MODELING SIDEWAYS OVERTURNING OF AGRICULTURAL TRACTORS

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ABSTRACT

A mathematical model was developed for the simulation of sideways overturning of agricultural tractors on slopes. The overturning motion was described as a combination of the rotational motions with respect to the first and second tipping axes using the principle of conservation of angular momentum. A stability criterion was also established in terms of angular acceleration of tractor about the second tipping axis. Verification of the proposed model was provided by comparing the stability boundaries predicted by computer simulation with those observed experimentally for an equivalent 1/6 scaled model tractor. A good agreement was shown between the simulation and experimental results.

Key Words : Sideways overturning, Stability criterion, Computer simulation

INTRODUCTION

Sideways overturn has been recognized as one of the most probable tractor accidents. In Japan, about 40% of all deaths by agricultural machinery accidents each year was related with farm tractors. Of such tractor-related deaths more than 70% was accounted for sideways overturning (Shin-Norin Sha co. , 1988). In the United states, three-fourths of all farm machinery fatalities from 1975 through 1981 were associated with farm tractors and 47.6% of all the tractor-related deaths were caused by overturning accidents (McKnight and Hetzel, 1985). Although no data on farm machinery fatalities are currently available in Korea, a similar trend in the number of deaths caused by tractor overturning accidents could be expected.

Due to the increased concerns with tractor stability, a considerable research on sideways overturn has been conducted over the last 40 years. McCormick(1941) and worthington(1949) discussed the sideways overturn caused by a sharp turning on level ground at high speeds. However, Larson and Liljedahl(1971) were among the frist who investigated more general dynamic behavior of sideways overturning using mathematical models. Davis and Rehkugler(1974) also developed a computer simulation of sideways overturning. A recent review of studies on the sideways overturn and simulation models of tractor motions has been presented by Kim and Rehkugler(1987)

Sideways overturning takes place in two stages: in the frist stage, a rotation about the frist tipping axis and second, a rotation about the second tipping axis, both of which were studied kinematically by Smith et al. (1974). Considering sideways overturning as a combination of rotational motions with respect to the first and second tipping axes much simplifies the model development and provides more practical implementations of the model with less time and efforts.

The objectives of this study were twofold: first, to develop a mathematical model for sideways overturning in terms of its rotational motions with respect to the first and second tipping axes and secondly, to approximate the stability boundaries for safe tractor works by computer simulation under the given conditions of ground, operation, and obstacle.

MATERIALS AND METHODS

Basic concept and assumptions

During the initial overturning, tractor tips sideways about the first tipping axis which is an axis connecting the hinge point of the front-end and ground contact point of a rear wheel. When the front-end assembly reaches a rotation stop, the second overturning takes place as the entire tractor rotates about the second tipping axis, which is an axis connecting the ground contact points of the front and rear wheels on the same side of tractor about which the initial overturning has developed.

Initial overturning may arise whenever one of the rear wheels traverses over an obstacle or ground disturbances. Whether or not the initial overturning may lead to the second tipping depends on tractor speed, size of obstacle, and ground slope.

In this study, the initial overturning was assumed to be developed when the up-hill side rear wheel collides with an obstacle on slopes. This motion can be described with respect to the frist tipping axis. Transformation of the frist tipping into the second one at the rotation stop of the front-end was performed by applying the principle of conservation of angular momentum. In addition, following assumptions were made for the development of the mathematical model:

1) Tractor is rigid and symmetrical about the vertical plane passing through

its mass center and perpendicular to the wheel axle.

2) Tractor speed is constants.

3) Deformation of wheels is neglected

4) Effects of pitch and yaw motions on overturning are neglected.

5) Ground surface is flat and non-deformable.

6) Obstacle is of trapezoidal shape and non-deformable.

Equation of motion

Four reference coordinate systems as shown in Fig. 1 were used for the description of overturning motions. Each of the systems was defined as follow:

X_G-Y_G-Z_G = a space-fixed coordinate system in which X_G-Y_G plane becomes a horizontal plane. Positive direction for Z_G is vertical downward.

X_g-Y_g-Z_g = a coordinate system attached to the ground surface with its origin at the ground contact point of down-hill side rear wheel. Positive directions for X_g, Y_g, and Z_g are respectively surface forward, surface to the driver's right, and vertical downward to ground surface.

X_f-Y_f-Z_f = a space-fixed coordinate system in which its X_f axis coincides with the first tipping axis and its origin is located at the ground contact point of down-hill side rear wheel. Y_f axis is parallel to the perpendicular drawn from the rotational center of the up-hill side rear wheel to X_f. The positive direction for Z_f axis is vertical downward to the X_f-Y_f plane.

X_s-Y_s-Z_s = a ground-fixed coordinate system in which its X_s axis coincides with the second tipping axis and its origin is located at the ground contact point of down-hill side rear wheel. Positive directions for X_s and Z_s are respectively to X_s's right and vertical downward to the ground surface.

The X_f-Y_f-Z_f and X_s-Y_s-Z_s may be termed as the first and second tipping coordinate systems because the X_f and X_s axes coincide respectively with the first and second tipping axes. Vector transformations between the coordinate systems can be accomplished by using transfer matrices which will be detailed later. In addition, following symbols and notations were also used in deriving the equations of motion :

W_b = weight of tractor excluding the front axle.

W_t = weight of entire tractor including the front axle.

I_{xx}^{f} = moment of inertia about the first tipping axis of tractor excluding the front axle.

I_{xx}^{s} = moment of inertia about the second tipping axis of the entire tractor

(X_1, X_2, X_3) = coordinates of hinge point of the front-end in X_g-Y_g-Z_g system

Y_0 = horizontal distance from the ground contact point of the front wheel to the hinge point of the front-end.

Y_2 = distance between the ground contact points of rear wheels.

(F_1, F_2, F_3) = coordinates of mass center of the front axle in X_g-Y_g-Z_g system

(C_1, C_2, C_3) = coordinates of mass center of tractor excluding the front axle in X_g-Y_g-Z_g system.

R_{wc} = radius of rear wheel.

C = distance from the mass center (C_1, C_2, C_3) to the first tipping axis.

ϕ_f = angle of rotation with respect to the first tipping axis.

$\phi_{f_{max}}$ = angle of rotation stop of the front-end

ϕ_t = angle between the X_f-Y_f plane and the line representing C when $\phi_f=0$

ϕ_s = angle of rotation with respect to the second tipping axis.

ϕ_{ts} = fixed angle between the X_s-Y_s plane and the line representing C_s when $\phi_f = \phi_{f_{max}}$

X_0 = length of obstacle

H_0 = height of obstacle

(H_x^f, H_y^f, H_z^f) = coordinates of the surface contact point of the up-hillside rear wheel in X_f-Y_f-Z_f system.

θ_t = incline angle of obstacle.

α = heading angle of tractor.

β = slope angle.

D = distance from the first tipping axis to the ground contact point of the up-hill side rear wheel when $\phi_f = 0$

V = forward velocity of tractor.

If the up-hill side rear wheel takes off the ground at the top of the obstacle, tractor will rotate about the first ripping axis and its motion can be expressed as

$$C\, W_{by}{}^f \sin(\phi_g - \phi_f) + C\, W_{bz}{}^f \cos(\phi_b - \phi_f) = I_{xx}{}^f \frac{d^2\phi_f}{dt^2} \qquad (1)$$

where $W_{by}{}^f$ and $W_{bz}{}^f$ are respectively the Y_f and Z_f axis components of W_t. In Equation (1), it should be noted that ϕ_f is always negative by the right-hand rule in rotational vector notations. The initial conditions for Equation (1) can be determined from the relationships between the hinge point position of the front-end , rear wheel diameter, rear wheel distance, obstacle height, obstacle slope, and forward velocity of tractor. As shown in Figure 2, the ground contact point of the wheel at the top of the obstacle before taking off can be defined by the coordinates ($H_x{}^f$, $H_y{}^f$, $H_z{}^f$). Then, the initial conditions, $\phi_f(0)$ and $d\phi_f(0)/dt$ are computed as follows:

$$\phi_f(0) = \sin t^{-1}\left(\frac{H_z{}^f}{D}\right) = \sin^{-1}\left(-\frac{Y_2 X_x H_0}{ABD}\right)$$

$$\frac{d\phi_f(0)}{dt} = -\frac{Y_2 X_1 V \tan\theta_t}{ABD}$$

where $A = \sqrt{X_1{}^2 + Y_1{}^2 + Z_1{}^2}$

$$B = \sqrt{\left[\frac{(Y_1 Y_2 - R_{wc} Z_1) X_1}{A^2}\right]^2 + \left[Y_2 - \frac{(Y_1 Y_2 - R_{wc} Z_1) Y_1}{A^2}\right]^2 + \left[-R_{wc} - \frac{(Y_1 Y_2 - R_{wc} Z_1)}{A^2}\right]^2}$$

For details in the derivation of initial conditions, author's thesis(Hyun, 1989) should be consulted.

When ϕ_f reaches $\phi_{f_{max}}$, the first tipping terminates and the second tipping begins if the first tipping motion is great enough to develop a rotation about the second tipping axis of the entire tractor. This second tipping motion can be expressed as follows:

$$C_s\, W_{ty}{}^s \sin(\phi_{bt} - \phi_s) + C_s\, W_{ty}{}^s \cos(\phi_{bt} - \phi_s) = I_{xx}{}^s \frac{d^2\phi_s}{dt^2} \tag{2}$$

where $W_{ty}{}^s$ and $W_{tz}{}^s$ are respectively the X_s and Z_s axis components of W_t . It should be noticed that ϕ_s is also negative in Equation (2). The initial value for ϕ_s was assumed to be zero, that is $\phi_s(0)=0$, and the initial velocity for the second tipping motion was calculated by applying the principle of conservation of angular momentum that the angular momentums about the first and second tipping axis are conserved. This can be stated by the equation as

$$I_{xx}{}^f \left.\frac{d\phi_f}{dt}\right|_{\phi_f = \phi_{f_{max}}} = I_{xx}{}^s \frac{d\phi_s(0)}{dt} \qquad \text{where} \quad \left.\frac{d\phi_f}{dt}\right|_{\phi_f = \phi_{f_{max}}}$$

indicates the velocity about the first tipping axis at $\phi_f = \phi_{f_{max}}$. Since the pitch and yaw motions of the tractor during the motion transfer are negligibly small, their effects on the second tipping motion were neglected.

Equations (1) and (2) with their associate initial conditions constitute the equations of motion for sideways overturning.

Stability criterion

A complete sideways overturning will occur when the mass center of the entire tractor passes over the vertical X_s-Z_s plane. Whether or not such a situation will develop depends upon the angular acceleration of the tractor about the second tipping axis. If the directions of the angular acceleration and velocity are the same, the complete sideways overturning will result. Thus, a stability criterion for the sideways overturning can be established as follows:

$$\frac{d^2\phi_s}{dt^2} \leq 0 \tag{3}$$

Coordinate transformations

To determine the values of coefficients in Equations (1) and (2) , coordinate transformations must be peformed.

In Figure 1, let i_g, j_g and k_g be the unit vectors in the positive directions of X_g, Y_g and Z_g axes respectively. Then, the unit vector in the positive $X_f$axis, denoted by i_f, can be described by

$$i_f = \frac{X_1 i_g + Y_1 j_g + Z_1 k_g}{A}$$

The X_f axis component of the position vector from the origin to the rotation center of the up-hill side rear wheel, denoted by $\vec{R}$, is obtained by taking the dot product of vectors i_f and $\vec{R}$. Let a be the component. Then,

$$\begin{aligned} a &= \vec{R} \cdot i_f = (Y_2 j_g - R_{wc} k_g) \cdot i_f \\ &= (Y_2 j_g - R_{wc} k_g) \cdot \frac{(X_1 i_g + Y_1 j_g + Z_1 k_g)}{A} \\ &= \frac{(Y_1 Y_2 - Z_1 R_{wc})}{A} \end{aligned}$$

A vector in the direction of Y_f axis is then given by

$$\begin{aligned} \vec{Y_f} &= \vec{R} - a\, i_f \\ &= (Y_2 j_g - R_{wc} k_g) - a \frac{X_1 i_g + Y_1 j_g + Z_1 k_g}{A} \\ &= E i_g + F j_g + G k_g \end{aligned}$$

where $E = -\frac{(Y_1 Y_2 - Z_1 R_{wc}) X_1}{A^2}$

$$F = Y_2 - \frac{(Y_1 Y_2 - Z_1 R_{wc}) Y_1}{A^2}$$

$$G = -R_{wc} - \frac{(Y_1 Y_2 - Z_1 R_{wc}) Z_1}{A^2}$$

Let j_f be the unit vector in the direction of Y_f . Then,

$$j_f = \frac{\vec{Y_f}}{B} = \frac{E i_g + F j_g + G k_g}{B}$$

where $B = \sqrt{E^2 + F^2 + G^2}$

Since i_f and j_f are perpendicular to each other, a third vector, k_f, the unit vector in the positive Z_f axis direction , can be found by taking the cross product of unit vectors i_f and j_f, giving

$$\begin{aligned} k_f &= i_f \times j_f \\ &= \left(Y_1 \frac{G}{AB} - Z_1 \frac{F}{AB}\right) i_g + \left(Z_1 \frac{E}{AB} - X_1 \frac{G}{AB}\right) j_g + \left(X_1 \frac{F}{AB} - Y_1 \frac{E}{AB}\right) k_g \\ &= \frac{(-Y_1 R_{wc} - Y_2 Z_1) i_g + X_1 R_{wc} j_g + X_1 Y_2 k_g}{AB} \end{aligned}$$

Now , the coordinate transformation form X_g-Y_g-Z_g to X_f-Y_f-Z_f coordinate systems can be expressed by a matrix equation

$$\{V\}^f = [T_{fg}]\{V\}^g$$

where $\{V\}^f$ and $\{V\}^g$ are respectively the vectors in X_f-Y_f-Z_f and X_g-Y_g-Z_g systems, and $[T_{fg}]$ is a transfer matrix defined as follows:

$$[T_{fg}] = \begin{bmatrix} X_1/A & Y_1/A & Z_1/A \\ E/B & F/B & G/B \\ -(R_{wc}Y_1 + Y_2Z_1)/AB & X_1R_{wc}/AB & X_1Y_2/AB \end{bmatrix}$$

The transfer matrix $[T_{fg}]$ then transforms a vector $\{V\}^g$ into a vector $\{V\}^f$. Taking the inverse of $[T_{fg}]$, denoted by $[T_{fg}]^{-1}$ or $[T_{gf}]$, the inverse matrix becomes a transfer matrix from X_f-Y_f-Z_f to X_g-Y_g-Z_g coordinate systems.

The transformation from X_G-Y_G-Z_G and X_g-Y_g-Z_g systems can be accomplished by two successive rotations α and β, heading and slope angles respectively, of the axes about, first the Y_g axis and, second the Z_g axis. These successive transformations can be represented by a single transfer matrix $[T_{gG}]$ which can be written as :

$$[T_{gG}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha & -\cos\alpha\sin\beta \\ -\sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

The same method is applied for the transformation from X_g-Y_g-Z_g to X_s-Y_s-Z_s coordinate systems. Since the transfer involves only a single rotation of the axes through the angle θ about Z_g axis, the transfer matrix $[T_{sg}]$ contains θ only and becomes

$$[T_{sg}] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{where } \theta = \cos^{-1}\frac{X_1}{\sqrt{(X_1{}^2 + (Y_1 - Y_0)^2}}$$

The transfer matrix for the successive transformations between the coordinate systems can now be made by properly taking successive multiplications of the related transfer matrices. Followings are such transfer matrices used for the computation of coefficients in the equations of motion.

$$[T_{fgG}] = [T_{fg}][T_{gG}]$$

$$[T_{sgf}] = [T_{sg}][T_{gf}] = [T_{sg}][T_{fg}]^{-1}$$

$$[\ T_{sgG}\ ]=[\ T_{sg}\ ][\ T_{gG}\ ]$$

Determination of coefficient values

The distance, C, form the mass center (C_1, C_2, C_3) to the first tipping axis is given by the cross product of the position vector, $C_1\ i_g + C_2\ j_g + C_3\ k_g$, and the unit vector, i_f , giving

$$C = |(C_1 i_g + C_2 j_g + C_3 k_g) \times i_f|$$
$$= \sqrt{(C_2 Z_1 - C_3 Y_1)^2 + (C_1 Z_1 - C_3 X_1)^2 + (C_1 Y_1 - C_2 X_1)^2}\ /\ A$$

Transformation of the coordinates (C_1, C_2, C_3) from X_g-Y_g-Z_g to X_f-Y_f-Z_f can be accomplished by using transfer matrix $[T_{fg}]$ as

$$\begin{bmatrix} C_x^{\ f} \\ C_y^{\ f} \\ C_z^{\ f} \end{bmatrix} = [\ T_{fg}] \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

where (C_x^f, C_y^f, C_z^f) indicates the coordinates of mass center of the tractor excluding the front-end with respect to X_f-Y_f-Z_f. The angle, ϕ_b, is then computed as follows:

$$\phi_b = \tan^{-1}|\ C_z^{\ f}\ /\ C_y^{\ f}\ |$$

Components of tractor weight, W_b, in X_f, Y_f, and Z_f axis directions are also obtained by taking successive transformations of the weight vector from X_G-Y_G-Z_G to X_f-Y_f-Z_f through X_g-Y_g-Z_g coordinate systems. In equation form

$$\begin{bmatrix} W_{bx}^{\ f} \\ W_{by}^{\ f} \\ W_{bz}^{\ f} \end{bmatrix} = [\ T_{fgG}] \begin{bmatrix} 0 \\ 0 \\ W_b \end{bmatrix} = [\ T_{fg}]\ [\ T_{gG}] \begin{bmatrix} 0 \\ 0 \\ W_b \end{bmatrix}$$

The same approach can be applied for the determination of the components of total weight, W_t, in the X_s, Y_s, and Z_s axis directions. Thus

$$\begin{bmatrix} W_{tx}^{\ s} \\ W_{ty}^{\ s} \\ W_{tz}^{\ s} \end{bmatrix} = [\ T_{sgG}] \begin{bmatrix} 0 \\ 0 \\ W_t \end{bmatrix} = [\ T_{sg}]\ [\ T_{gG}] \begin{bmatrix} 0 \\ 0 \\ W_t \end{bmatrix}$$

The coordinates of mass centers of the front-end and tractor body were transformed from X_g-Y_g-Z_g to X_s-Y_s-Z_s systems to yield respectively

$$\begin{bmatrix} C_{ax}^{\ s} \\ C_{ay}^{\ s} \\ C_{az}^{\ s} \end{bmatrix} = [\ T_{sg}] \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}, \quad \begin{bmatrix} C_{bx}^{\ s} \\ C_{by}^{\ s} \\ C_{bz}^{\ s} \end{bmatrix} = [\ T_{sg}] \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

Then, the coordinates of mass center of the entire tractor with respect to

X_s-Y_s-Z_s can be described by

$$C_x^s = \frac{(W_t - W_b)C_{ax}^s + W_b C_{bx}^s}{W_t}$$

$$C_y^s = \frac{(W_t - W_b)C_{ay}^s + W_b C_{by}^s}{W_t}$$

$$C_z^s = \frac{(W_t - W_b)C_{az}^s + W_b C_{bz}^s}{W_t}$$

Using the coordinates (C_x^s, C_y^s, C_z^s), the distance, C_s , from the mass center of the entire tractor to the second tipping axis and the angle ϕ_{bt} are determined as follows :

$$C_s = \sqrt{(C_y^s)^2 + (C_z^s)^2}$$

$$\phi_b = \tan^{-1} | C_z^s / C_y^s |$$

Matrix notations provide a convenient shorthand for handling the coordinate transformations and calculating the values of the coefficients by computer programs.

Scale model tractor

A battery powered two-wheel drive model tractor was built to provide the data for use in both the computer simulation and verification of the mathematical model. The model tractor was modified from a commercially available toy car so as to approximate 1/6 scaled geometrical properties of a full-sized TS 2500 Iseki tractor. However, the speed control, rear wheels and front-end were completely rebuilt to improve the motion controls and to satisfy the scale requirements.

Geometric properties and weight of the model tractor were measured for each of the front-end and tractor body. Moments of inertia about the first and second tipping axes were calculated from the periods of oscillations of the model tractor measured around each axis by the suspension method. Details of the property measurements were given by Hyun(1989). Table 1 shows the measured properties of the model tractor.

Verification of mathematical model

A Fortran computer program was coded to implement the mathematical model represented by Equations (1) and (2) with their respective initial conditions for the simulation of sideways overturning of the model tractor. The model tractor was assumed to operate with a constant forward velocity of 1 m/s and collide with an obstacle on slopes. The angles of slope and heading of the tractor were varied continuously until the angular acceleration about the second tipping axis meets the stability criterion given by Equation (3). The slope angle satisfying the stability criterion may be termed as the critical slope angle. The critical slope angle is that angle of the slope in which the tractor is likely to sideways overturn if it collides with an obstacle in a given heading direction. Connecting the critical slope angles in all the heading directions yields a curve representing a stability boundary in polar diagram. Examples of such stability boundary for the model tractor were shown in Figures 3 and 4. In these figures, the concentric circles indicate the slope angles and radii the heading angles of the tractor. It also should be noted that the stability boundary curve is symmetrical due to the assumed symmetric properties of the model tractor.

Verification of the mathematical model is provided by comparing the critical slope angles predicted by the computer simulation to those observed experimentally for the equivalent model tractor and overturning conditions. The overturn test for the model tractor was conducted by operating the tractor in different combinations of the slope and heading angles, and allowing it to collide with an obstacle. The surface of the slope was covered with sand paper to assume uniform surface characteristics.

Eight overturn tests were also depicted in Figures 3 and 4 to compare with the simulation results. As shown in Figures 3 and 4, a good similarity between the simulation and experimental values of the critical ground slopes was found enough to verify the mathematical model.

Application of computer simulation

Having been verified, the mathematical model becomes useful in studying details of sideways overturning behavior of tractors. The effects on the overturing stability of various design, operation, and terrain conditions can be investigated by implementing the mathematical model as required for the specific purposes.

One of the examples for such implementation using a computer program could be to establish safe or unsafe regions for tractor works. The safe region can be defined as the region where a tractor is not likely to sideways overturn under normal operation conditions. In polar diagrams shown in Figures 3 and 4, the safe region can be represented by the inside of the stability boundaries.

The computer program developed by Hyun(1989) includes various design and operation parameters of tractor that assist in interpretation and analysis of its sideways overturning motions.

CONCLUSIONS

A mathematical model has been developed to simulate the sideways overturning of agricultural tractors when traversing over an obstacle on slopes. The overturning motion was considered as a combination of tipping motions with respect to the first and second tipping axes. One of the important applications of the mathematical model would be to establish safe of unsafe work regions for a tractor of concern by computer simulations. Other implementations of the model by computer simulation also include the acquisition of detailed information on safe design and operation of tractors with respect to sideways overturning.

Verification of the model was provided by comparing the stability boundaries predicted by computer simulations with those observed experimentally for the equivalent 1/6 scaled model tractor. A good agreement was shown between the simulation and experimental results.

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Table 1. Properties of the model tractor.

Parameters	Values	
W_a	5.89	N
W_b	42.18	N
I_{xx}^t	0.088	kg · m²
I_{xx}^s	0.139	kg · m²
(X_1, Y_1, Z_1)	(0.272, 0.141, -0.06)	m
Yo	0.088	m
Y2	0.259	m
(F_1, F_2, F_3)	(0.274, 0.141, -0.053)	m
(C_1, C_2, C_3)	(0.075, 0.141, -0.091)	m
Φ_{fmax}	15	degrees

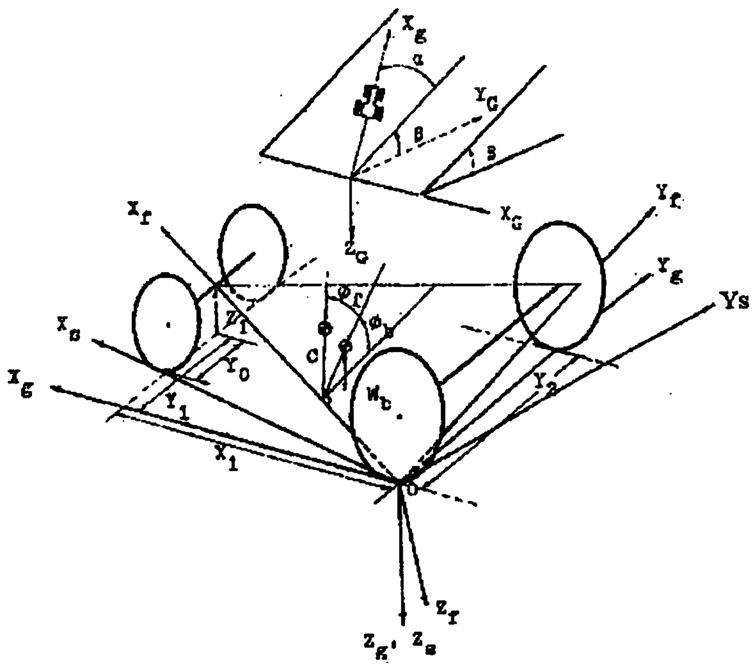

Fig. 1 Reference coordinate systems.

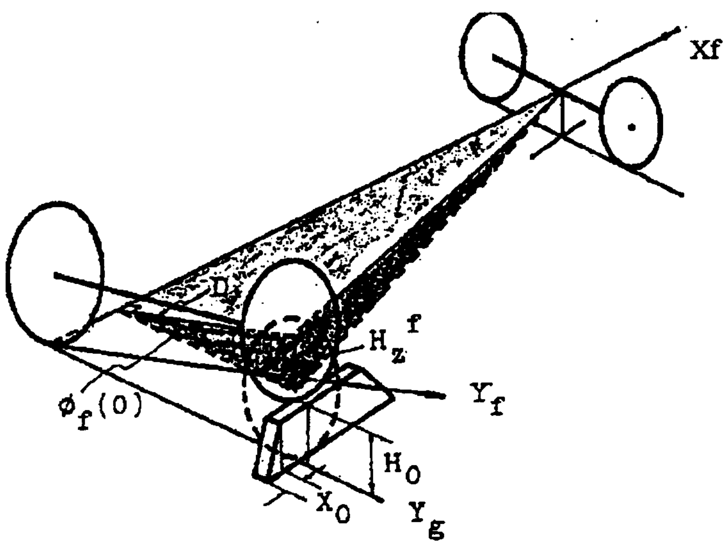

Fig. 2 Determination of initial conditions for the first tipping motion.

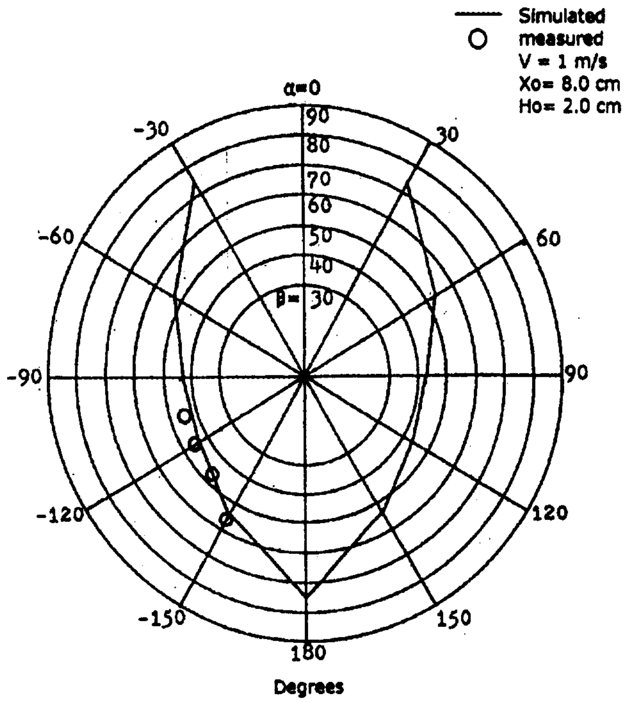

Fig. 3 Stability boundary for model tractor at θt = 14°

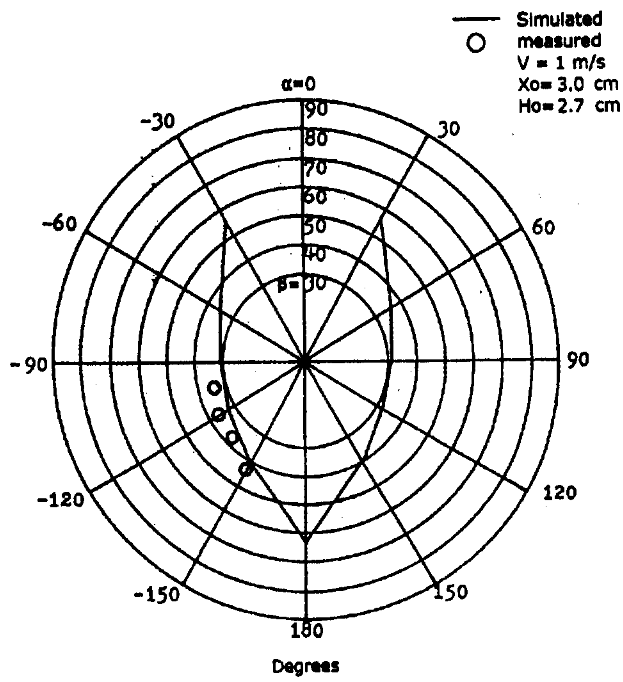

Fig. 4 Stability boundary for model tractor at θt = 42° .