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불확실한 계측 조건하에 변수에 의한 샘플링 검사를 위한 새로운 방법

(A New Method for Sampling Inspection by Variables under Undesired Measurement Conditions)

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ABSTRACT

This paper presents a new approach to modify the traditional method for sampling inspection by variables to suit undesired measurement conditions. On the basis of a systematic analysis of the effects of measurement errors on different types of acceptance schemes, this paper gives two modified equations for revising the sampling size n and the acceptability constant k to form a new acceptance scheme (n',k') under undesired measurement conditions. It has proved that the new method is suitable to different types of sampling inspection by variables including that for mean value and that for percent defective. The new acceptance scheme based on the proposed technique provides an engineering feasibility to replace the traditional scheme to suit undesired measurement conditions.

NOMENCLATURE

- σ the standard deviation of measured values
- σ_p the standard deviation of the true value of the product
- σ_m the standard deviation of measurement errors
- k the acceptability constant
- Q_u quality index
- Q_ℓ quality index
- μ the expected value of distribution mean
- $\bar{x}$ sample mean
- U upper specification limit
- L lower specification limit
- S estimated standard deviation of the lot
- t a value to be determined

1. INTRODUCTION

The growth of knowledge of statistical quality control techniques has led to a considerable increase in the industrial use of acceptance sampling by variables. Sampling by variables requires that the critical quality characteristic be a measurable quality. By using the actual values measured in the sample, a more sensitive sampling scheme can be developed, the main advantage of which is that more information is obtained about the quality characteristics in question[1]. Bowker and Goode's "Sampling Inspection by Variables" is the pioneer treatise on this aspect[2]. MIL-STD-414 is the first basic standard about sampling inspection by variables[3], which is still the generally recognized standard in the world and is applied to many countries standard of this aspect. The standards about the sampling inspection by variables are generally based on desired measurement conditions. The actual measurement conditions, however, are not always perfect, especially in geometric measurement under workshop conditions. Therefore, we should deal with the problem of how to apply the standards of the acceptance sampling by variables if the measured data are affected by measurement errors that can not be neglected.

The variability observed in measured values of dimensions, hardness, and other quality characteristics of industrial product are due in part to the variability of the quality characteristic and in part to the variability inherent in the method of measurement. As these two variables are likely to be independent of one another, the formula for the standard deviation of the sum of two independent variables may be employed. For this purpose, it might be written as:

$$\sigma = \sqrt{\sigma_p^2 + \sigma_m^2}$$

In desired measurement conditions, the value of σ_m is so small compared with σ_p that it can be neglected. In this case, the σ from the measured data can be employed to represent the standard deviation of the product dot (σ_p). If the measurement condition are undesired, that is $\sigma_m / \sigma_p \geq 1/4$, the value of σ_m can not be neglected and the value of σ based on measured data is obviously larger than σ_p. The acceptance schemes (n,k) based on desired measurement conditions is not reasonable to be directly because the measurement errors will inevitably affect the result of sampling by variables and in certain conditions, even cause wrong judgments, which will be discussed in the following section. In this paper, a new approach is presented to modify the traditional method for sampling inspection by variables in order to suit undesired measurement conditions.

2. EFFECTS OF MEASUREMENT ERRORS ON SAMPLING BY VARIABLES

Measurement errors are mainly devised into two kinds : random errors and systematic errors. Their influences on sampling inspection by variables will be discussed in this section.

2.1 Influence of Random Errors on Sampling by Variables

If two instruments with different accuracy are employed to do sampling inspection by variables under the measurement condition of no systematic errors, it is obvious that the sample means $\bar{x}$ are same while the standard deviations from the data may be different. The difference in standard deviations will affect the result of sampling inspection. Followed are the detailed analyses for different types of acceptance schemes.

2.1.1 Acceptance schemes for one sided-specifications

In the case of sampling by variables for percent defective, if there is an upper specification limit U, then the hypothesis that the fraction defective is less or equal to P_o (expected value of percent defective) is equivalent to the hypothesis that the distribution mean $\bar{x}$ is less or equal to μ_o (the mean of the distribution whose percent defectives is P_o). That is :

$$\bar{x} \le \mu_o = U - k\sigma \qquad (1)$$

Thus,

$$\bar{x} \le U - k\sigma \qquad (2)$$

Also, we have a set of equations:
For upper limit of known-Sigma

$$k = \frac{U-\mu}{\sigma} \quad and \quad Q_u = \frac{U-\bar{x}}{\sigma} \qquad (3)$$

For lower limit of known-Sigma

$$k = \frac{U-\mu}{\sigma} \quad and \quad Q_\ell = \frac{\bar{x}-L}{\sigma} \qquad (4)$$

For upper limit of known-Sigma

$$k = \frac{U-\mu}{S} \quad and \quad Q_u = \frac{U-\bar{x}}{S} \qquad (5)$$

For lower limit of known-Sigma

$$k = \frac{U-\mu}{S} \quad and \quad Q\ell = \frac{\bar{x}-L}{S} \qquad (6)$$

when Q_u or $Q_\ell \ge k$ the lot should be accepted. Otherwise it should be rejected.

In the above formulas, σ or S is a main factor to determine Q_u or Q_ℓ. If the value of σ or S increases by the measurement errors in undesired measurement conditions, the value of Q_u or Q_ℓ will decrease. In this situation, the lot of products that should be accepted in desired measurement conditions may be rejected, thus resulting in unnecessary loss.

2.1.2 Acceptance schemes for two-sided specifications

In the sampling by variables with two-sided specification limits, the upper and lower quality index Q_u or Q_ℓ should be calculated firstly. If $Q_u \ge k_u$ and $Q_{\ell 1} \ge k_l$, then, the lot should be accepted. Otherwise, it should be rejected. Therefore, the larger σ or S, the smaller Q_u or Q_ℓ, then the more possibility of rejecting it.

2.1.3 Acceptance schemes for comprehensive two-sided specifications

In this case, besides the procedure of section2.1.2, the standard deviation of lot σ_p should be compared with the maximum sampling standard deviation (MSSD). If the value of σ_p is less than MSSD and both Q_u or Q_ℓ agree with the criteria mentioned in Section 2.1.2, then the lot should be accepted. Obviously, the measurement errors will enlarge the value of σ and will probably result in the wrong judgment on the actually qualified product lot.

2.2 Influence of Systematic Errors on Sampling Inspection by Variables

Systematic errors, especially constant systematic errors, will obviously affect the means of samples $\bar{x}$. The analysis of its influence on the results of inspection of product quality will be discussed according to the following several cases.

Case 1 : Acceptance scheme with upper specification limit

Positive systematic error may result in that the mean of samples goes up to $\bar{x} + \Delta$. In the scheme with upper specification limit, as shown in Fig 1(a), the qualified lot of product will probably be judged as unqualified.

Negative systematic error makes the $\bar{x}$ shift to $x-\Delta$, as shown in Fig 1(b). In this case, the unqualified lot will probably be accepted.

Case 2 : Acceptance scheme with lower specification limit

In the acceptance schemes with single lower specification limit, the situations are just contrary to that of Case 1. The positive error increases the possibility of acceptance to the unqualified lot while the negative error increases the possibility of rejection to the qualified lot, as shown in Fig. 2(a) and (b).

Case 3 : Acceptance scheme with two-sided specification limits

There are two possibilities of increasing or decreasing the acceptance as shown in Fig. 3. If the true mean of samples $\bar{x}$ is closed to the lower limit of specification, the negative systematic error will decrease the possibility of acceptance. If the true mean of samples $\bar{x}$ is closed to the upper limit, the positive systematic errors will increase the possibility of rejecting.

3. MODIFIED SCHEME OF SAMPLING BY VARIABLES AND ITS MATHEMATICAL PROOF

It can be seen from the above analysis that both random errors and systematic errors have obvious but different influences on the sampling inspection by variables. Therefore, theses two kinds of errors must be dealt with differently based on their own characteristics. Since systematic errors directly affect the value of $\bar{x}$, it should be reduced or eliminated by using some well-established methods[4-5] that are effective for reducing systematic measurement errors to their minimum degree. For the random errors, when the measurement conditions are not desired, they have obvious effect on measured values. In order to keep the same Acceptable Quality Level (AQL) for undesired measurement conditions, a new approach is proposed in this paper, that is, to modify the traditional schemes of sampling inspection by variables (n,k) by enlarging the sample size n to n' and the acceptability constant k to k'. The sampling inspection by variables mainly includes two-types : sampling by variables for mean and that for percent defective. The modification on the two types is discussed and the mathematical proof is carried out in this section.

3.1 Modified Scheme of Sampling by Variables for Percent Defective (n',k')

The modified scheme (n',k') should be adopted when σ instead of σ_p is used under undesired measurement conditions. The formulas of n' and k' are expressed as follows:

$$n' = n\left(1+\frac{\sigma_m^2}{\sigma_p^2}\right) \tag{7}$$

$$k' = k\frac{\sigma_p}{\sigma} \tag{8}$$

The mathematical proof is given below for the case of the scheme with single upper specification limit. Assume that individual values of quality characteristic x obey the normal distribution (μ,σ^2) as shown in Fig. 4. The products whose quality characteristic values surpass the upper specification limit U are defective products. The ratio of defective products in the lot is percent of non conforming P_o. Otherwise, the lot whose percent of non conforming is larger than P_1 is bad lot, and should be rejected[6]. Considering the randomness of inspection, we define α as the probability of good lot (percent defective being smaller than P_o) being wrongly judged as bad lot, β as the probability of bad lot (percent defective being larger than P_1) being wrongly judged as good lot. So α represents the risk of producers, β represents the risk of consumers. Fig. 5(a) shows the traditional scheme of sampling by variables (n,k) under desired measurement conditions. Fig. 5(b) shows the modified scheme (n',k') under undesired measurement condition.

Proof for the modified Eqs (7) and (8):

According to Fig. 4, U (the upper limit of x) can be written as:

From the mean of good lot

$$U = \mu_0 + K_{p0}\sigma_p \tag{9}$$

From the mean of bad lot

$$U = \mu_1 + K_{p1}\sigma_p \tag{10}$$

According to Fig. 5, $\bar{x}_u$ (the upper limit of $\bar{x}$) can be expressed as:

From the mean of good lot

$$\bar{x}_u = \mu_0 + K_\alpha\frac{\sigma}{\sqrt{n'}} \tag{11}$$

From the mean of bad lot

$$\bar{x}_u = \mu_1 - K_\beta\frac{\sigma}{\sqrt{n'}} \tag{12}$$

To eliminate μ_0, let Eq. (9) - Eq. (11)

$$U - \bar{x}_u = K_{p0}\sigma_p - K_\alpha \frac{\sigma}{\sqrt{n'}} \quad (13)$$

To eliminate μ_0, let Eq. (10) - Eq. (12)

$$U - \bar{x}_u = K_{p1}\sigma_p - K_\beta \frac{\sigma}{\sqrt{n'}} \quad (14)$$

To compare Eq. (13) with Eq. (14)

$$K_{p0}\sigma_p - K_\alpha \frac{\sigma}{\sqrt{n'}} = K_{p1}\sigma_p + K_\beta \frac{\sigma}{\sqrt{n'}}$$

$$(K_{p0} - K_{p1})\sigma_p = (K_\alpha + K_\beta)\frac{\sigma}{\sqrt{n'}}$$

$$\sqrt{n'} = \frac{K_\alpha + K_\beta}{K_{p0} - K_{p1}} \cdot \frac{\sigma}{\sigma_p}$$

$$n' = (\frac{K_\alpha + K_\beta}{K_{p0} - K_{p1}})^2 \frac{\sigma_p^2 + \sigma_m^2}{\sigma_p^2}$$

$$n = (\frac{K_\alpha + K_\beta}{K_{p0} - K_{p1}})^2$$

$$n' = n(1 + \frac{\sigma_m^2}{\sigma_p^2})$$

Thus, Eq (7) has been proved.
The equations for the upper limit of $\bar{x}$ based on Fig. 5(a) are given below:
From the mean of good lot

$$\bar{x}_u = \mu_0 + K_\alpha \frac{\sigma_p}{\sqrt{n}} \quad (15)$$

From the mean of bad lot

$$\bar{x}_u = \mu_0 - K_\beta \frac{\sigma_p}{\sqrt{n}} \quad (16)$$

Let Eq. (9) - Eq. (15)

$$U - \bar{x}_u = (K_{po} - \frac{K_\alpha}{\sqrt{n}})\sigma_p \quad (17)$$

Let Eq. (10) - Eq. (16)

$$U - \bar{x}_u = (K_{p1} - \frac{K_\beta}{\sqrt{n}})\sigma_p \quad (18)$$

Compare Eq. (17) with Eq. (18)

$$K_{po} - \frac{K_\alpha}{\sqrt{n}} = K_{p1} + \frac{K_\beta}{\sqrt{n}} = \frac{U - \bar{x}_u}{\sigma_p} = k \quad (19)$$

Eq. (17) can be simplified as

$$U - \bar{x}_u = k\sigma_p \quad (20)$$

When using σ instead of σ_p, k should be changed to k' in Eq. (20)

$$U - \bar{x}_u = k\sigma_p = k'\sigma$$

$$k' = k\frac{\sigma_p}{\sigma}$$

Thus, Eq. (8) has been proved.

3.2 Modified Scheme of Standard Sampling by Variables for Mean

Some products are weighed by the mean of their one quality characteristics, such as the mean of stress, weight, etc. Let consider sampling by variables with lower specification limits for means. Suppose the characteristics x obey normal distribution and a sample of n is taken from the lot and measured.
Judgment rule is

If $\bar{x} \geq t$, the lot is accepted.
If $\bar{x} < t$, the lot is rejected.

The number t and sample size n are to be determined subject to the risk of producer and consumer.
If the lot mean is μ_o (the lower limit of $\bar{x}$), the probability of accepting the lot is $1-\alpha$. That is:

$$P\mu_o(\bar{x} \geq t) = P\mu_o(\frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}} \geq \frac{t - \mu_o}{\sigma / \sqrt{n}}) = 1 - \alpha \quad (21)$$

As $\bar{x}$ obeys normal distribution N($\mu, \sigma^2 / n$), $\frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}}$ also obeys standard normal distribution (0,1). Therefore, Eq (21) is as follows:

$$\phi(\frac{t - \mu_o}{\sigma / \sqrt{n}}) = \alpha \quad (22)$$

$$\frac{t-\mu_o}{\sigma/\sqrt{n}} = \phi^{-1}(\alpha) = K_\alpha \qquad (23)$$

If the lot mean is μ_1 (a value smaller than μ_o), the probability of accepting the lot is β. That is:

$$P\mu_1(\bar{x} \geq t) = 1 - P\mu_1(\bar{x} < t) = \beta \qquad (24)$$

$$P\mu_1(\bar{x} < t) = \phi(\frac{t-\mu_1}{\sigma/\sqrt{n}}) = 1-\beta \qquad (25)$$

$$\frac{t-\mu_1}{\sigma/\sqrt{n}} = \phi^{-1}(1-\beta) = \phi^{-1}(\beta) = -K_\beta \qquad (26)$$

From Eqs (23) and (26), n and t are

$$n = (\frac{K_\alpha + K_\beta}{\mu_1 - \mu_o}\sigma)^2 \qquad (27)$$

$$t = \frac{K_\alpha\mu_1 + K_\beta\mu_o}{K_\alpha + K_\beta} \qquad (28)$$

Let $Q_l = \frac{\bar{x}-\mu_o}{\sigma}$ and $k = \frac{t-\mu_o}{\sigma}$ (29)

The judgment rule now is changed to the follows:

If $Q_l \geq k$, the lot is accepted.
If $Q_l < k$, the lot is rejected.

σ_p instead of σ is used in the formulas of calculating the sample size n and acceptability constant k under desired measurement conditions. The acceptance scheme (n,k) of standard type sampling inspection by variables with lower specification limits for means can be worked out from the following formulas

$$n = \left[\frac{K_\alpha + K_\beta}{\mu_1 - \mu_2}\sigma_p\right]^2 \qquad (30)$$

$$k = \frac{t-\mu_o}{\sigma p} \qquad (31)$$

When the measurement conditions are undesired, that is $\sigma_m/\sigma_p \geq 1/4$, a modified scheme ($n',k'$) can be derived based on Eqs (27) and (29), given as follows:

$$n' = \left[\frac{K_\alpha + K_\beta}{\mu_1 - \mu_0}\sigma\right]^2 = \left[\frac{K_\alpha + K_\beta}{\mu_1 - \mu_0}\right](\sigma_p^2 + \sigma_m^2)$$

$$= \left[\frac{K_\alpha + K_\beta}{\mu_1 - \mu_0}\right]^2 \sigma_p^2 (1 + \frac{\sigma_m^2}{\sigma_p^2})$$

$$n' = n(1 + \frac{\sigma_m^2}{\sigma_p^2})$$

$$k' = \frac{t-\mu_o}{\sigma} = \frac{(t-\mu_o)\sigma_p}{\sigma_p \sigma} = \frac{\sigma_p}{\sigma}k$$

Thus, the new acceptance scheme (n',k') modified based on the traditional acceptance scheme (n,k) is mathematically proved to suit the undesired measurement conditions. It can be proved that the two formulas, i.e. Eqs (1) and (2) also suit the cases for two-sided specifications under undesired measurement conditions.

4. CONCLUSIONS

1. Measurement errors, both systematic and random, have obvious effects on the result of sampling inspection by variables. Therefore, the measurement conditions should be considered whenever the sampling inspection by variables is applied.
2. When the measurement conditions are undesired and unable to be improved, the traditional acceptance schemes of sampling by variables should not be used directly. Instead, the modified schemes discussed in this paper should be employed.
3. The new method for sampling inspection by variables presented in this paper aims at enlarging the application of sampling by variables, but not at encouraging to keep undesired measurement conditions. If possible, the first selection is always to improve the measurement conditions. Only in the case where the improvement of the measurement conditions is not available or relatively expensive, the modified acceptance scheme proposed in this paper is of significance for avoiding the unnecessary lost due to the measurement errors.

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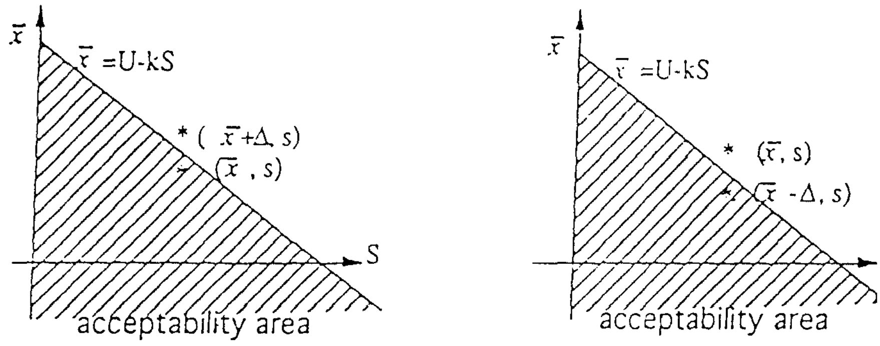
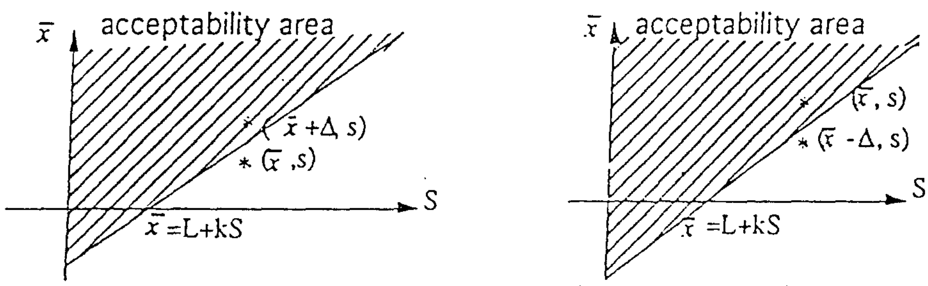


(a) positive systematic error

(b) negative systematic error

Fig. 1 Systematic errors resulting in wrong judgements in the scheme with upper specification limit



(a) positive systematic error

(b) negative systematic error

Fig. 2 Systematic errors resulting in wrong judgements in the scheme with lower specification limit

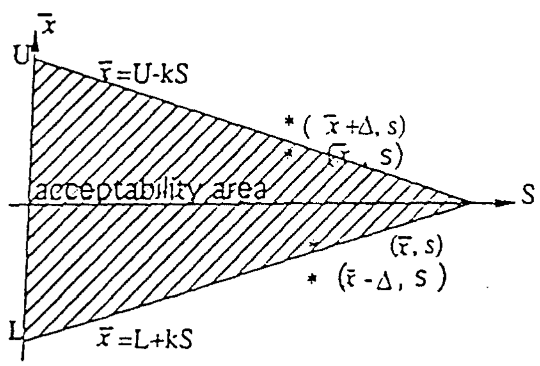
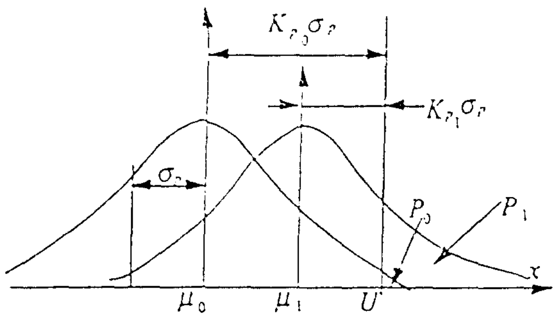


Fig. 3 Systematic errors resulting in wrong judgements in the scheme with two-sided specification limit



μ_0: the mean of the lot whose percent defective is P_0.

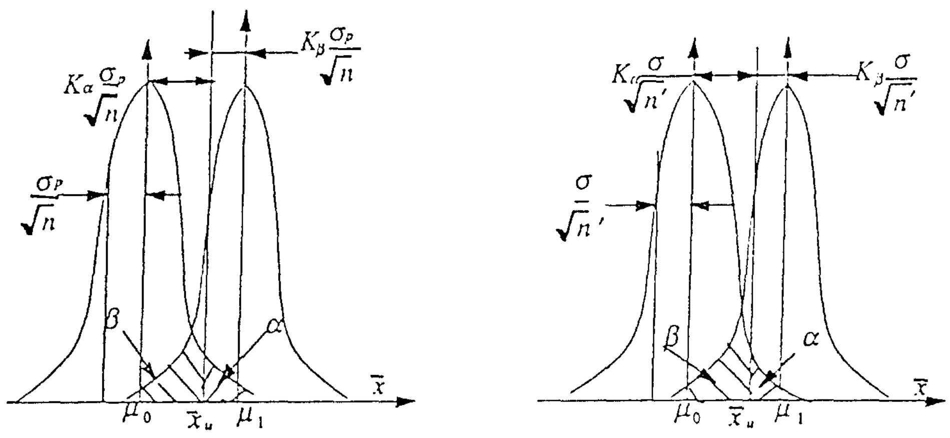
μ_1: the mean of bad the lot whose percent defective is larger than P_1

$K_{p_0} = \phi^{-1}(p_0)$

$K_{p_1} = \phi^{-1}(p_1)$

$\phi^{-1}(x)$: the inverse of the standardised normal distribution function

Fig. 4 Distribution curves for individual values



(a) Under desired measurement conditions

(b) Under undesired measurement conditions

Fig. 5 Comparison between the traditional and modified acceptance schemes

($\bar{x}_u$=the upper limit of $\bar{x}$, $K_\alpha = \phi^{-1}(\alpha)$, $K_\beta = \phi^{-1}(\beta)$)