m/s and maximum value is about 0.2m/s.

With this result, we can calculate orbit element and analysis orbit motion.

As a result, the computer simulation of the orbit determination using the observation data from GPS satellite tracking system converge within position error and velocity error bound. This is similar result when we treat observations data from satellite tracking ground radar system.

6. Conclusion

Especially, for the narrow land of Korea, the ground tracking system has very limited time of operation. And we can know GPS tracking system also powerful and economical method for near earth satellites in this study.

Therefore we have better to pay attention to the

research of satellites of orbit determination by use of GPS.

This result is just near earth user satellite. If we want to apply to the geostationary user satellite, it can be modified.

Also, We will be able to examine the adaptation of GPS tracking techniques to satellite in highly elliptical and geostationary orbits.

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Table 1. data of the user satellite

data of the user satellite	
mass	: 358.84 kg
intersection area	: 3 m ²
drag coefficient(C _D)	: 2

Table 2. orbit element of the user satellite

orbit element	value
semi-major axis (a)	6937 (km)
eccentricity (e)	0.001
inclination (i)	97.631(deg)
ascending node (Ω)	354.878(deg)
argument of periapsis (ω)	180 (deg)
mean anomaly (M)	0.0 (deg)

Table 3. case 1 (data of the noise)

case 1 : data of the noise	
pseudo range noise	: 2.5 m
pseudo range rate noise	: 0.02 m/s
clock drift rate	: 0.005 m/s ²

Table 4. case 2 (data of the noise)

case 2 : data of the noise	
pseudo range noise	: 10 m
pseudo range rate noise	: 0.05 m/s
clock drift rate	: 0.005 m/s ²

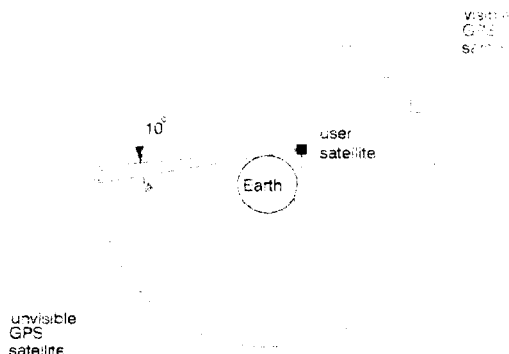
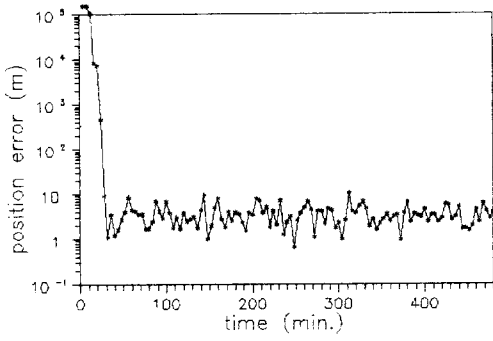
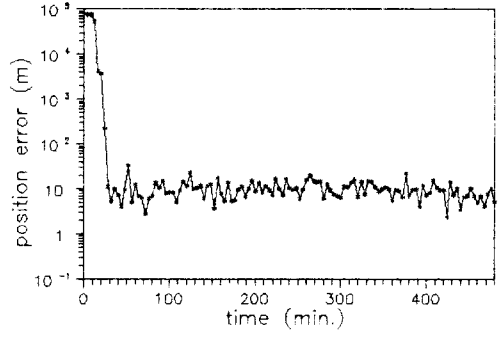


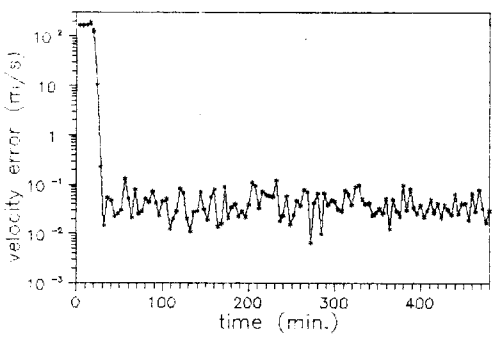
Fig.1 Select GPS Satellite to the User Satellite



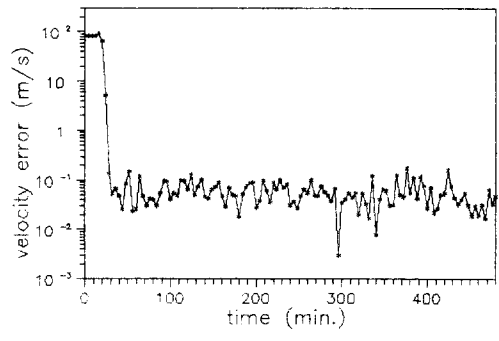
a) position error



a) position error



b) velocity error



b) velocity error

Fig.2 Error of the case 1

Fig.3 Error of the case 2