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복수 압전 가진기의 최적 설계를 통한 판구조물의 소음제어

°김재환*

Noise Control of Plate Structures with Optimal Design of Multiple Piezoelectric Actuators

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ABSTRACT

Noise control of a plate structure with multiple disk shaped piezoelectric actuators is studied. The plate is excited by an acoustic pressure field produced by a noise source located below the plate. Finite element modeling is used for the plate structure that supports a combination of three dimensional solid,flat shell and transition elements. The objective function,in the optimization procedure,is to minimize the sound energy radiated onto a hemispherical surface of given radius and the design parameters are the locations and sizes of the piezoelectric actuators as well as the amplitudes of the voltages applied to them. Automatic mesh generation is addressed as part of the modeling procedure. Numerical results for both resonance and off resonance frequencies show remarkable noise reduction and the optimal locations of the actuators are found to be close to the edges of the plate structure. The optimized result is robust such that when the acoustic pressure pattern is changed,reduction of radiated sound is still maintained. The robustness of an optimally designed structure is also tested by changing the frequency of the noise source using only the actuator voltages as design parameters.

요 약

본 연구에서는 여러개의 원판형 압전소자가 부착된 판구조물의 소음제어를 다루었다. 판재의 아래에는 소음원이 위치하고 이 소음원은 판재를 가진한다. 구조물 및 압전소자는 3차원 요소, 구조물 요소 및 천이요소의 조화로 이루어지는 유한요소로 모델링 되었다. 최적화 과정의 목적함수는 구조물로부터 복사되는 소음 에너지이고 설계변수는 원판형 압전소자의 위치, 크기 및 인가되는 전압이 사용되었다. 최적설계과정에 요구되는 자동격자형성이 언급되었다. 구조물의 공진 및 비공진 주파수에서 최적설계가 행해졌으며 괄목할 만한 소음감소를 얻었다. 이 때에 압전소자의 최적위치는 판재의 모서리에 가깝게 되는 것이 바람직하다. 이 결과는 음향 하중의 형태가 다르게 변하더라도 크게 변하지 않는 것이 밝혀졌다. 또한 한 주파수 뿐 아니라 넓은 주파수 영역에서도 압전가진기의 전압을 조정함으로써 좋은 감소를 얻을 수 있다.

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1. INTRODUCTION

Noise radiated from structures such as automobiles, aircrafts and buildings are bombarding our living environments and much research has been done on methods for combating noise pollution. In controlling noise, there are two distinctive approaches: passive and active methods. Passive approaches are based on designing of material properties or shapes of the structure so as to minimize radiated noise. Exploiting damping layers in the structure is a good example in passive noise control [1, 2].

Smart materials or structures have recently emerged as a very promising technique to reduce the radiated sound field. In such structures, piezoelectric ceramics are widely used as active devices on the structures. Much research has been done analytically and experimentally for piezoelectric active, adaptive or intelligent structures for many applications [3, 4, 5]. However, the design of smart structures for minimal sound radiation is a multi-disciplinary and challenging problem which involves complex models of the structures with active devices such as piezoelectric materials, understanding of structural acoustics, large number of design variables and computationally expensive calculations.

In modeling plate structure with piezoelectric devices, many approaches have been proposed: pure bending deformation model [6], symmetrical piezoelectric thick shells model in finite element analysis [7], a hybrid equivalent single layer theory or layerwise theory [8] and a finite element method by using three-dimensional/transition/shell elements [9]. In this paper, the finite element method proposed in reference 9 is adopted, which includes three dimensional solid elements for modeling piezoelectric actuator and neighboring region, shell elements approximated by many flat-shell elements for plate structure and transition elements to connect the three dimensional solid and shell elements.

Noise distribution on the structure is hard to determine in a realistic situation because the structure interacts with the surrounding acoustic medium. In order to find the real structural response, the structure, the acoustic medium which encloses the noise source and the exterior of the structure with appropriate boundary condition should be included in the analysis. This results in a complicated problem. Even though real acoustic pressure distribution on the structure is not known, if a design of smart structure is robust for many different patterns of acoustic loading, this result might be acceptable for the practicality. Therefore, Kim *et al.* [10] and Varadan *et al.* [11] have considered acoustic pressure loading in a comparative manner and investigated the effects of size, actuator gain and location of piezoelectric actuators on plate structures to reduce the radiated sound using finite element and optimization techniques.

In designing smart structures, many factors affect the performance of the system, for example, the number, sizes, locations and voltages applied to the actuators. Optimal placement of the actuators on the structure has been studied for the last two decades [12, 13, 14]. Clark and Fuller[13] studied the location of a piezoelectric actuator and both the size and location of a PVDF sensor for active structural acoustic control. Wang *et al.* [14] have presented a formulation of the optimization problem for the placement and sizing of piezoelectric actuators in adaptive LMS control systems. The optimal location of piezoelectric actuators for sound radiation control at the on and off-resonance frequencies have been investigated.

Optimal configurations at on and off-resonance frequencies have been found and the result at on-resonance was examined for a band of frequencies up to 200 Hz to verify the robustness of the design [10, 11]. With a disk shaped piezoelectric actuator, it was found that radiation from the first few natural frequencies can be controlled. To control radiation from higher modes, the use of multiple piezoelectric actuators is indicated. However, when several disk shaped piezoelectric devices are considered, the locations of the devices should be movable during the optimization process and the finite element meshes have to be automatically generated. When two disk shaped piezoelectric elements are considered, the entire mesh has to be rebuilt according to the locations of the devices. One could think of using ready-made mesh generation packages. However, the mesh generation program has to be closely linked with the analysis program such that the given data should be conveyed to the mesh generation program, which needs special efforts to link them and consumes much time in computing. Further more, electric boundary conditions on the piezoelectric devices are so complicated that it is difficult to generate these conditions using general mesh generation programs To cope with these features, a program for automatic mesh generation has been developed which is customized for arbitrary locations of piezoelectric devices on flat plate structures.

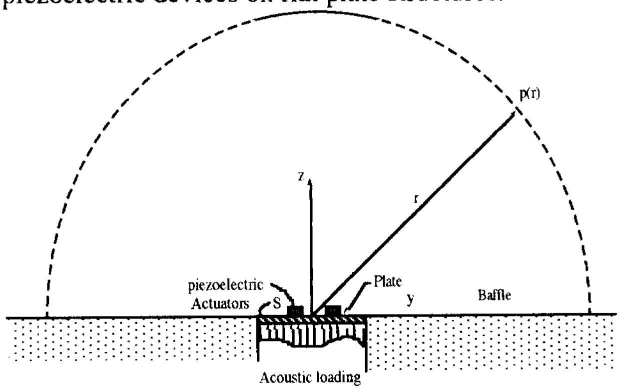

Figure 1. Plate structure with PZT actuator in an infinite baffle.

In this paper, two piezoelectric devices are considered to cover a higher range of frequency in active control of active radiation of sound from a plate structure by changing the size, location and voltage applied to the piezoelectric actuators (Figure 1). A flat plate structure is built into an infinite baffle and a noise field excites the plate from a noise source below the plate. The finite element method is used to model the actuators and the plate structure taking into account all the relevant dynamical fields in the piezoelectric actuators and the structure. The acoustic pressure distribution on the plate due to the noise source is assumed to be a constant or of a given functional form [10, 11]. This distribution is converted into an equivalent nodal force distribution on the structure. The structural response is computed with the acoustic loading on the plate structure and an RF voltage applied to the piezoelectric actuator by using the finite element method. At this stage, the effect of infinite acoustic medium above the plate is assumed to be negligible and traction free boundary conditions are applied. The acoustic field radiated into the exterior region is then computed using a Helmholtz integral representation and the surface nodal values of the displacement and pressure resulting from the finite element method. An optimization technique is used to minimize the radiated sound by means of rearranging the sizes, the locations of the piezoelectric actuators as well as the amplitudes of the voltages applied to the actuators. The cost function to be minimized by the routine is the integrated value of the radiated sound energy on a hemisphere of desired radius whose base includes the structure. After finding an optimal design configuration, several kinds of acoustic field distributions are examined to check the robustness of the optimal results and a bandwidth of the excitation frequency is tested.

2. MODELING AND THEORY

A rectangular plate with piezoelectric actuators is built into an infinite baffle and an acoustic pressure disturbance is applied to the bottom of the plate (Figure 1). Controlling the vibrations of the plate structure by activating the piezoelectric actuators may reduce the sound radiated into the upper half space. In the structure model, three dimensional solid elements are used in the piezoelectric regions and their neighbors because these can take into account the coupling of electric and displacement fields very accurately. Flat shell elements are used in the remaining part of the plate structure using transition elements which connect the shell elements and three dimensional solid elements (Figure 2).

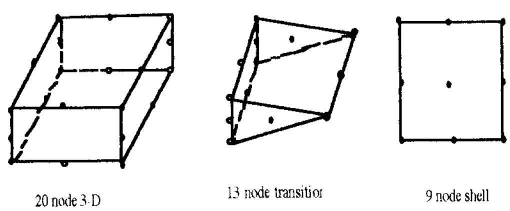

Figure 2. Three dimensional, Transition and Shell finite elements .

2.1 Formulation for the piezoelectric actuators and structure.

To solve the response of the structure, a finite element method is adopted because this method can be used for arbitrary geometry of the model and include the anisotropic properties of the model. Finite element equations of a piezoelectric actuator with structure has already been formulated and can be written as [15, 16]

$$\left(-\omega^2\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{uu} & \mathbf{K}_{u\phi}\\ \mathbf{K}_{u\phi}^T & \mathbf{K}_{\phi\phi}\end{bmatrix}\right)\begin{Bmatrix}\mathbf{U}\\ \mathbf{\Phi}\end{Bmatrix}=\begin{Bmatrix}\mathbf{F}\\ \mathbf{Q}\end{Bmatrix} \quad (1)$$

where $\mathbf{M}$ and $\mathbf{K}_{uu}$ is the mass and stiffness matrices respectively, $\mathbf{K}_{u\varphi}$ is the piezoelectric coupling matrix and $\mathbf{K}_{\varphi\varphi}$ is the dielectric stiffness matrix. $\mathbf{U}$ is the displacement, $\mathbf{\Phi}$ is the electrical potential, $\mathbf{F}$ is point force on the structure and $\mathbf{Q}$ is point charge on the piezoelectric actuator. There is no distinction between piezoelectric medium and structure to apply equation (1) except that the piezoelectric coupling matrix and the dielectric stiffness matrix are zero in the structure. The stiffness and mass matrices of flat shell and transition elements for the structure are already described in the previous work [9]. This work referred to finite element analysis texts and papers [17, 18, 19].

2.2 Acoustic pressure loading.

The actual distribution of acoustic pressure disturbance on a structure is hard to determine. So, different patterns of acoustic load of which have same total input power have been investigated: constant, random, Bessel function of zero order, Bessel function of first order and the sound field distribution produced in an acoustically hard box [13]. The pressure distribution in the acoustically hard box is expressed in terms of a modal superposition and evaluated on the top surface of the box of dimensions 1' x 1' x 1' which has a point source, Q, at the bottom . In applying five different acoustic loading conditions, the same amount of input power is used by changing the parameters that determine the acoustic pressure distributions.

2.3 Automatic mesh generation

In this paper, by moving a mesh template enclosing a piezoelectric device and its vicinity, we have developed an automatic mesh generation program. The plate structure is assumed to be of flat, rectangular shape and two disk shaped piezoelectric elements are considered to be mounted on one side of the plate (Figure 3). Most of the plate structure is meshed by using a 9-node plate element except for the piezoelectric devices and a small neighboring region which are modeled by the element template. The template includes the most complicated part of the mesh with a combination of elements -- 20-node solid, 13-node transition and 9-node plate elements. The size of template is determined by the length BSIDE which is proportional to the radius of piezoelectric disk. Two piezoelectric elements are allowed to move on the flat plate with an assumption that the templates never overlap, so that the template shape is not changed. Under these conditions, the automatic mesh generation algorithm is developed.

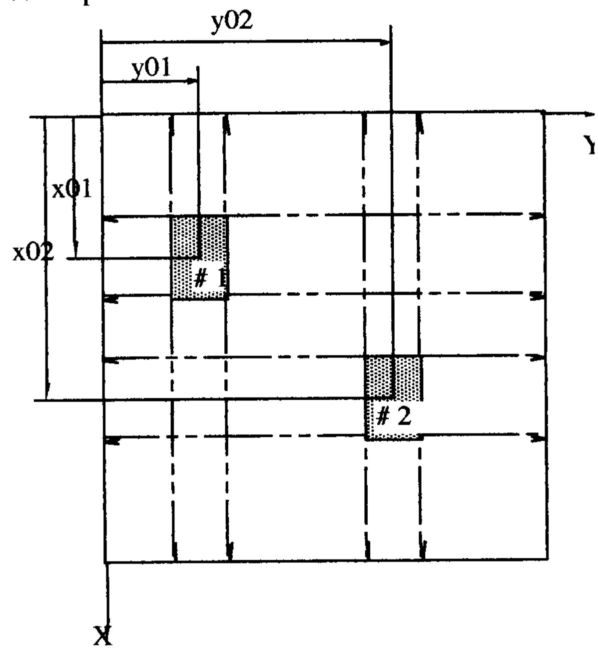

Figure 3. Finite elements for plate structure with two piezoelectric disks.

2.4 Optimization.

The objective function in optimal design is the total radiated sound power,

$$\Psi = min\ W_{tot}(\mathbf{b},\mathbf{s}). \quad (2)$$

where W_{tot} is total radiated sound power, $\mathbf{b}$ is the design variable vector and $\mathbf{s}$ is state variable vector which is needed to describe the system. W_{tot} is written as

$$W_{tot} = \int_0^{2\pi}\int_0^{\pi/2}\frac{|p(r)|^2}{2\rho c}r^2\sin\theta d\theta d\phi \quad (3)$$

where r is the radius of hemispherical surface. This equation is resolved by using an extended Simpson's numerical integration rule for the dual integration on the θ and ϕ variables on a surface of radius, r. The radiated acoustic pressure, $p(r)$ can be expressed by using the normal displacement on the surface S of a vibrating structure as

$$p(r) = -\frac{\rho\omega^2}{4\pi}\int_S u(r_s)\frac{e^{ikR}}{R}ds \qquad (4)$$

where $R = |r - r_s|$, r_s is the location vector on the surface S, k is the wave number in the acoustic medium, ρ is the density of the acoustic medium, $u(r_s)$ is the normal displacement at point r_s on the surface S and ω is the excitation frequency.

When the piezoelectric actuators are used for active noise control, there are many undetermined factors: number, locations, sizes and excitation voltages of the actuators. These can be taken as design variables in the optimization procedure. For simplicity, the number of piezoelectric devices is restricted to two. Thus, a total ten design variables are taken as following,

b1=x01 (x coordinate of first piezo element),
b2=y01 (y coordinate of first piezo element),
b3=x02 (x coordinate of second piezo element),
b4=y02 (y coordinate of second piezoelement),
b5=r1 (radius of first piezo element),
b6=r2 (radius of second piezo element),
b7=t1 (thickness of first piezo element),
b8=t1 (thickness of first piezo element),
b9=ψ1 (amplitude of applied voltage on the first piezo element),
b10=ψ2 (amplitude of applied voltage on the second piezo element).

Design variables are automatically changed to achieve the goal and these variables should be restricted in some manner to be practical. For this, inequality conditions are used such as:

$$L_i \le b_i \le U_i \qquad (5)$$

where b_i is i-th design variable, U_i ,L_i are lower and upper bounds of i-th design variable respectively. Above inequality conditions can be expressed as two inequality constraints

$$g_{(i-1)*2+1} = b_i - L_i, \qquad g_{i*2} = U_i - b_i. \quad (6)$$

Another inequality constraint is that the templates for the piezoelectric devices do not overlap. Therefore, a total of 21 inequality constraints are involved.

Outline of the optimal design procedure is following. With the constant acoustic pressure loading and activated piezoelectric actuators, the response of the structure is found using the finite element method. From this response, the sound radiated from the structure and the total radiated sound power is calculated. Optimization is performed until the result has converged yielding the required value of the cost function. For the constrained optimization problem, an optimization program named by PCON [20] is used. The finite element analysis program is directly merged with the optimization program.

3. NUMERICAL EXAMPLES

An aluminum square plate in an infinite baffle is considered (Figure 1). The size of Aluminum plate is 1' x 1' and the thickness is 0.8 mm. The material properties of PZT-5 (Lead Zirconate Titanate - 5) are used for the piezoelectric actuators. Clamped boundary conditions are used for the outside edges of the plate. To model the plate, shell and three dimensional solid elements with transition element which can connect shell and solid regions are adopted.

3.1 Automatic mesh generation.

To allow a variation of the locations of piezoelectric devices, an automatic mesh generation program is developed and included in the finite element program. When the locations, radii and thicknesses of piezoelectric devices with the size ratio of template versus radius of piezoelectric devices are given, nodal coordinates and element connectivity are generated automatically.

3.2 Optimal design at f=270Hz.

In the previous studies [13, 14], it is found that optimal design with one piezoelectric device gives good results within a limited frequency, 200 Hz. To cover higher frequency, an optimal design with two piezoelectric actuators is proposed and at the fourth resonance frequency, 270 Hz, the design is tried. Design variables are bounded as following:

50 mm < b1, b2, b3, b4 < 250 mm
5 mm < b5, b6 < 15 mm
0.5 mm < b7, b8 < 1.5 mm

-150 V < b9, b10 < 150 V.
The applied voltages on the piezoelectric actuators is out-of-phase with respect to the noise source which means negative gain in feedback. In the optimal design, a constant pressure load (p=2 Pa) is assumed first and after finding an optimal configuration, other loading conditions are examined with the optimal result.

Table 1. Initial and optimal result at f=270Hz.

design vars	initial	optimal result
b1 (x01)	90 mm	75.47 mm
b2 (y01)	90 mm	82.92 mm
b3 (x02)	210 mm	204.9 mm
b4 (y02)	210 mm	206.1 mm
b5 (r1)	10 mm	9.51 mm
b6 (r2)	10 mm	9.57 mm
b7 (t1)	1 mm	1.0 mm
b8 (t2)	1 mm	1.24 mm
b9 (ψ1)	free	39.25 V
b10 (ψ2)	free	4.94 V
object fcn (dB)	0.2651xE-6 (34.76 dB)	0.9108xE-9 (10.1 dB)

Table 1 shows the initial values of the design parameters and their optimized values. In the initial guess, the electrodes of the actuators are set to free electrodes, in other words, the system is presumed to be passive.

It is well known fact that in the sound radiation of flat plate structure, corners or edges contribute significantly to the radiation [21]. Therefore, from the initial design stage, it is recommendable to reduce the four peripheral peaks rather than the central peak of the plate for minimizing the radiated sound energy. The optimal result in Table 1 proven this fact. In the optimal design result, the sizes of the PZT actuators are not changed very much. Another encouraging result is that the amplitudes of the voltages needed for the PZT actuators are not so high and it still minimizes the radiated sound. By optimally designing the PZT actuators, the total radiated power at an on-resonance frequency is reduced by 25 dB.

To check the robustness of the optimized result, four different loading conditions previously mentioned are examined with the optimized configuration (Table 2). In the constant loading case, 25 dB reduction is achieved while in the random and Bessel function cases, 24-26 dB reduction is observed. In the acoustically hard box case, the reduction same as the constant loading case. The reason why more reduction is obtained for the acoustically hard box case rather than the random and Bessel function cases is that the excitation frequency, 270 Hz is close to the natural frequency of fourth mode in the plate and the acoustic pressure distribution in the hard box is described as a trigonometric function of the frequency [22]. The close coupling between the structural vibrations and the radiated acoustical field results in the reduction of noise by structural vibration control. In the optimal design result, the minimum reduction that we can get is 25 dB.

Table 2. Effect of different acoustic loading conditions on the optimal result.

Loading condition	Total Radiated Power
A (Constant)	0.9108xE-9 (10.10 dB)
B (Random)	0.9596xE-9 (10.34 dB)
C (Bessel 0)	0.9558xE-9 (10.32 dB)
D (Bessel 1)	0.7178xE-9 (9.08 dB)
E (Hard Box)	0.6934.xE-9 (8.92 dB)

3.3 Optimal design at f=300 Hz.

Another attempt has been made to see the feasibility of the radiated sound reduction at off-resonance frequency which is known being difficult to suppress radiated sound from the structure. The excitation is tried at 300 Hz. Table 3 shows the initial and optimal results.

Table 3. Optimal design result at f=300 Hz.

design vars	initial design	optimal result
b1 (x01)	90 mm	65.75 mm
b2 (y01)	90 mm	59.28 mm
b3 (x02)	210 mm	220.4 mm
b4 (y02)	210 mm	242.6 mm
b5 (r1)	10 mm	11.19 mm
b6 (r2)	10 mm	11.76 mm
b7 (t1)	1 mm	1.10 mm
b8 (t2)	1 mm	1129 mm
b9 ($|\phi|$1)	free	66.82 V
b10 ($|\phi|$2)	free	19.1 V
object fcn. (dB)	0.1403E-5 (41.98 dB)	0.9253E-9 (10.18 dB)

The radii and thicknesses of piezoelectric actuators in the optimal result show a slight increase. As mentioned previously regarding optimal locations of the piezoelectric actuators, when the excitation frequency is off-resonance, 300 Hz, same philosophy in reducing radiated sound energy can be applied. The locations of the actuators moved toward outside. In off-resonance case, more than 31 dB reduction in radiated sound power is achieved.

3.4 Robustness of the design configuration in a bandwidth of excitation frequencies.

The result of optimal design gives a fixed configuration: the size and location of the piezoelectric actuators can not be adjustable. And the noise sound source, in general, is not a single frequency rather it has multiple frequency contents. Therefore, another attempt for checking the robustness of the optimal result in different frequencies has been made by changing the excitation frequency from 200 to 400 Hz with a chosen optimal configuration (the optimal result in Table 1 when f=270Hz was selected). The actuator voltages are optimally adjusted on each frequency to see the frequency bandwidth in which the radiated sound can be reduced with two actuators. Figure 4 represents the difference between the results when the optimized actuator voltages are applied and the results without excitation at different excitation frequencies.

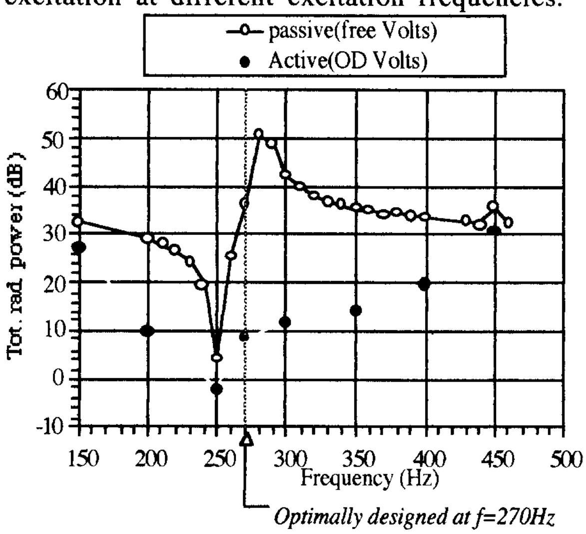

Figure 4. Robustness of optimal design result at different frequencies.

With the configuration optimally designed at f=270 Hz, the radiated noise can be reduced 30 to 10 dB in the frequency range of 200 to 400 Hz. In conclusion, when the frequency is changed, by adjusting the applied voltages, significant reduction in the radiated sound output can be achieved.

4. CONCLUSIONS

Optimal design of the sizes, locations and applied voltages of two disk shaped piezoelectric actuators on a flat plate structure is studied for reducing the radiated sound. The finite element program that supports a combination of three dimensional solid, shell and transition elements in the plate structure is developed. Locations, thicknesses and radii, of the piezoelectric actuators as well as the amplitudes of the voltages applied to them are taken as design variables in the optimization procedure. To support a variation of the locations of piezoelectric actuators on flat plate, an automatic mesh and boundary condition generation program is developed based upon an idea of moving mesh templates for complicated regions near piezoelectric actuators. The objective function is to minimize the total sound power radiated from the structure. At both of on/off-resonance frequencies, significant reductions are observed. Optimal locations of the piezoelectric actuators tend to be near the corners of the plate rather than the center of the plate. Also, to find the validity of the design result at a bandwidth of frequency, the excitation frequency is varied and it is observed that by adjusting the applied voltages on the actuators, reduction of the radiated sound from the structure is still possible. Due to the difficulty of finding the true acoustic pressure loading in structural acoustic problems, the load distribution is assumed to be a constant pressure and to verify the validity of the result, several different patterns of pressure loading conditions are examined. From this procedure, a little bit degradation of the objective function is observed but the results are still very acceptable.

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