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DESIGN SENSITIVITY ANALYSIS FOR MULTIBODY SYSTEMS

다물체 시스템의 민감도 해석

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ABSTRACT

This paper presents a 'mixed' method for performing the sensitivity analysis for multibody dynamics. The mixed method uses both the analytical derivation and the numerical evaluation, in which premitive derivations rely on the analytical process and their associated individual terms are evaluated by the numerical process. Therefore, this method can eliminate difficulty in derivation of the direct differentiation. Furthermore, by using the joint coordinate formulation for the equations of motion, computational efficiency and numerical accuracy are achieved.

1 INTRODUCTION

High speed digital computers allows engineers to simulate the dynamics of a mechanical system under different input conditions and to modify its design prior to actual production. Ultimately, this capability allows the engineer to pursue the optimal design of mechanical systems. Before the era of computer analysis, a manufacturer had to construct and test a series of prototypes, which was not only time-consuming but also costly.

Optimal design of multibody systems is performed based on iterative process of dynamic analysis and senstivity analysis. When large scale multibody systems, such as ground vehicles, are considered to be optimized, a large amount of computation time is required. Furthermore, if the sensitivity analysis is purely based on explicit analytical derivations, developing a sensitivity analysis software for ground vehicles also becomes a major pain. Because of this difficulty, general purpose of optimal design analysis code is not avalable in sensitivity analysis of multibody systems. Therefore, simplicity in programming and computational speed are critical issues in performing a sensitivity analysis.

For the sensitivity analysis, researchers have mostly used one of the following three methods: the adjoint variable method, the direct differentiation method, and the finite difference method. Advantages and drawbacks of each method have been reported over the years.

This paper covers a method for performing sensitivity analysis refered as a 'mixed' method. The mixed method combines some processes of the two existing sensitivity analysis methods (direct differentiation method and finite difference method) in order to overcome the shortcomings in each of the existing methods.

2 EQUATIONS OF MOTION

The equations of motion for a multibody system can be described in terms of different sets of coordinates. One of the most effective coordinate system is the joint coordinate system. In the joint coordinate system, a vector of velocities for body i is defined as $\mathbf{v}_i$, which contains a 3-vector of translational velocities $\dot{\mathbf{r}}_i$ and a 3-vector of angular velocities ω_i. A vector of accelerations for this body is denoted by $\dot{\mathbf{v}}_i$, which contains $\ddot{\mathbf{r}}_i$ and $\dot{\omega}_i$. For a multibody system containing b bodies, the vector of coordinates $\mathbf{q}$, velocities $\mathbf{v}$, and accelerations $\dot{\mathbf{v}}$, contain the elements of $\mathbf{q}_i$, $\mathbf{v}_i$, and $\dot{\mathbf{v}}_i$, respectively, for $i = 1,\cdots, b$.

The relative configurations of two adjacent bodies can be defined by one or more joint coordinates which are equal in number to the number of relative degrees of freedom between these bodies. For a multibody system with open-loops, the vector of joint coordinates is denoted by θ containing all of the joint coordinates and the absolute coordinates of a base body if the base body is not the ground. Therefore, the vector θ has k elements, equal to the number for degrees of freedom of the system. The vector of joint velocities is defined as $\dot{\theta}$. It can be shown that there is a linear transformation between $\dot{\theta}$ and $\mathbf{v}$ as [4]

$$\mathbf{v} = \mathbf{B}\dot{\theta} \tag{1}$$

where $\mathbf{B}$ is a $n \times k$ matrix. The generalized equations of motion for an open-loop multibody system, when the number of selected coordinates is equal to the number of degrees of freedom, can be written as

$$M\ddot{\theta} = f \tag{2}$$

where

$$M = \mathbf{B}^T\mathbf{M}\mathbf{B}$$

$$f = \mathbf{B}^T(\mathbf{g} - \mathbf{M}\dot{\mathbf{B}}\dot{\theta})$$

3 COST SENSITIVITY ANALYSIS

A general dynamic response optimization problem is defined as a process to minimize a cost function ψ which is based on the equations of motion of a given system. The cost function can be written as a function of system state variables and a set of design parameters as

$$\psi = \psi(\mathbf{b}, \theta(\mathbf{b}; t), \dot{\theta}(\mathbf{b}; t), \ddot{\theta}(\mathbf{b}; t)) \tag{3}$$

where θ, $\dot{\theta}$, and $\ddot{\theta}$ are vectors of joint coordinates, velocities, and accelerations, respectively. The design parameter vector $\mathbf{b}$ is assumed to have r elements which may be bounded as $b_i^L \le b_i \le b_i^U$, where b_i^L and b_i^U are the lower and upper bounds of the ith design parameter.

In certain applications, the cost function may be subjected to some constraints. The constraints can also be expressed in the general form of Eq. (3) as function of θ, $\dot{\theta}$, $\ddot{\theta}$, $\mathbf{b}$, and t. Therefore, the term "cost function ψ" refers to both the cost function and the constraints. Furthermore, this study is not concerned with any particular optimization algorithm but is concentrated on methods for evaluating the cost sensitivity.

The cost sensitivity associated with the ith design parameter is described as the rate of a variation of the cost ψ due to the variation of the associated design parameter as

$$\frac{d\psi}{db_i} = \lim_{\Delta b_i \to 0} \frac{\psi(b_i + \Delta b_i) - \psi(b_i)}{\Delta b_i} \tag{4}$$

By using analytical expression, this can be written as

$$\psi_b = \frac{\partial\psi}{\partial\mathbf{b}} + \frac{\partial\psi}{\partial\theta}\theta_b + \frac{\partial\psi}{\partial\dot{\theta}}\dot{\theta}_b + \frac{\partial\psi}{\partial\ddot{\theta}}\ddot{\theta}_b \tag{5}$$

where $\frac{\partial\psi}{\partial b}$, $\frac{\partial\psi}{\partial\theta}$, $\frac{\partial\psi}{\partial\dot{\theta}}$ and $\frac{\partial\psi}{\partial\ddot{\theta}}$ can be easily obtained through symbolic operation, because ψ is generally defined as a simple function of θ, $\dot{\theta}$, $\ddot{\theta}$, and $\mathbf{b}$ in most applications. Since the sensitivity of vectors θ_b and $\dot{\theta}_b$ are obtained by numerically intergration $\dot{\theta}_b$ and $\ddot{\theta}_b$, the focus in the sensi

tivity analysis is on methods for evaluating $\ddot{\theta}_b$.

4 MIXED METHOD

This method uses the sensitivity equations from the formulation of the direct differentiation method, however, the individual terms in the sensitivity equations are obtained by the finite difference method. The method can then reduce the analytical difficulty encountered in the direct differentiation method and can utilize the numerical efficiency in the direct differentiation method.

The joint coordinate formulation produces a minimal number of equations of motion. By using this coordinate system, the size of the senstivity equation can also be reduced. This mixed method, here, uses the joint coordinate formulaton. For an open-loop system, the residual of equations of motion Eq. (2) can be defined as

$$\Gamma = \mathbf{B}^T(\mathbf{M}\mathbf{B}\ddot{\theta} - \mathbf{g} + \mathbf{M}\dot{\mathbf{B}}\dot{\theta}) = \mathbf{0} \tag{6}$$

By differentiating Eq. (6) with the ith design parameter, it can be written as

$$\mathbf{B}^T\mathbf{M}\mathbf{B}\ddot{\theta}_{b_i} = \frac{\partial\Gamma}{\partial b_i} + \frac{\partial\Gamma}{\partial\theta}\theta_{b_i} + \frac{\partial\Gamma}{\partial\dot{\theta}}\dot{\theta}_{b_i} = \Gamma_{b_i} + \Gamma_\theta\theta_{b_i} + \Gamma_{\dot{\theta}}\dot{\theta}_{b_i} \tag{7}$$

In the mixed method, the terms Γ_θ and $\Gamma_{\dot{\theta}}$ are computed by using the finite difference method. The term Γ_{b_i} is obtained numerically or analytically depending on a chosen design parameter. When the terms Γ_θ and $\Gamma_{\dot{\theta}}$ are numerically evaluated each of the state vectors θ and $\dot{\theta}$ are individually perturbed. Hence, the number of evaluations of Γ is the same as the total element number of the state vectors θ and $\dot{\theta}$. This element number of the state vectors is dependent on the degrees of freedom of the given system. The matrices Γ_θ and $\Gamma_{\dot{\theta}}$ are evaluated once, they are used in deriving the sensitivity vectors associated to other design parameters.

In this method, the integrations of the state sensitivity and the nominal state are completely separated. Thus, in the computation of the state of the norminal system need not use the same integration algorithm or the same time step used in the state sensitivity computation. For the case of a crude sensitivity analysis, the computation time can be reduced by using a large time step in the same integration routine or a relatively crude integration algorithm in the integration of the state sensitivity matrices. Furthermore, once the coefficient matrix in the equations of motion is evaluated, the same matrix can be used in each sensitivity equations corresponding to other design parameters.

4.1 Formulation of Γ_θ

The elements in matrix Γ_θ consist of the column vectors Γ_{θ_i}'s corresponding to the ith element of the selected joint coordinates θ. From Eq. (7), each column vector Γ_{θ_i} is written as

$$\Gamma_{\theta_i} = \frac{\partial\Gamma}{\partial\theta_i} = \frac{\partial}{\partial\theta_i}[\mathbf{B}^T(\mathbf{g} - \mathbf{M}\dot{\mathbf{B}}\dot{\theta}) - \mathbf{B}^T\mathbf{M}\mathbf{B}\ddot{\theta}] \tag{8}$$

where $\dot{\theta}$ and $\ddot{\theta}$ are independent from the variation of θ. The operation $\frac{\partial}{\partial\theta}$ is performed by using numerical perturbation.

The value of Γ for the nominal system is zero and for the perturbed system it is the same as the residual of the equations of motion. Hence, the numerical calculation of operation $\frac{\partial}{\partial\theta}$ can be performed by using perturbed θ. The term Γ_{θ_i} can be written as

$$\Gamma_{\theta_i} = \frac{\Gamma_i^*}{\Delta\theta_i} = \mathbf{B}^{*T}(\mathbf{g}^* - \mathbf{M}\dot{\mathbf{B}}^*\dot{\theta}) - \mathbf{B}^{*T}\mathbf{M}\mathbf{B}^*\ddot{\theta} \tag{9}$$

where superscript * is a term corresponding to the perturbed $\theta_i^* = \theta_i + \Delta\theta_i$. The column vector Γ_{θ_i} fills the ith column of the matrix Γ_θ.

4.2 Formulation of $\Gamma_{\dot{\theta}}$

The numerical calculation of this term is performed by using the perturbed $\dot{\theta}$. Note that the term $\dot{\theta}$ only appears in $\mathbf{g}$ and $\dot{\mathbf{B}}$ of Γ. Therefore, the calculation of $\Gamma_{\dot{\theta}}$ is concerned with the terms that are a function of $\dot{\theta}$. These terms can be expressed as

$$\mathbf{c} = \mathbf{g} - \mathbf{M}\dot{\mathbf{B}}\dot{\theta} \tag{10}$$

It can be obtained directly from the calculation of the nominal system. From the same way of obtaining Eq.(9), the term $\Gamma_{\dot{\theta}_i}$ can be written as

$$\Gamma_{\dot{\theta}_i} = \frac{\mathbf{c}_i^* - \mathbf{c}}{\Delta\dot{\theta}_i} = \frac{\mathbf{g}^* - \mathbf{g} - \mathbf{M}(\dot{\mathbf{B}}^*\dot{\theta}^* - \dot{\mathbf{B}}\dot{\theta})}{\Delta\dot{\theta}_i} \tag{11}$$

4.3 Formulation of Γ_b

The most commonly considered design parameters in the design process of multibody systems are inertial properties, spring and damper locations and characteristics, and the locations of the kinematic joint. The inertial properties simply appear as linear factors in the mass matrix $\mathbf{M}$ and the force vector $\mathbf{g}$ (not in the velocity transformation matrix $\mathbf{B}$). The partial derivatives of $\mathbf{f}$ can be written as

$$\frac{\partial\Gamma}{\partial m_i} = \mathbf{B}_{i(\mathbf{r})}^T\left(\begin{bmatrix}0\\0\\-1\end{bmatrix}g_c - \dot{\mathbf{B}}_{i(\mathbf{r})}\dot{\theta} - \mathbf{B}_{i(\mathbf{r})}\ddot{\theta}\right) \tag{12}$$

where the subscripts i and $(\mathbf{r})$ are the ith body and its translational coordinates, respectively, and $\mathbf{B}_{i(\mathbf{r})}$ is the corresponding row block submatrix of the velocity transformation matrix $\mathbf{B}$. The gravitational constant g_c is applied when the given system is in the gravitational field.

Spring and damper coefficients, and their attachment position vectors with respect to a body-fixed coordinate system, are embedded in vector $\mathbf{g}$ in Eq. (6). Then, the partial derivatives of Γ with respect to those design parameters can be written as

$$\frac{\partial\Gamma}{\partial K_{ij}} = [\mathbf{B}_i^T, \mathbf{B}_j^T]\begin{bmatrix}\mathbf{I}\\\tilde{\mathbf{s}}_i\\-\mathbf{I}\\-\tilde{\mathbf{s}}_j\end{bmatrix}(l_{ij} - l_{ij}^0)\mathbf{u}_{ij} \tag{13}$$

$$\frac{\partial\Gamma}{\partial C_{ij}} = [\mathbf{B}_i^T, \mathbf{B}_j^T]\begin{bmatrix}\mathbf{I}\\\tilde{\mathbf{s}}_i\\-\mathbf{I}\\-\tilde{\mathbf{s}}_j\end{bmatrix}\dot{l}_{ij}\mathbf{u}_{ij} \tag{14}$$

$$\frac{\partial\Gamma}{\partial\mathbf{s}'_i} = [\mathbf{B}_i^T, \mathbf{B}_j^T]\left\{\begin{bmatrix}\mathbf{I}\\\tilde{\mathbf{s}}_i\\-\mathbf{I}\\-\tilde{\mathbf{s}}_j\end{bmatrix}\frac{\partial\mathbf{g}^k}{\partial\mathbf{s}'_i} + \begin{bmatrix}\mathbf{0}\\-\tilde{\mathbf{g}}^k\\\mathbf{0}\\\mathbf{0}\end{bmatrix}\right\} \tag{15}$$

where

$$\mathbf{g}^k = K_{ij}(l_{ij} - l_{ij}^0)\mathbf{u}_{ij}, \quad \frac{\partial\mathbf{g}^k}{\partial\mathbf{s}'_i} = -K_{ij}\left[\mathbf{I}\left(1 - \frac{l_{ij}^0}{l_{ij}}\right) + \frac{l_{ij}^0}{l_{ij}}\mathbf{u}_{ij}\mathbf{u}_{ij}^T\right]\mathbf{A}_i \tag{16}$$

The subscript i and j are the two connected bodies by the spring and the damper. The terms l and l_0 are the total length and the undeformed length of the spring or the damper, and $\mathbf{u}$ is the direction of the spring or the damper. The prime (′) stands for a vector in a body fixed coordinate system ($\xi\eta\zeta$).

The position vectors with respect to a body-fixed coordinate system is implicitly embedded in matrices $\mathbf{B}$ and $\dot{\mathbf{B}}$. Since, an analytical derivation of the term Γ_{b_i} can be complicated, the term Γ_{b_i} will be evaluated by numerical perturbation as

$$\frac{\partial\Gamma}{\partial\mathbf{s}'_i} = \left[\frac{\Gamma_\xi^*}{\Delta(\mathbf{s}_i)_\xi}, \frac{\Gamma_\eta^*}{\Delta(\mathbf{s}_i)_\eta}, \frac{\Gamma_\zeta^*}{\Delta(\mathbf{s}_i)_\zeta}\right] \tag{17}$$

4.4 Numerical Example

In order to demonstrate the feasibility of this method, cost sensitivity results of a simple mechanical system are compared against those obtained from the finite difference method in terms of accuracy and also computational efficiency.

The example system is a quarter car with a double A-arm suspension system shown in Figure 1. For the sake of convenience in the simulation process, the chassis is constrained to have motion in the z-direction. Hence, this system has four degrees of freedom. The tire is mounted on the ground subject to a sine wave given by $z_g = A\sin(2\pi f_n t)$. In this model, the sine wave is assigned with an amplitude $A = 0.02\ m$ and a frequency $f_n = 3\ Hz$.

The selected cost function of this system is described as

$$\psi = \int_0^1 \ddot{z}^2 dt \tag{19}$$

where $\ddot{z}$ is the z-directional acceleration of the chassis. The considered design parameters are masses of body 1 and 3, the stiffness and the damping coefficient of the suspension, the radial stiffness of the tire, and the attachment point of the link of joint 5. Hence, the design parameter vector is described as $\mathbf{b}^T = [m_1, m_3, K, D, C_z, s_z^p]$. The unperturbed design parameters are given as [200.0, 5.0, 40.0×10³, 10.0×10³, 1.5×10⁵, 0.0].

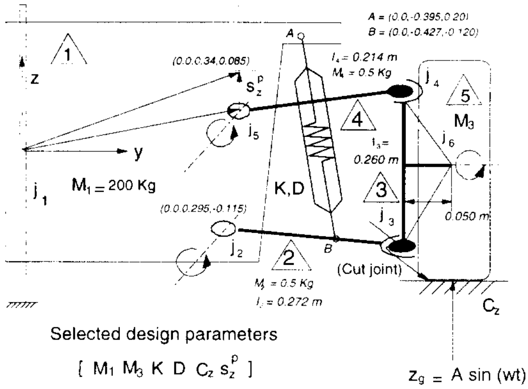

Figure 1 Schematic diagram of a quarter car model.

The simulation of this example uses the Multi-Body System Simulation computer package (MBOSS) [13]. This is a general purpose computer program for the dynamic analysis of multi-body systems which uses the joint coordinate formulation of equations of motion and comprehensive anlytical model for a tire and its interaction with the ground [14-15]. This mixed method for the sensitivity analysis is incorporated with this program.

The cost sensitivity can be obtained analytically from Eq. (19) as

$$\psi_b = \int_0^t 2\ddot{z}\ddot{z}_b dt \qquad (20)$$

where $\ddot{z}$ is obtained from solving equations of motion and $\ddot{z}_b$ is obtained from solving the sensitivity equations. In this method, the Gear integration algorithm is used to solve the nominal system and the Euler integration algorithm is used to solve the state sensitivity and the cost sensitivity.

Table 1 shows the sensitivity results from the finite difference method and the mixed method. It can be seen that the sensitivity results from the mixed method are in good agreement with those from the finite difference method. The CPU times with six design parameters and with single design parameter in Sun sparc 10 are 40 sec and 35 sec, respectively.

Table 1. Comparison of results of the mixed method and the finite difference method (F.D.M.).

	Mixed Method	F. D. M.
$\frac{\partial\psi}{\partial m_1}$	-8.06×10^{-2}	-7.40×10^{-2}
$\frac{\partial\psi}{\partial m_3}$	4.32×10^{-2}	4.72×10^{-2}
$\frac{\partial\psi}{\partial K}$	-4.14×10^{-6}	-4.45×10^{-6}
$\frac{\partial\psi}{\partial D}$	1.55×10^{-3}	1.54×10^{-3}
$\frac{\partial\psi}{\partial C_z}$	-1.00×10^{-5}	-1.09×10^{-5}
$\frac{\partial\psi}{\partial s_z^p}$	23.47	3.51

5 CONCLUSION

One example of the sensitivity analysis is a quarter car with a double A-arm suspension system. The sensitivity analysis is performed using the mixed method. The results are checked through the finite difference method. In this example, overall deviation of the sensitivity results of the two methods is less than *10 percent*. The computation time of the mixed method is largely dependent on the degrees of freedom of the given system, but it is slightly dependent on the number of design parameters as observed in the result. The total computational time is 40 sec which is relatively small compared to the type of problem. It can be concluded that the mixed method based on joint coordinate formulation can not only reduce computational time but also numerical errors. Hence this method has some advantage in the optimal design problem such as a complex and large number of parameter optimization.

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