몇가지 비모수 검파기의 성능 비교

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Performance Comparisons of some nonparametric detectors

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Abstract

In this paper, we propose a new detector based on the median-shift sign. We call it the ***median-shift sign (MSS)*** detector, which is an extension of the classical sign detector. We first analyze the problem of detecting a dc signal in noise of known probability density function (pdf). The MSS detector with the optimum median-shift value, the optimum MSS detector, performs better than the sign detector in Gaussian noise: it has the best performance among the detectors compared in Laplacian and Cauchy noise. It is shown that the MSS detectors with constant median-shift values are nearly equal to the optimum MSS detector. We also analyze the problem of detecting a dc signal when only partial information is available on the noise. The MSS detectors with constant median-shift values are almost equal to the sign detector in Gaussian noise: they perform better than the sign and Wilcoxon detectors for most signal ranges in Laplacian and Cauchy noise.

I Introduction

The problem of signal detection can be considered as a parameter test problem of a null hypothesis against an alternative hypothesis [1]-[2]. As a consequence, the knowledge of *a priori* information on the parameter is required for establishing the hypothesis testing problem. Unfortunately, it is very difficult to exactly estimate the value of the parameter in practice. If we are not able to get a priori information on the distribution of the parameter, we cannot design an optimum parametric detector. Although we can estimate the parameters in some cases, small deviations of the parameters from the theoretic model in the real environment may lead to a significant performance degradation of the optimum parametric detector. In such cases, we shall need a *nonparametric* detector [3]-[5].

For signal detection problems, the use of a nonparametric detector results in systems with constant probabilities of false alarm for large classes of noise distributions, classes satisfying only a few mild conditions. A nonparametric detector is therefore useful in statistical environments where detailed information on the statistics of the noise is not available. In addition to ***insensitivity to environment***, a nonparametric detector exhibits ***simplicity in implementation*** at the expense of some deterioration in performance compared to an optimum detector. Therefore, nonparametric detectors have been used in diverse areas [6]-[7] which include radar, sonar, pattern recognition, fault detection, biomedical signal processing, and so on.

Among the representative nonparametric detectors are the ***sign***, ***linear rank (Wilcoxon)***, ***normal scores (Fisher-Yates)***, ***polarity coincidence correlator (PCC)***, and ***Mann-Whitney*** detectors. Other nonparametric detectors can also be found in the literature [8]-[13].

While the linear rank detectors exhibit excellent performance relative to the optimum parametric detector, especially in Gaussian noise, they have more complicated structure and poorer performance in impulsive (Cauchy or Laplacian) noise than the sign detector. The sign detector, on the other hand, is quite simple to implement, but does not perform well in Gaussian noise.

We propose a nonparametric detector acquired by modifying the classical sign detector. The detector is quite simple to implement, performs better than the sign detector in Gaussian noise, and performs better than the rank and sign detectors for most signal ranges in non-Gaussian noise.

II The Observation Model

Consider the binary hypothesis testing problem: given an observation vector $\underline{X}_n = (X_1, X_2, ..., X_n)$, a decision has to be made between a null hypothesis H and an alternative hypothesis K. The pair H and K are defined as

$$\begin{aligned} H : X_i &= N_i, \\ K : X_i &= N_i + S, \qquad i = 1, 2, \cdots, n. \end{aligned} \quad (1)$$

In (1), n is the sample size, $S > 0$ is a constant representing the signal, and N_i are the independent and identically distributed random variables representing noise components, each with zero-mean symmetric probability density function (pdf) f.

In the usual fixed-sample-size detection scheme, the detector is based on a critical function $\phi(\underline{x}_n)$, a function of the specific realization $\underline{x}_n$ of $\underline{X}_n$, and takes one value in $[0, 1]$ depending on the outcome of a threshold comparison. The function $\phi(\underline{x}_n)$ gives the probability with which the alternative hypothesis is to be accepted, and is defined by

$$\phi(\underline{x}_n) = \begin{cases} 1 & , \text{if } T(\underline{x}_n) > \lambda_{n,\alpha}, \\ \gamma_{n,\alpha} & , \text{if } T(\underline{x}_n) = \lambda_{n,\alpha}, \\ 0 & , \text{if } T(\underline{x}_n) < \lambda_{n,\alpha}. \end{cases} \quad (2)$$

In (2), the test statistic $T(\underline{x}_n)$ is a function of the observation appropriate for testing H versus K, and the threshold $\lambda_{n,\alpha}$ and the randomization parameter $\gamma_{n,\alpha}$ are constants chosen to achieve the desired false alarm probability α.

III The Median-Shift Sign Test Statistic

Now we consider the test statistic of the detector proposed in this paper,

$$T_{MSS}(\underline{X}_n) = \sum_{i=1}^{n} U(X_i + V), \quad (3)$$

where

$$U(z) = \begin{cases} 1 & , \text{if } z > 0, \\ 0 & , \text{if } z < 0 \end{cases} \quad (4)$$

and V is the *median-shift value*. We call the detector based on the test statistic (3) the *median-shift sign (MSS)* detector. The *optimum median-shift value* V_{op} is so obtained as to make the detection probability maximum once the sample size, false alarm rate, signal strength, and noise statistics are fixed. When $V = V_{op}$, the MSS detector will be called the *optimum MSS* detector. The MSS detector with the median-shift value V is written as the MSS (V) detector: note that the MSS (0) detector is the sign detector.

First we obtain the optimum median-shift value. The probability that the input data plus median-shift value V is positive under the alternative hypothesis is given by

$$\begin{aligned} P_1 &= \int_0^{\infty} f(x - (S + V))dx \\ &= \int_{-\infty}^{S+V} f(x)dx \\ &= F(S + V), \end{aligned} \quad (5)$$

where F is the cumulative distribution function of f. Under the null hypothesis, $S = 0$ and we have

$$\begin{aligned} P_0 &= P_1|_{S=0} \\ &= F(V). \end{aligned} \quad (6)$$

We can now obtain the threshold and randomization parameter. The threshold λ is the minimum integer which satisfies

$$\sum_{k=\lambda+1}^{n} \binom{n}{k} P_0^k (1 - P_0)^{n-k} \leq \alpha, \quad (7)$$

where $1 \leq \lambda \leq n$ and α is the false alarm rate. The randomization parameter γ is given by

$$\gamma = \frac{\alpha - \sum_{k=\lambda+1}^{n} \binom{n}{k} P_0^k (1 - P_0)^{n-k}}{\binom{n}{\lambda} P_0^{\lambda} (1 - P_0)^{n-\lambda}}, \quad (8)$$

where $0 \leq \gamma < 1$.

Then, using λ and γ, we can compute the detection probability. If the test statistic is greater than the threshold, we choose the alternative hypothesis with probability one. If the test statistic is equal to the threshold, we choose the alternative hypothesis with probability γ. Otherwise, we reject the alternative hypothesis with probability one. The detection probability is therefore given by

$$\begin{aligned} P_D &= \sum_{k=\lambda+1}^{n} \binom{n}{k} P_1^k (1 - P_1)^{n-k} \quad (9) \\ &\quad + \gamma \binom{n}{\lambda} P_1^{\lambda} (1 - P_1)^{n-\lambda}. \quad (10) \end{aligned}$$

The optimum median-shift value is then obtained from

$$V_{op} = arg \max_{V} P_D, \quad (11)$$

which is a function of the sample size n, false alarm rate α, signal strength S, and the noise pdf f.

Figure 1 shows a block diagram of the optimum MSS detector.

IV The Optimum Median-Shift Value

In this section, we consider some properties of the optimum median-shift value. Consider the Gaussian, Laplacian, and Cauchy distributions, whose pdfs are given by

$$f_G(x) = \frac{1}{\sqrt{2\pi}\sigma_G}\exp^{-\frac{x^2}{2\sigma_G^2}}, \quad (12)$$

$$f_L(x) = \frac{1}{2\sigma_L}\exp^{-\frac{|x|}{\sigma_L}}, \quad (13)$$

and

$$f_C(x) = \frac{1}{\pi\sigma_C}\frac{1}{1+(\frac{x}{\sigma_C})^2}, \quad (14)$$

respectively [14], where

$$\sigma_G = \sigma, \quad (15)$$

$$\sigma_L = \sqrt{\frac{\pi}{2}}\sigma, \quad (16)$$

and

$$\sigma_C = \sqrt{\frac{2}{\pi}}\sigma. \quad (17)$$

Here σ is the *common deviation parameter* introduced to make $f_G(0) = f_L(0) = f_C(0) = f(0)$.

Figure 2 shows the median-shift value versus the detection probability, when $n = 100$, $\alpha = 0.01$, $S = 0.5$, and $\sigma = 1.0$. In this figure, the solid line represents the Gaussian noise case, the dashed line the Laplacian noise case, and the dotted line the Cauchy noise case. Since the detection probability is roughly symmetric about the optimum value, the MSS (V) detector performs better than the sign detector approximately when $2V_{op} < V < 0$.

To show the effect of the sample size variation on the optimum median-shift value, we plotted some results in Figure 3, when $\alpha = 0.01$ and $\sigma = 1.0$. In this figure, the solid line represents the case $n = 20$, the dashed line the case $n = 50$, and the dotted line the case $n = 100$. From the results, we can say the following properties. As the signal strength increases, the absolute value of the optimum median-shift value increases stairwise. In Gaussian noise, as the sample size increases, the absolute value of the optimum median-shift value decreases. In non-Gaussian noise, the optimum median-shift value is rather insensitive to the sample size variation. The height ΔV of the stair strongly depends on the sample size and can be approximately calculated as, in the weak signal case,

$$\begin{aligned} 1 &= E\{T_{MSS}|H \textit{ and } V = V_1\} \quad (18) \\ &\quad -E\{T_{MSS}|H \textit{ and } V = V_2\} \\ &= nP_{01} - nP_{02} \\ &\approx n(0.5 + f(0)V_1) - n(0.5 + f(0)V_2) \\ &= nf(0)\Delta V, \\ \Delta V &\approx \frac{1}{nf(0)}, \quad (19) \end{aligned}$$

where $V_1 = V_2 + \Delta V$.

Next, to show the effect of the false alarm rate variation on the optimum median-shift value, we plotted some results in Figure 4, when $n = 50$ and $\sigma = 1.0$. In this figure, the solid line is the case $\alpha = 0.001$, the dashed line the case $\alpha = 0.008$, and the dotted line the case $\alpha = 0.010$. From the results, we can say that the optimum median-shift value is insensitive to the false alarm rate variation in non-Gaussian noise.

V Finite Sample-Size Performance

In general, there are two methods in measuring the performance of detectors. One is the finite sample-size performance analysis, which is the exact analysis. The other is the asymptotic performance analysis based on the asymptotic relative efficiency (ARE) which is used when the sample size is very large. In this paper, we use the *Monte-Carlo* method for the simulation of finite sample-size performance comparisons.

1 Detection of Signals in Noise of Known Distribution

In this section, we consider the problem of detecting a dc signal when we know the pdf of noise. The detectors compared in the simulations are the linear, sign, Wilcoxon, and MSS detectors, whose test statistics are

$$T_L(\underline{X}_n) = \sum_{i=1}^{n} X_i, \quad (20)$$

$$T_S(\underline{X}_n) = \sum_{i=1}^{n} U(X_i), \quad (21)$$

$$T_W(\underline{X}_n) = \sum_{i=1}^{n} Z_i U(X_i), \quad (22)$$

$$(23)$$

where Z_i is the rank of X_i in the set $\{|X_1|, |X_2|, \cdots, |X_n|\}$, and

$$T_{MSS}(\underline{X}_n) = \sum_{i=1}^{n} U(X_i + V), \quad (24)$$

respectively.

In Figures 5-7, we show the detection probabilities as a function of the signal strength in Gaussian, Laplacian, and Cauchy noise, respectively, when $n = 50$, $\alpha = 0.01$, and $\sigma = 1.0$. In these figures, the solid line represents the MSS (V_{op}) detector, the circle the MSS (-0.1) detector, the star the MSS (-0.3) detector, the dotted line the classical sign detector, the dashed line the linear detector, and the dashdot line the Wilcoxon detector.

In the Gaussian noise case, the linear detector is optimum and the Wilcoxon detector is nearly optimum. The MSS (V_{op}) detector slightly performs better than the sign detector: the MSS detectors with constant median-shift values also slightly perform better than the sign detector, but slightly perform worse than the MSS (V_{op}) detector.

In the Laplacian noise case, the MSS (V_{op}) detector has the best performance among the detectors compared. The sign and Wilcoxon detectors have almost the same performance: when the signal is weak, the sign detector slightly performs better than the Wilcoxon detector. When the signal is strong, on the other hand, the sign detector slightly performs worse than the Wilcoxon detector. The linear detector has the worst performance among the detectors compared. An interesting fact is that the MSS detectors with constant median-shift values have quite good performance. The MSS (-0.1) detector is nearly equal to the MSS (V_{op}) detector when the signal is weak. When the signal is strong, it performs worse than the MSS (V_{op}) detector, but still performs better than the sign and Wilcoxon detectors. The MSS (-0.3) detector is nearly equal to the MSS (V_{op}) detector when the signal is strong. When the signal is weak, it slightly performs worse than the sign and Wilcoxon detectors, but still performs better than the linear detector.

In the Cauchy noise case, observations similar to those made in the Laplacian noise can be made, except that the sign detector performs better than the Wilcoxon detector, since the Cauchy noise is more impulsive than the Laplacian noise.

2 Detection of Signals in Noise of Nearly Unknown Distribution

In this section, we consider the problem of detecting a dc signal in nearly unknown noise: all we know is that the noise pdf f satisfies $f(x) = f(-x)$ and

$$F(z) = \int_{-\infty}^{z} f(x)dx \qquad (25)$$

is fixed for some constant z. The false alarm rate is constant for the MSS (V = z) detector: that is, the MSS (V = z) detector is a *nonparametric* or *constant false alarm rate (CFAR)* detector.

Figures 8-10 show the detection probability as a function of the signal strength in Gaussian, Laplacian, and Cauchy noise, respectively, when $n = 50$, $\alpha = 0.01$, $z = -0.1$, and $F(-0.1) = 0.46017$. In these figures, the circle represents the MSS (-0.1) detector, the star the MSS (-0.3) detector, the dotted line the classical sign detector, the dashdot line the Wilcoxon detector, and the dashed the linear detector.

In Gaussian noise, the linear detector is optimum and the Wilcoxon detector is nearly optimum. The sign detector performs worse than the optimum detector. The MSS (-0.1) and MSS (-0.3) detectors slightly perform better than the sign detector.

In Laplacian noise, the MSS (-0.1) and MSS (-0.3) detectors perform better than the sign and Wilcoxon detectors for most signal ranges. When the signal is weak, the MSS (-0.1) and MSS (-0.3) detectors slightly perform worse than the sign detector, because the sign detector is the *locally optimum (LO)* detector for the weak signal. For the linear detector, we assumed that the noise pdf is known so that the linear detector can satisfy the given constant false alarm rate.

In Cauchy noise, the MSS (-0.1) and MSS (-0.3) detectors perform better than the sign and Wilcoxon detectors for most signal ranges. For the linear detector, we assumed that the noise pdf is known so that the linear detector can satisfy the given constant false alarm rate. When the signal is not strong, the detection probabilities of the linear detector is almost equal to the false alarm rate.

VI Conclusion

In this paper, we proposed a new detector based on the median-shift sign. This detector was a modification and an extension of the classical sign detector. In Section IV, we showed some properties of the optimum median-shift value.

In Section V, we analyzed the problem of detecting a dc signal in known noise. The MSS detector with the optimum median-shift value, i. e., the optimum MSS detector, performed better than the sign detector in Gaussian noise: it had the best performance among the detectors compared in Laplacian and Cauchy noise. It was shown that the MSS detectors with constant median-shift values were nearly equal to the optimum MSS detector.

Next, we analyzed the problem of detecting a dc signal when only partial information was available on the noise. In this case, the MSS detector with the constant median-shift value was used. The

MSS detector was almost equal to the sign detector in Gaussian noise: it performed better than the sign and Wilcoxon detectors for most signal ranges in Laplacian and Cauchy noise. Because the false alarm rate of the MSS detector is constant, the MSS detector is a *nonparametric* detector.

It is noteworthy that the MSS detectors with small constant median-shift values have simple construction and perform better than the sign and Wilcoxon detectors for most signal ranges in non-Gaussian noise.

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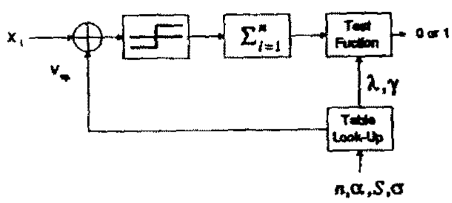


Figure 1: A block diagram of the optimum MSS detector

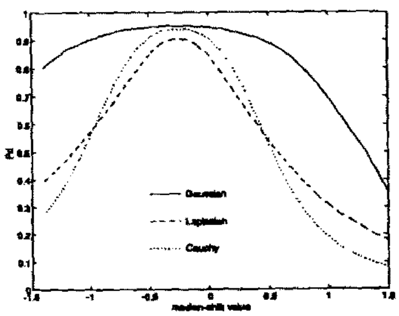


Figure 2: Median-shift value versus detection probability when $n = 100$, $\alpha = 0.01$, $S = 0.5$, and $\sigma = 1.0$

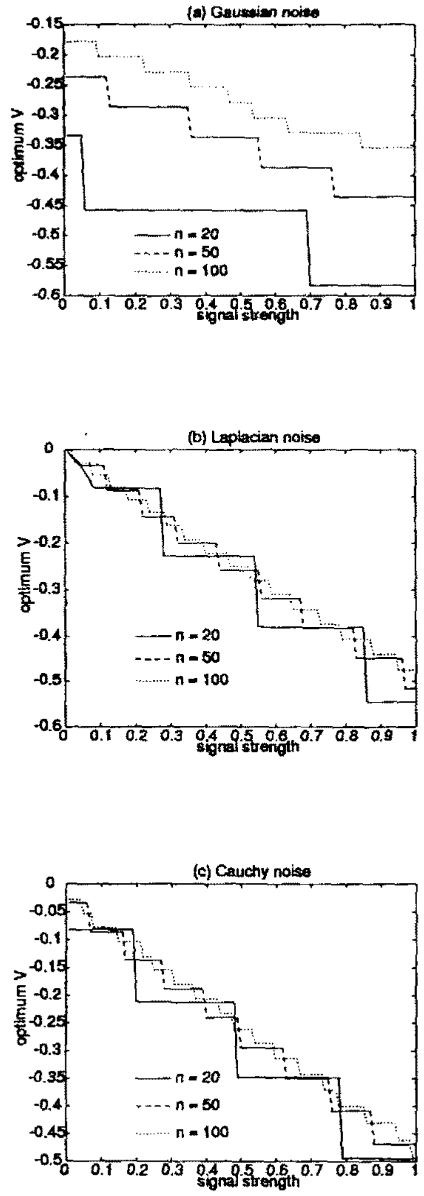
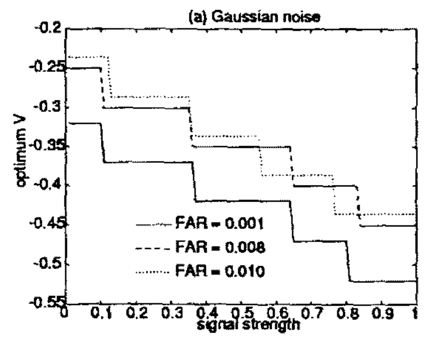


Figure 3: The effect of sample size variation when $\alpha = 0.01$ and $\sigma = 1.0$



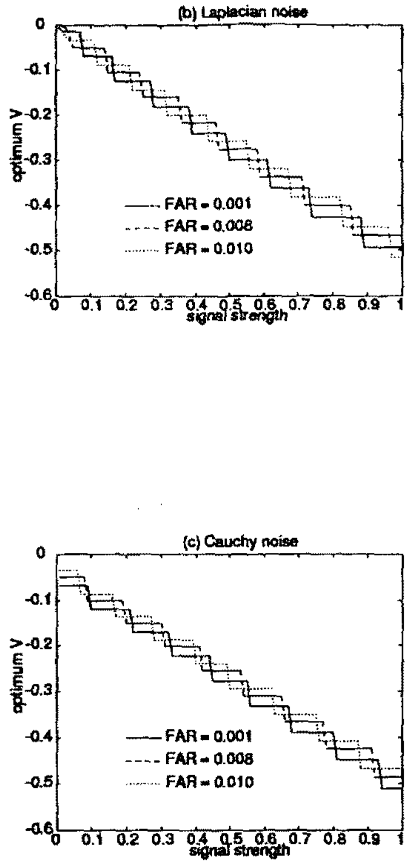


Figure 4: The effect of false alarm rate variation when $n = 50$ and $\sigma = 1.0$

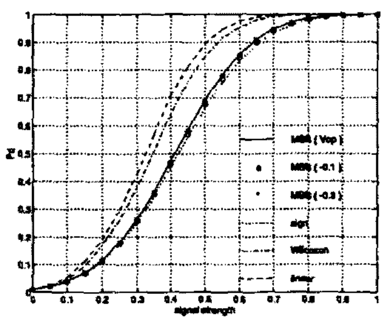


Figure 5: Detection probabilities in Gaussian noise when $n = 50$, $\alpha = 0.01$, and $\sigma = 1.0$ (known noise case)

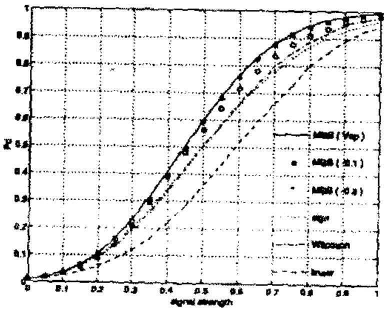


Figure 6: Detection probabilities in Laplacian noise, when $n = 50$, $\alpha = 0.01$, and $\sigma = 1.0$ (known noise case)

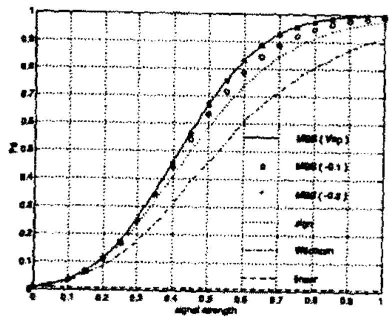


Figure 7: Detection probabilities in Cauchy noise, when $n = 50$, $\alpha = 0.01$, and $\sigma = 1.0$ (known noise case)

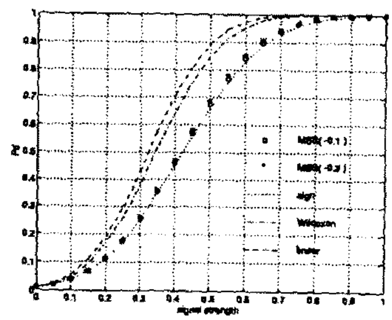


Figure 8: Detection probabilities in Gaussian noise, when $n = 50$, $\alpha = 0.01$, $z = -0.1$, and $F(-0.1) = 0.46017$ (nearly unknown noise case)

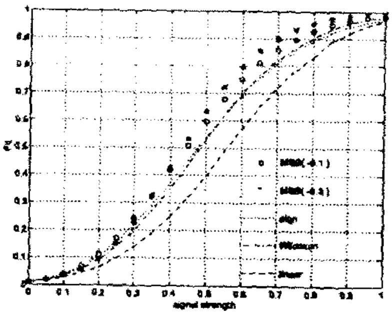


Figure 9: Detection probabilities in Laplacian noise, when $n = 50$, $\alpha = 0.01$, $z = -0.1$, and $F(-0.1) = 0.46017$ (nearly unknown noise case)

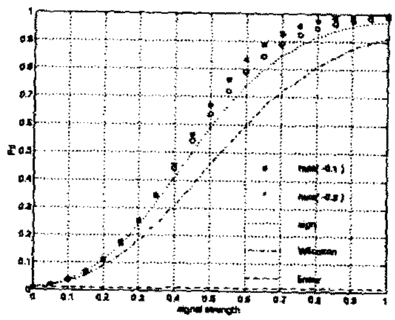


Figure 10: Detection probabilities in Cauchy noise, when $n = 50$, $\alpha = 0.01$, $z = -0.1$, and $F(-0.1) = 0.46017$ (nearly unknown noise case)