이동 통신 채널에서 직교 주파수 분할 다중 시스템의 성능 연구

김 윤 희, 송 익 호, 김 상 우, 방 영 조
한국과학기술원 전기 및 전자공학과

On the Performance of an Orthogonal Frequency Division Multiplexing System in a Mobile Radio Channel

Yun Hee Kim, Iickho Song, Sang Wu Kim, and Young Jo Bang
Dept. of EE, KAIST

Abstract

In this paper, we first analyze the influence of interference due to the time variation and delay spread of the mobile channel on an orthogonal frequency division multiplexing (OFDM) system. With the result, we obtain the bit error rate performance of the 16-QAM OFDM system. Second, we investigate the performance of the Reed-Solomon (RS) coded 16-QAM OFDM system when the number of subcarriers varies. In the investigation, we assume that the information transmission rate and the total bandwidth expansion due to coding, guard interval, and the number of subcarriers are fixed. Under this condition, it is observed that there are optimum numbers of subcarriers that minimize the post decoding symbol error probability of RS code for various channel states.

1 Introduction

In current and future mobile communications systems, high bit rate data transmission is required for many services such as video, high quality audio, and mobile ISDN. Many mobile radio channels are characterized as time-varying multipath Rayleigh fading channels. When high bit rate data are transmitted over such a channel, the channel impulse response can extend over many symbol periods and this leads to intersymbol interference (ISI).

OFDM is an effective techinique to mitigate ISI. The main idea is to send data in parallel over a number of subchannels which are spectrally overlapping yet orthogonal in time. Several aspects of potential systems using OFDM have been studied in [1]-[6]. The method to reduce the ISI in OFDM is to increase the number of subcarriers: The ISI can be also effectively eliminated by adding a guard interval. However, the time variation of the channel results in more interchannel interference (ICI) caused by a loss of orthogonality among the subchannels as the number of subcarriers increase. The ICI of OFDM system in a time-varying multipath fading channel was analyzed under the assumption that the guard interval eliminates the ISI perfectly [5].

In this paper, we first analyze the OFDM system when the guard interval does not eliminate the ISI perfectly in time-varying multipath Rayleigh fading channel. We use the same discrete-time OFDM model as in [5]. With the result, the performance of the 16-QAM OFDM is analyzed.

Second, we investigate the tradeoff between the code rate and the bandwidth expansion due to the guard interval with the number of subcarriers for the RS coded 16-QAM OFDM system. In the investigation, the available transmission bandwidth and information transmission rate are fixed. Under this condition, we compare the post decoding symbol error probability (SER) of RS code as the number of subcarriers varies. It is shown there exist optimum numbers of subcarriers for various channel states.

The discrete time lowpass equivalent OFDM system model and channel model is defined in Section 2. In Section 3, the performance of the OFDM system is analyzed. Then with this result, the performance of the RS coded 16-QAM OFDM system is investigated for various numbers of subcarries in Section 4. We make some concluding remarks in Section 5.

2 System Model

The lowpass equivalent discrete-time complex-valued model considered in this paper is depicted in Fig. 1.

2.1 Transmitter

Input data streams are coded at the code rate r and mapped to one of the symbols by the symbol generator. The symbol generator outputs the complex symbol whose duration is T. The serial-to-parallel converter buffers a block of symbols for the N-point IDFT to modulate them onto the N subchannels with orthogonal subcarriers in baseband. The set $\{a_i(n),\ 0 \leq n \leq N-1\}$ is the set of symbols to be modulated in the i-th block interval. These

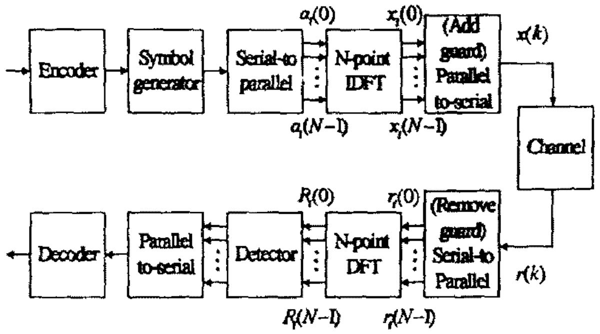

Fig. 1: The lowpass equivalent discrete-time system model

symbols are assumed to be independent and identically distributed (i.i.d.).

To reduce the ISI between the blocks, the guard interval of length G can be appended to the beginning of the block [7]. The transmitted sequence with guard interval of the i-th block is written as

$$x_i^g(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a_i(n) e^{j\frac{2\pi nk}{N}}, \quad -G \le k \le N-1, \tag{1}$$

which corresponds to the samples kT_s of an OFDM modulated signal, where $(N+G)T_s = NT$. The first G elements of (1) constitute the guard samples. When guard samples are added, the sampling rate should be increased by the factor of $\frac{(N+G)}{N}$ to transmit $(N+G)$ samples during one block period. Therefore, the bandwidth of the system increases to $\frac{1}{T_s} = \left(\frac{N+G}{N}\right)\frac{1}{T}$. If $x_i^g(k)$ is assumed zero for $k < -G$ and $k \ge N$, the total transmitted (baseband) sequence may be written as

$$x(k) = \sum_{i=-\infty}^{\infty} x_i^g(k - i(N+G)). \tag{2}$$

Since coding at the code rate r expands a system bandwidth by $1/r$, the total bandwidth expansion ratio is

$$F = \frac{1}{r}\left(1 + \frac{G}{N}\right). \tag{3}$$

We will fix the value of F to some constant so that the information transmission rate and the bandwidth of the system may not vary for different N values. Therefore, as N increases, r decreases and the error correction capability of code increases.

2.2 Channel Model

The channel model used in this system is a time-varying diffuse multipath model. This model can be represented as a tapped delay line model with time varying coefficients and fixed tap spacings T_s, if the transmitted signal is band-limited as $1/T_s$ [7]. If the multipath delay spread T_m does not exceed MT_s, the channel output with additive white Gaussian noise (AWGN) is written as

$$r(k) = \sum_{m=0}^{M-1} h_m(k) x(k-m) + n(k). \tag{4}$$

The channel is assumed as a wide-sense stationary uncorrelated scattering (WSSUS), Rayleigh fading channel.

2.3 Receiver

We assume that the channel impulse response does not extend over more than one block, i.e., $M < N+G$. The multipath delay spread T_m can be larger than the guard interval $T_G = GT_s$, then the i-th block can have interference from the $(i-1)$-th block. Here, we assume that $0 \le G \le M-1$: if $G = 0$, no guard interval is appended, and if $G = M-1$, the guard interval eliminates the ISI from the adjacent block perfectly. After removing the guard samples, the received sequence is written as

$$r_i(k) = \begin{cases} \sum_{m=0}^{k+G} h_{m,i}(k) x_i^g(k-m) \\ \quad + \sum_{m=k+G+1}^{M-1} h_{m,i}(k) x_{i-1}^g(N+G-m+k) + n_i(k) \\ \qquad\qquad \text{if } 0 \le k \le M-G-1 \\ \sum_{m=0}^{M-1} h_{m,i}(k) x_i^g(k-m) + n_i(k) \\ \qquad\qquad \text{if } M-G \le k \le N-1, \end{cases} \tag{5}$$

where $h_{m,i}(k) = h_m(i(N+G)+k)$.

The demodulator performs DFT with $\{r_i(k), 0 \le k \le N-1\}$ and the output of DFT becomes

$$\begin{aligned} R_i(l) = {} & \frac{1}{N} \sum_{n=0}^{N-1} a_i(n) \left[\sum_{k=0}^{M-G-1} \sum_{m=0}^{k+G} h_{m,i}(k) e^{-j\frac{2\pi mn}{N}} e^{j\frac{2\pi k(n-l)}{N}} \right. \\ & \left. + \sum_{k=M-G}^{N-1} \sum_{m=0}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi mn}{N}} e^{j\frac{2\pi k(n-l)}{N}} \right] \\ & + \frac{1}{N} \sum_{n=0}^{N-1} a_{i-1}(n) \sum_{k=0}^{M-G-1} \sum_{m=k+G+1}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi n(G-m)}{N}} e^{j\frac{2\pi k(n-l)}{N}} \\ & + \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} n_i(k) e^{-j\frac{2\pi kl}{N}}, \qquad 0 \le l \le N-1. \end{aligned} \tag{6}$$

Here, $R_i(l)$ represents the transmitted signal through the l-th subchannel (l-th subcarrier) during the i-th block interval. From $R_i(l)$, the symbol detector makes a decision about which symbol is transmitted, then the detector ouputs are decoded through the decoder.

3 Performance Analysis of the OFDM System

The DFT output (6) can be separated into the signal part with multiplicative noise, interferences, and AWGN as

$$R_i(l) = H_i(l) a_i(l) + C_i(l) + S_i(l) + N_i(l), \quad 0 \le l \le N-1, \tag{7}$$

where

$$H_i(l) = \frac{1}{N} \left[\sum_{k=M-G}^{N-1} \sum_{m=0}^{M-1} h_{m,i}(k) e^{-j\frac{2\pi lm}{N}} \right.$$

$$+\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G} h_{m,i}(k)e^{-j\frac{2\pi lm}{N}}\Bigg], \tag{8}$$

$$C_i(l)=\frac{1}{N}\sum_{n\neq l} a_i(n)\Bigg[\sum_{k=M-G}^{N-1}\sum_{m=0}^{M-1} h_{m,i}(k)e^{-j\frac{2\pi lm}{N}}e^{j\frac{2\pi k(n-l)}{N}}$$
$$+\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G} h_{m,i}(k)e^{-j\frac{2\pi lm}{N}}e^{j\frac{2\pi k(n-l)}{N}}\Bigg], \tag{9}$$

$$S_i(l)=\frac{1}{N}\sum_{n=0}^{N-1} a_{i-1}(n)\sum_{k=0}^{M-G-1}\sum_{m=k+G+1}^{M-1} h_{m,i}(k)e^{-j\frac{2\pi n(G-m)}{N}}e^{j\frac{2\pi k(n-l)}{N}}, \tag{10}$$

and

$$N_i(l) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} n_i(k)e^{-j\frac{2\pi kl}{N}}, \quad \text{for } 0\le l\le N-1. \tag{11}$$

In (8)-(11), $H_i(l)$ is the fading term on the l-th subchannel, and $N_i(l)$ is AWGN of the l-th subchannel. The term $C_i(l)$ is the interference from the other subchannels of the same block and $S_i(l)$ is the interference from the adjacent block. We will call $C_i(l)$ and $S_i(l)$ the ICI and ISI, respectively.

With WSSUS Rayleigh fading assumptions, the tap coefficients $h_m(k)$ are independent zero mean complex Gaussian random processes. Therefore, the fading term $H_i(l)$ becomes a zero mean complex Gaussian random variable, i.e., each subchannel is a Rayleigh fading channel. The noise $N_i(l)$ is also white Gaussian noise because it is the sum of white Gaussian variables. The random variables $C_i(l)$ and $S_i(l)$ are not easy to analyze exactly because they are sums of many interferences which are not independent. Interferences are, however, at least uncorrelated and the mean of $C_i(l)$ and $S_i(l)$ is zero, because random symbols $a_i(n)$ are i.i.d. and have zero mean with variance E_s. We will thus model $C_i(l)$ and $S_i(l)$ as Gaussian random variables to obtain the performance bound : of all additive noise processes with fixed variance and mean, independent Gaussian noise results in the smallest capacity. With WSSUS and isotropic scattering assumption, the crosscorrelation of channel tap coefficients is

$$E\{h_{m,i}(k)h^*_{v,i}(u)\} = \delta_{mv}\sigma_m^2 J_0(2\pi f_D T_s(k-u)), \tag{12}$$

where $J_0(\cdot)$ is the zero order Bessel function.

The variance of $C_i(l)$ can thus be derived as

$$\sigma_C^2 = E\{C_i(l)C_i^*(l)\}$$
$$=\frac{E_s}{N^2}\sum_{n\neq l}\Bigg[\sum_{k,u=M-G}^{N-1}\sum_{m=0}^{M-1}\sigma_m^2 J_0(2\pi f_D T_s(k-u))e^{j\frac{2\pi(k-u)(n-l)}{N}}$$
$$+\sum_{u=M-G}^{N-1}\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G}\sigma_m^2 J_0(2\pi f_D T_s(k-u))e^{j\frac{2\pi(k-u)(n-l)}{N}}$$
$$+\sum_{u=M-G}^{N-1}\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G}\sigma_m^2 J_0(2\pi f_D T_s(k-u))e^{-j\frac{2\pi(k-u)(n-l)}{N}}$$
$$+\sum_{k,u=0}^{M-G-1}\sum_{m=0}^{\min(u,k)+G}\sigma_m^2 J_0(2\pi f_D T_s(k-u))e^{-j\frac{2\pi(k-u)(n-l)}{N}}\Bigg]$$

$$=\frac{E_s}{N^2}\Bigg[N(N-M-G)\sum_{m=0}^{M-1}\sigma_m^2 + N\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G}\sigma_m^2$$
$$-\sum_{k=M-G}^{N-1}\sum_{u=M-G}^{N-1}\sum_{m=0}^{M-1}\sigma_m^2 J_0(2\pi f_D T_s(k-u))$$
$$-2\sum_{u=M-G}^{N-1}\sum_{k=0}^{M-G-1}\sum_{m=0}^{k+G}\sigma_m^2 J_0(2\pi f_D T_s(k-u))$$
$$-\sum_{k=0}^{M-G-1}\sum_{u=0}^{M-G-1}\sum_{m=0}^{\min(u,k)+G}\sigma_m^2 J_0(2\pi f_D T_s(k-u))\Bigg], \tag{13}$$

where we use the property

$$\sum_{\substack{n=0\\ n\neq l}}^{N-1} e^{-j\frac{2\pi(k-u)(n-l)}{N}} = N\delta_{ku}-1. \tag{14}$$

The variance of $H_i(l)$ and $S_i(l)$ can similarly be calculated and are written as

$$\sigma_S^2 = \frac{E_s}{N}\sum_{k=0}^{M-G-1}\sum_{m=k+G+1}^{M-1}\sigma_m^2, \tag{15}$$

and

$$\sigma_H^2 = \frac{1}{N^2}\Bigg[\sum_{k,u=M-G}^{N-1}\sum_{m=0}^{M-1}\sigma_m^2 J_0(2\pi f_D T_s(k-u))$$
$$+2\sum_{k=0}^{M-G-1}\sum_{u=M-G}^{N-1}\sum_{m=0}^{k+G}\sigma_m^2 J_0(2\pi f_D T_s(u-k))$$
$$+\sum_{k,u=0}^{M-G-1}\sum_{m=0}^{\min(k,u)+G}\sigma_m^2 J_0(2\pi f_D T_s(u-k))\Bigg]. \tag{16}$$

The variance of AWGN does not change after DFT and is given by

$$E\{N_i(l)N_i^*(l)\} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{u=0}^{N-1}E\{n_i(k)n_i^*(u)\} = N_o, \tag{17}$$

where N_o is the one sided power spectral density of AWGN.

If we assume that $H_i(l)$ is estimated perfectly, SER or bit error probability (BER) of several modulation schemes can be derived with the average SNR which is written as

$$\bar{\gamma} = \frac{E_s\sigma_H^2}{\sigma_C^2+\sigma_S^2+N_o}. \tag{18}$$

The average SNR without AWGN is plotted in Fig. 2 when N=64. We assume that the multipath intensity profile has the exponential profile that well characterizes a mobile radio channel [8]. Therefore, the variance of channel tap coefficients can be written as $\sigma_m^2 = A\exp(-mT_s/\tau_0)$, for $0\le m\le M-1$, where A is the constant that normalizes the sum of variances such that $\sum_{m=0}^{M-1}\sigma_m^2 = 1$. It is assumed in these figures that T_s=5 μs, the mean delay spread τ_0 is T_s, and the multipath delay spread T_m is $5T_s$ (25 μs). When the guard interval is not added, i.e.,

T_G =0 s ($G = 0$), the average SNR is very small due to the ISI. As we increases the guard interval, the average SNR increases. When $T_G = T_m$=25 μs ($G = 5$), the ISI from the adjacent block is eliminated completely and only ICI remains. In such a case, the average SNR decreases rapidly with the maximum Doppler frequency.

Using the average SNR, let us investigate the performance of the 16-QAM OFDM system (the symbol generator generates 16-QAM symbols). The BER averaged over Rayleigh fading [7] for gray-mapped 16-QAM is

$$P_{b,Q} = \int_0^\infty \left[\frac{3}{4}Q\left(\sqrt{\frac{\gamma}{5}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{9\gamma}{5}}\right) - \frac{1}{4}Q\left(\sqrt{5\gamma}\right)\right]\frac{1}{\bar\gamma}e^{-\frac{\gamma}{\bar\gamma}}d\gamma$$
$$= \frac{3}{8}\left(1-\sqrt{\frac{\bar\gamma}{\bar\gamma+10}}\right) + \frac{1}{4}\left(1-\sqrt{\frac{9\bar\gamma}{9\bar\gamma+10}}\right)$$
$$-\frac{1}{8}\left(1-\sqrt{\frac{5\bar\gamma}{5\bar\gamma+2}}\right). \quad (19)$$

Some results of the BER for the 16-QAM OFDM system are shown in Figs. 3 - 6, for various N and channel states. Two extreme cases, when $T_G = 0$ and $T_G = T_m$, are examined. It is assumed in these figures that T_s=5 μs and T_m= 25 μs. To accelerate the simulation, FFT and IFFT are used instead of DFT and IDFT.

Figs. 3 and 4 show the BER of the 16-QAM OFDM system *without guard interval*, $T_G = 0$, for various N when $E_b/N_o = \infty$ and E_b/N_o =15 dB, respectively. The results show that the analytically found BER upper-bounds the BER from simulation, and both of them show similar performance tendency. As N increases, the ISI decreases, however, the ICI due to the time variation of the channel increases. Therefore, we can see there are optimum numbers of subcarriers which minimize the BER of the system.

Figs. 5 and 6 show the BER of the 16-QAM OFDM *with guard interval* , $T_G = T_m$, for various N when $E_b/N_o = \infty$ and E_b/N_o=15 dB. As N increases, the peformance becomes worse, it should be noted, however, that the required bandwidth decreases as N increases, which can be used for error correction coding to compensate for the performance degradatin. In next section, we will compare the performance after decoding for various N.

4 Performance of the RS Coded 16-QAM OFDM System

For the (n, k) RS code, RS SER after decoding is given by [7]

$$P_{d,R} = \sum_{i=\lfloor\frac{n-k}{2}\rfloor+1}^{n} P_R^i(1-P_R)^{n-i}, \quad (20)$$

where P_R is the RS SER before decoding. If we use the RS code of 255 block size, one RS symbol is mapped to two 16-QAM symbols. If we assume that 16-QAM symbols are perfectly interleaved, the RS SER before decoding is written as

$$P_R = 1-(1-P_{s,Q})^2, \quad (21)$$

where $P_{s,Q}$ is the 16-QAM SER and

$$P_{s,Q} \approx 4P_{b,Q} \quad (22)$$

for gray mapped 16-QAM symbols.

When $F = 2$ and E_b/N_o= 15dB, $P_{d,R}$ is plotted for various N and guard interval at some Doppler frequencies in Figs. 7 and 8. We can thus see that there exist optimum numbers of subcarriers and guard interval that minimize the RS SER for fixed total available bandwidth. An observation from these figures is that, when the total available bandwidth is fixed, we should give first priority to guard interval. The number of subcarriers effects a great change of the performace when Doppler frequency is high. A careful choice of the number of subcarriers is thus demanded for high Doppler frequency.

5 Conclusion

In this paper, we first analyzed the performance of OFDM system in time-varying multipath Rayleigh fading channel. With the result, we compared the performance of the 16-QAM OFDM system when the number of subcarriers varies. In the no guard interval case, the ISI decreased and the ICI increased as the number of subcarriers increased. It was observed that there were optimum numbers of subcarriers for various channel states. In the case of perfect ISI elimination with the guard interval, the performance became worse as the number of subcarriers increased. However, the bandwidth expansion due to the guard interval was small for large number of subcarriers.

Second, we investigated the performance of the RS coded 16-QAM OFDM system when the number of subcarriers varied. In the investigation, we assumed that the information transmission rate and the total bandwidth expansion due to coding, guard interval, and the number of subcarriers were fixed. Under this condition, it was observed there were optimum numbers of subcarriers for various channel states. As the time variation of a channel increases, a proper choice of the number of subcarriers is demanded.

References

[1] L. C. Cimini, Jr., "Analysis and Simulation of Digital Mobile Channel", *IEEE Tr. Comm.*, Vol. COM-33, pp. 665-675, July 1985.

[2] J. A. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come", *IEEE Comm. Mag.*, Vol. 28, pp. 5-14, May 1990.

[3] M. Okada, S. Hara, and N. Morinaga, "Bit Error Rate Performances of Orthogonal Multicarrier Modulation Radio Transmission Systems", *IEICE Tr. Comm.*, Vol. E76-B, pp. 113-119, Feb. 1993.

[4] G. Santella, "Bit Error Rate Performances of M-QAM Orthogonal Multicarrier Modulation in Presence of Time-Selective Multipath Fading", *Proc. IEEE Int. Conf. Comm.*, Vol. 3, pp. 1683-1688, Seattle, USA, June 1995.

[5] M. Russell and G. L. Stuber, "Interchannel Interference Analysis of OFDM in a Mobile Environment", *Proc. IEEE Vehic. Techn. Conf.*, Vol. 2, pp. 820-824, Chicago, USA, July 1995.

[6] M. Gudmunson and P. Anderson, "Adjacent Channel Interference Analysis in an OFDM System", *Proc. IEEE Vehic. Techn. Conf.*, Vol. 2, pp. 918-922, Atlanta, USA, April 1996.

[7] J. G. Proakis, *Digital Communications*, McGraw-Hill, 1995.

[8] M. C. Jeruchim, P. Balaban, and K. S. Shanmugan, *Simulation of Communication Systems*, Plenum Press, 1992.

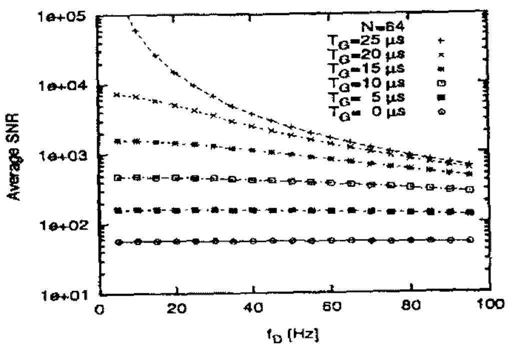

Fig. 2: Average SNR versus f_D, where N=64

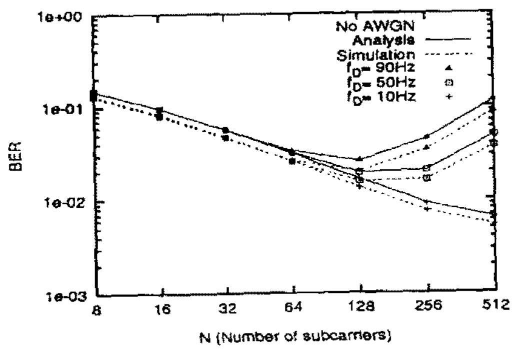

Fig. 3: BER versus N, where T_G= 0 s and $E_b/N_o = \infty$

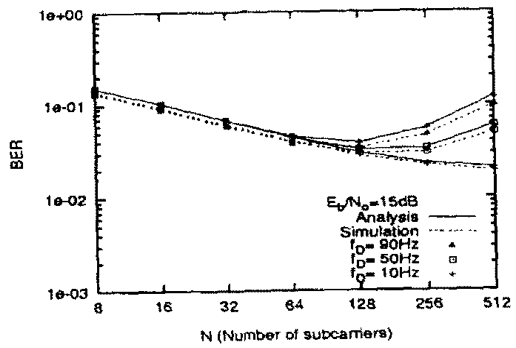

Fig. 4: BER versus N, where T_G= 0 s and E_b/N_o=15 dB

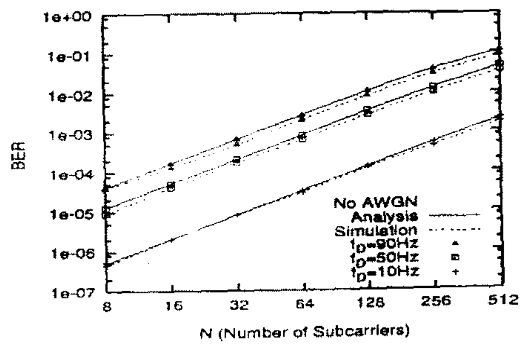

Fig. 5: BER versus N, where T_G= 25 μs and $E_b/N_o = \infty$

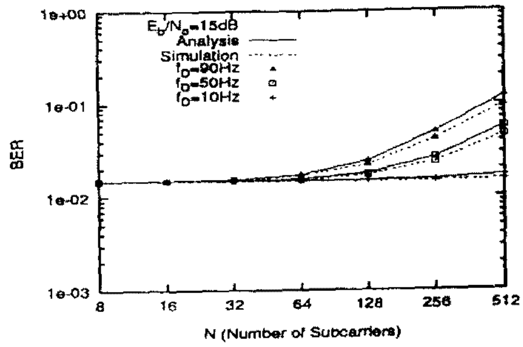

Fig. 6: BER versus N, where T_G= 25 μs and E_b/N_o =15 dB

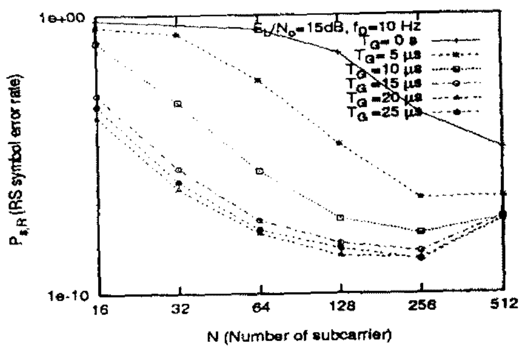

Fig. 7: RS SER versus N, where f_D=10 Hz

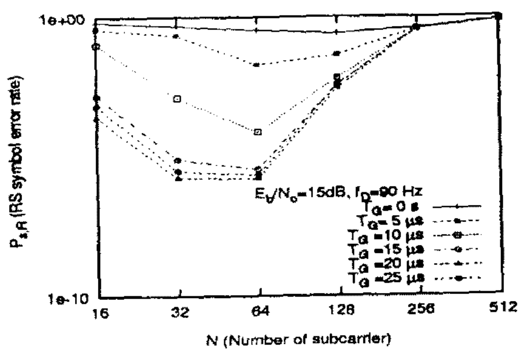

Fig. 8: RS SER versus N, where f_D=90 Hz