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# PATH CONTROL FOR NONLINEAR VEHICLE MODELS

## 비선형 차량모델 모의실험의 경로제어

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## ABSTRACT

This paper presents a steering control strategy applicable to vehicle path following problems. This control strategy is based on realistic nonlinear equations of motion of multibody systems described in terms of relative joint coordinates. The acceleration of the steering angle is selected as a control input of the system. This input is obtained by considering position and slope errors at current and at advance times. This steering control strategy is tested in circular and lane change maneuvers with a nonlinear vehicle model.

## 1 INTRODUCTION

This paper presents a steering control strategy applicable to any vehicle system simulation. Vehicle systems are generally modelled as large-size nonlinear equations of motion. The equations may include complex suspension systems and analytical tire models. Hence, it is difficult to develop a control strategy that simulates a vehicle model to follow a given path.

In order to avoid this difficulty, most of researches have used a linear vehicle system producing a small set of linear equations of motion [1-3]. Based on these equations, path control strategies adopting the linear control and the optimal control theories have studied. However, these control strategies are not applicable to realistic vehicle models. Hence, a general control strategy based on the nonlinear multibody systems is required.

The method presented herein uses joint coordinate formulation of equations of motion to reduce the size of equations and independent state variables and to increase computational efficiency and numerical accuracy. From these equations of motion, the acceleration of the steering angle is selected as a control input. Since in realistic driving situations not only a vehicle position but also its moving direction needs to be considered in the control strategy for a vehicle to follow a given path. Furthermore, a response delay of the lateral motion with respect to a steering input and driver's anticipation capability need alos to be considered in the control strategy. Hence, this method uses the errors of the positions and the moving directions at current and in advanced times. This steering control strategy is tested in circular and lane change maneuvers with a nonlinear vehicle model.

## 2 STEERING CONTROL STRATEGY

The vehicle maneuver can be generally classified into two types, a lane change and a circular maneuver. In order to control a vehicle to perform these maneuvers, a steering control strategy is required. As the control strategy, an open and a closed loop path control methods can be considered and introduced into the equations of motion.

A closed loop path control uses the feedback of some states of the system. It is commonly used for linear systems with other linear control theories. However, the vehicle model is generally modeled as a complex nonlinear equations which may includes a comprehensive analytical or experimental tire model or a complex kinematics of a suspension system. Hence, it is difficult to consider this system as a linear system and to apply the linear control theory to this system. For the path control of the complex nonlinear vehicle model, a modified control strategy which considers the dynamic characteristic of the vehicle is required. The proper control method is developed through a nonlinear bicycle model.

### 2.1 Equations of Motion for the Vehicle Model

The nonlinear bicycle model can be described as consisting of a main chassis, a front wheel and a rear wheel. The front wheel is connected to the main chassis by a revolute-revolute joint and the rear wheel is connected to the main chassis by a revolute joint. Hence, this system has nine degrees of freedom. The configuration of the system is shown in Figure 1. The necessary data to formulate the equations of motion are taken from Appendix. The suspension system used here is simplified since the kinematics of the suspension system in the equation of motion does not have crucial influence in the path control strategy. Since the complexity of the tire model is considered in this model, this control strategy can also be used for any other complex vehicle model.

The equations of motion for multibody systems can be formulated by several ways. Out of many ways, it is known that equations of motion described in joint coordinates is simple and the size of equations are small [4,5]. This control strategy is investigated through the equations formulated in the joint coordinates in this paper. It can also be applied to any other formulation of equations of motion.

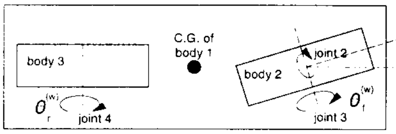


Figure 1 Nonlinear bicycle model with joint coordinates.

In order to describe the equations of motion of this system, a set of joint coordinates can be defined as

$$\underline{\theta} = \begin{bmatrix} \underline{\theta}_1 \\ \theta_r^{(w)} \\ \theta_f^{(w)} \\ \theta_s \end{bmatrix} \qquad (1)$$

where $\underline{\theta}_1$ contains six absolute coordinates of the chassis, such as $[x\ y\ z\ \phi_z\ \phi_x\ \phi_z]$. $\theta_r^{(w)}$ and $\theta_f^{(w)}$ are the rear and the front wheel joint angles, respectively. $\theta_s$ is the steering angle of the front wheel. The equations of motion in joint coordinates are written in compact form as

$$\begin{bmatrix} \underline{M}_a & \underline{m}_{as}^T \\ \underline{m}_{as} & m_s \end{bmatrix} \begin{bmatrix} \ddot{\underline{\theta}}_a \\ \ddot{\theta}_s \end{bmatrix} = \begin{bmatrix} \underline{f}_a \\ f_s \end{bmatrix} \qquad (2)$$

where $\ddot{\theta}_s$ is the acceleration of the steering angle in the joint coordinates which will be used as a control input, and $\ddot{\underline{\theta}}_a$ contains other eight joint accelerations. The terms $\underline{M}_a$ and $\underline{f}_a$ are the generalized mass matrix and the generalized force vector corresponding to $\ddot{\underline{\theta}}_a$, which are derived in Appendix. The terms $m_s$ and $f_s$ are the generalized mass and the generalized force corresponding to the steering angle $\theta_s$. The term $\underline{m}_{as}$ is a generalized mass vector corresponding to coupling between the steering angle and the remaining joint coordinates. The equations of motion can then be reorganized for path control as

$$\underline{M}_a \ddot{\underline{\theta}}_a = \underline{f}_a - \underline{m}_{as}^T \ddot{\theta}_s \qquad (3)$$

where $\ddot{\underline{\theta}}_a$ is the system state vector dependent on $\ddot{\theta}_s$ which is the input variable of the system. This means that a vehicle can be controlled to follow a given path using the input $\ddot{\theta}_s$ in Eq. (3).

## 2.2 Path Control Strategy

In a closed loop path control, the steering angle command can be determined by monitoring one or more error measurements between a trajectory of the vehicle and the given path during a maneuver. The errors are a position error and a moving direction error (later it will be referred to as the slope error) which will be utilized in the feedback path control. The moving direction error can be presented as the error between the moving direction of the vehicle to the tangential direction at the given path. The feedback control method is studied first in the lane change maneuver for convenience. Then, the feedback control method can be applied to the circular maneuver.

### 2.2.1 Lane Change Maneuver

The lane change maneuver is the most frequent maneuver occurring in real driving situations. It can be described as the shifting of a vehicle from one lane to another lane within a certain distance at a given speed. The intermediate path of the vehicle may not be of importance. However, for the feedback control, a prescribed path is required as a reference to measure errors. Hence, the prescribed path needs to be given as a continuous explicit function which connects the start and the end points. As the continuous explicit function, a polynomial function is proper to describe a lane change maneuver.

It is necessary to determine a proper polynomial function which represents an ideal path of lane change. By considering the vehicle dynamics, sharp turns are undesiable behavior of the vehicle. In order to avoid the sharp turns at the start or the end point of the path, tangential slopes there can be specified to zero. For smoother path, the derivatives of the tangential slope at both points are also required to be zero. These conditions are used to determine the polynomial for the path and they are written as

$$Y(X_0) = Y_0, \quad \frac{dY}{dX}(X_0) = 0, \quad \frac{d^2Y}{dX^2}(X_0) = 0,$$

$$Y(X_f) = Y_f, \quad \frac{dY}{dX}(X_f) = \theta_f, \quad \text{and} \frac{d^2Y}{dX^2}(X_f) = 0 \qquad (4)$$

where $\theta_f$ is an exit angle of the lane change which must be given less than 90°. When $\theta_f$ is 0, it is an ordinary lane change and the six boundary conditions can lead a fifth order polynomial, which is written as

$$Y(X) = Y_0 + A_1(X - X_0)^3 + A_2(X - X_0)^4 + A_3(X - X_0)^5$$
$$for\ X_0 < X < X_0 + \Delta X \qquad (5)$$

By differentiating the path function, the tangential slope along the path is obtained as

$$\frac{dY}{dX}(X) = 3A_1(X - X_0)^2 + 4A_2(X - X_0)^3 + 5A_3(X - X_0)^4$$

$$for\ X_0 < X < X_0 + \Delta X \quad (6)$$

where, $\Delta X = X_f - X_0$, $\quad \Delta Y = Y_f - Y_0$.

$$A_1 = 10\frac{\Delta Y}{\Delta X^3},\quad A_2 = -15\frac{\Delta Y}{\Delta X^4},\quad \text{and} \quad A_3 = 6\frac{\Delta Y}{\Delta X^5}.$$

In order to follow this analytically produced path, the PID type control method is most commly used in the feedback control of general systems. However, for the path control problems, this PID control method needs to be properly modified. In the path control, a path error between a desired path and a current path is crucial information. Then, this path error can be considered as combination of the position error and the moving direction error.

The position error can be computed by comparing the $Y$ coordinate of the center of the mass of the vehicle ($y$) against the $Y$ coordinate of the prescribed path corresponding to the $X$ coordinate of the center of mass of the vehicle ($x$) as shown in Figure 2. This position error can be written as

$$e_1(t) = y(t) - Y(X = x(t)) \quad (7)$$

The moving dierction error is computed by comparing the moving direction of the vehicle to the corresponding tangential slope at the prescribed path. This error can be written as

$$e_2(t) = \frac{\dot{y}(t)}{\dot{x}(t)} - \frac{dY}{dX}(X = x(t)) \quad (8)$$

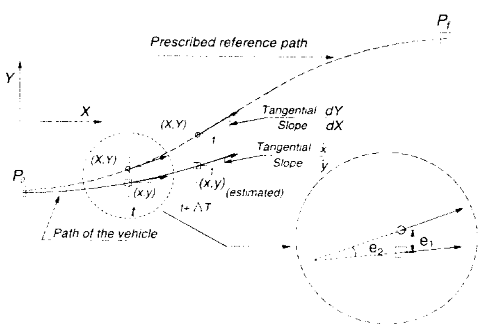


Figure 2 Schematic diagram of the positions and the slopes.

In the vehicle maneuvering simulation, the input command obtained from the error feedback is applied into the acceleration component of the steering angle. Hence, the detected error can not be immediately transfered to a proper steering angle compensating the error. The applied steering angle also produces a lateral force to the vehicle. Due to tire characteristics a steering angle of the wheel does produce a corrsponding lateral force not at instance but with a response delay from the steering input. Hence, the feedback control of the position and the tangential slope errors at the current position are not sufficient for the vehicle to follow a prescribed path. Therefore, the position and slope errors predicted ahead of an advance time are required to be added to the feedback control. In reality, actual drivers observe a path and initiate an action in advance.

The position and velocity of the center of the mass of a chassis at the advance time $\Delta T$ can be approximated by using Taylor expansion about its current positon and velocity. If acceleration is considered as the maximum order of accessible states, the predicted positon and velocity can be written as

$$x(t + \Delta T) = x(t) + \dot{x}(t)\Delta T + \frac{1}{2}\ddot{x}(t)\Delta T^2$$

$$y(t + \Delta T) = y(t) + \dot{y}(t)\Delta T + \frac{1}{2}\ddot{y}(t)\Delta T^2$$

$$\dot{x}(t + \Delta T) = \dot{x}(t) + \ddot{x}(t)\Delta T$$

$$\dot{y}(t + \Delta T) = \dot{y}(t) + \ddot{y}(t)\Delta T$$

By considering the current errors and the predicted errors .. advance time, the steering angle input is written as

$$\ddot{\theta}_s = K_1e_1(t) + K_2e_2(t) + K_3e_1(t + \Delta T) + K_4e_2(t + \Delta T)$$

where $e_1(t)$ and $e_2(t)$ are the errors of the position and the .. at an current time. On the other hand, $e_1(t+\Delta T)$ and $e_2(t+\Delta$ the errors of the position and the slope at an advanced time which are illustrated in Figure 2.3. The corresponding gains $K_1$, $K_2$, $K_3$, and $K_4$. In other words, this control scheme assumes the driver looks ahead a distance of $U_0\Delta T$, where $U_0$ is the sp… of vehicle in the direction of its motion.

In feedback path controls of the nonlinear vehicle, a ra… change of the steering angle may cause an excessive lateral s… of tires and may not control the vehicle but may increase … instability. These undesirable rapid changes in steering an… can be eliminated by introducing an artificial damper in t… feedback path controller. Hence, the smooth change of … steering angle will produce a proper lateral force from the ti… which will result in a smooth path control of the vehicle. Th… artificial damper is then added in the steering angle controller…

$$\ddot{\theta}_s(t) = K_1e_1(t) + K_2e_2(t) + K_3e_1(t + \Delta T) + K_4e_2(t + \Delta T)$$

$$+ K_4e_2(t + \Delta T) - K_5\dot{\theta}_s(t) \quad (1…)$$

which is expected to expand a range of acceptable feedba… control parameters for a given path.

### 2.2.2 Circular Path Maneuver

The lane change only can be defined when the exit angle … less than 90 degree. If the exit angle is greater than 90 degre… the path can be considered as circular path maneuver. When circular path is described using a position and a tangential slop…

in a Cartesian $X$-$Y$ reference frame, the magnitude of tangential slope along the circle would vary from zero to infinity. Hence, the control method using the tangential slope fails in controlling the vehicle to follow a prescribed circular path. In order to avoid this difficulty, a polar reference frame can be used in circular path maneuvers.

In a polar reference frame, the measurements used in the path control are the radius and the angle. The radius can be described as a constant parameter of a function of the angle. For convenience of the study, the radius is assumed to be constant. Then, the position errors $e_1(t)$ and $e_1(t+\Delta T)$ can be obtained from the distance between the center of the circular path and radius of the prescribed circular path.

For the slope errors, the angle in polar coordinates is used since this angle has uniform increment along the circle. Hence, the slope errors $e_2(t)$ and $e_2(t+\Delta T)$ are obtained from the angles at the current and advanced time.

## 3 SIMULATIONS AND RESULTS

By using the nonlinear bicycle model with the developed path control strategy, a lane change and a circular maneuvering simulations are performed. The simulation of this example uses the Multi-BOdy System Simulation computer package (MBOSS) [6]. This is a general purpose computer program for the dynamic analysis of multibody systems which uses the joint coordinate formulation of equations of motion and comprehensive anlytical model for a tire and its interaction with the ground [7]. This path control strategy is incorporated with MBOSS.

The lane change simulation is set by $X_i$ = 10 m, $Y_i$ = 0 m, $\Delta X = 40m$, and $\Delta Y = 6m$. The speed of the vehicle is *10.0 m/sec*. The results of the closed loop control simulations are illustrated in Figure 4. The feedback gains $K_1$, $K_2$, $K_3$, $K_4$ and $K_5$ are *10, 2, 100, 5* and *10*, respectively. The advanced observation time $\Delta T$ is *0.6* second. Figure 4ashows that the vehicle successfully follows the prescribed path and Figure 4b shows the corresponding steering angle input history.

Simulation results of the circular maneuver are illustrated in Figure 5. The simulation is set by $X_i$ = 10 m, $Y_i$ = 0 m, and R = 40 m. The speed of the vehicle is *10.0 m/sec*. The feedback gains $K_1$, $K_2$, $K_3$, $K_4$ and $K_5$ are *10, 2, 200, 20* and *10*, respectively. The advanced observation time $\Delta T$ is *0.6 second*. Figure 5a shows that the vehicle successfully follows the prescribed path and Figure 5b shows the corresponding steering angle input history. In the steering angle history, a slight overshooting is observed at the initial and final stages before the steering angle input reaches the steady values.

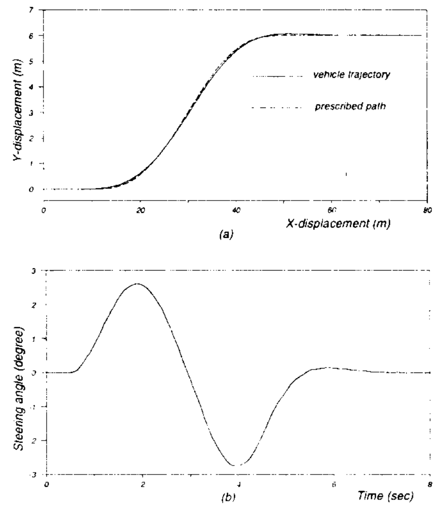


Figure 4 Lane change maneuver with a feedback controller; (a) trajectroy and (b) Steering angle history with $K_1$ = 10 , $K_2$ = 2 , $K_3$ = 100, $K_4$ = 5 , $K_5$ = 10 , $\Delta T$ = 0.6 , and $U_0$ = 10 *m/sec*.

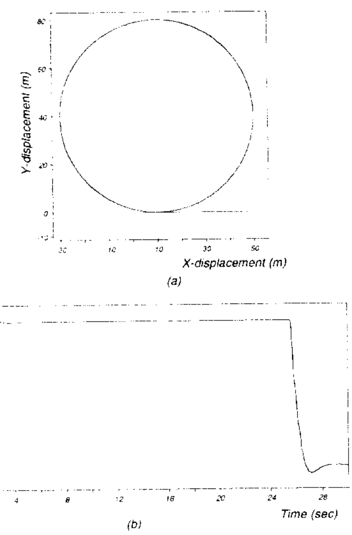


Figure 5 Lane change maneuver with a feedback controller; (a) trajectroy and (b) Steering angle history with $K_1$ = 10 , $K_2$ = 2 , $K_3$ = 200, $K_4$ = 20 , $K_5$ = 10 , $\Delta T$ = 0.6 , and $U_0$ = 10 *m/sec*.

## 4 CONCLUSION

This path control strategy offers the vehicle maneuvering simulation method for a given path. In order to complete the control strategy, six feedback gains need to be selected depending on the path and vehicle speed. The set of feedback gains are obtained by the trial and error for a given path and speed. As another way to determine a set of feedback gains, the optimization with the design sensitivity analysis for the feedback gains can be used, where the cost function can be the quardratic sum of error during the given maneuvering.

The properly selected advanced observation time $\Delta T$ in the set of feedback gains is 0.6 sec and it does not vary by path and vehicle speed. The selected set of gains are also valid in wide range of paths and speeds due to the artificial damping introduced in the controller. Therefore, if various sets of feedback gains for various paths and speeds are obtained in pre-process, this path control strategy can permit application to a broad range of path control simulations.

## 5 APPENDIX

Joint coordinate formulation of Equations of motion

The Equations of motion described in the joint coordinate system are written as follows

$$\mathbf{B}^T\mathbf{M}\mathbf{B}\ddot{\theta} = \mathbf{B}^T\mathbf{f}$$

where, $\mathbf{M} = \text{diagonal}[m_1\mathbf{I}, \mathbf{J}_1, m_2\mathbf{I}, \mathbf{J}_2, m_3\mathbf{I}, \mathbf{J}_3]$

$$\mathbf{J}_i = \begin{bmatrix} J_{\xi\xi} & J_{\xi\eta} & J_{\xi\zeta} \\ J_{\eta\xi} & J_{\eta\eta} & J_{\eta\zeta} \\ J_{\zeta\xi} & J_{\zeta\eta} & J_{\zeta\zeta} \end{bmatrix}_i$$

and $\xi\eta\zeta$ refers to body fixed coordinates.

$$\mathbf{B} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \tilde{\mathbf{d}}_{21} & \tilde{\mathbf{d}}_{22}\mathbf{u}_2 & \tilde{\mathbf{d}}_{23}\mathbf{u}_3 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{u}_2 & \mathbf{u}_3 & \mathbf{0} \\ \mathbf{I} & \tilde{\mathbf{d}}_{31} & \mathbf{0} & \mathbf{0} & \tilde{\mathbf{d}}_{34}\mathbf{u}_4 \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{u}_4 \end{bmatrix}$$

$\mathbf{d}_{ij}$ and $\mathbf{u}_i$ refers to a position vector from the center of $i$th body to the $j$th joint and a unit vector of $i$th joint axis.

Description of Kinematic Joints

| Joint # & type | Connected bodies | Position of the joint of refernce point 1 $\xi/\eta/\zeta$ | Position of the joint of reference point 2 $\xi/\eta/\zeta$ |
|---|---|---|---|
| 1 (RVRV) | (1,2) | (0.0/0.0/0.0, 0.0/0.0/0.0) | (0.0/0.0/0.1, 0.0/0.1/0.0) |
| 2 (RVLT) | (1,3) | (0.0/0.0/0.0, 0.0/0.0/0.0) | (0.0/0.1/0.0, 0.0/0.1/0.0) |

* RVLT: a revolute joint

* RVRV: a compound joint of revolut and revolute.

Description of Rigid Bodies

| Body # | Description | Mass ($Kg$) | Inertia ($Kg/m^2$) $\xi\xi/\eta\eta/\zeta\zeta$ | Initial position of c.g. $x/y/z$ ($m$) |
|---|---|---|---|---|
| 1 | Main Chassis | 800 | 125/1000/550 | 0/0/0 |
| 2 | Front wheel | 30 | 1.0/5.0/1.0 | 1.305/0.0/-0.349 |
| 3 | Rear wheel | 30 | 1.0/5.0/1.0 | -0.965/0.0/-0.351 |

Tire Characteristics

| | |
|---|---|
| Radius | 0.30 ($m$) |
| Radial stiffness $C_z$ | $1.5\text{x}10^5$ ($N/m$) |
| Longitudinal stiffness $C_s$ | $8.0\text{x}10^4$ ($N/slip$) |
| Cornering stiffness $C_\alpha$ | $7.0\text{x}10^4$ ($N/rad$) |
| Max. friction coeff. $\mu_{max}$ | 0.80 |
| Min. friction coeff. $\mu_{min}$ | 0.60 |

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