Proceedings of the 11th KACC, October 1996

네트워크를 이용한 제어 시스템의 안정도 및 스케쥴링에 관한 연구
Stability and a Scheduling Method for Network-based Control Systems

°김 용호†, 권 욱현†, 박 홍성‡

†서울대학교 전기공학부 (Tel. : 02-873-2279, Fax. : 02-878-8933, Email: yongho@isltg.snu.ac.kr)
‡강원대학교 제어계측공학과 (Tel. : 0361-50-6346, Fax. : 0361-51-6501, Email: hspark@cc.kangwon.ac.kr)

Abstracts This paper obtains maximum allowable delay bounds for stability of network-based control systems and presents a network scheduling method which makes the network-induced delay be less than the maximum allowable delay bound. The maximum allowable delay bounds are obtained using the Lyapunov theorem. Using the network scheduling method, the bandwidth of a network can be allocated to each node and the sampling period of each sensor and controller can be determined. The presented method can handle three kinds of data (periodic, real-time asynchronous, and non real-time asynchronous data) and guarantee real-time transmissions of real-time synchronous data and periodic data, and possible transmissions of non real-time asynchronous data. The proposed method is shown to be useful by examples in two types of network protocols such as the token control and the central control.

Keywords Maximum Allowable Delay Bounds, Network-Based Control Systems, Network Scheduling Method, Three Kinds of Data.

1. Introduction

In distributed control systems, a feedback control loop is often closed through a communication link, which is called a network-based control system(NBCS). Since all kinds of data are transmitted through a shared communication link, time-varying random delays exist in feedback control loops, which are called network-induced delays. Though a shorter sampling period is preferable in most control systems, for some purposes, it can be lengthened up to a certain bound within which stability of the system is guaranteed in spite of the performance degradation. We will call this certain bound as a maximum allowable delay bound (MADB).

In NBCS, a sampling period should be long enough to guarantee real-time transmissions of real-time synchronous data and periodic data, and possible transmissions of non real-time asynchronous data if time is allowed. However the sampling period should be within the MADB to guarantee the stability of the given system. Therefore it is important that the sampling period should be shortened by minimizing network-induced delays via certain methods. In this paper, we present only a network scheduling method to make the network-induced delays be within the obtained MADB and to guarantee real-time transmissions of real-time synchronous data and periodic data, and possible transmissions of non real-time asynchronous data.

There have been some papers on the stability of NBCS [1-3]. But the existing works on stability of NBCS were concerned about obtaining stability conditions of the system with a given delay. In this paper, a MADB is obtained for stability of the NBCS by modifying the existing results in the conventional delay systems.

Network scheduling algorithms have some different characteristics from those of the processor scheduling algorithms. Those processor scheduling algorithms have some limitations in applications to the NBCS because a retransmission of periodic data with suspended old values by other urgent data transmissions is meaningless. There have been some studies on the scheduling algorithms which can be applied to the NBCS [7-9]. A heuristic algorithm was presented only for periodic tasks [8]. A dynamic scheduling algorithm modified from the rate monotone scheduling algorithms was presented for periodic and asynchronous data in fieldbus networks [7]. A scheduling algorithm which can allocate the bandwidth of a network and determine sensor data sampling periods was presented [9]. In [9], a control system has only single input and single output(SISO), only the periodic data was considered, and the MADB was not obtained analytically.

In the next section, NBCS are analyzed and a MADB for stability of NBCS is derived. In Section 3, a scheduling method of a network medium is presented, which allocates the bandwidth and determines the sampling period for NBCS. In Section 4, examples are given to show that the presented method is valid. Finally, this paper is concluded in Section 5.

2. A Maximum Delay Bound for Stability in a Single Loop

Generally, the NBCS deals with 3 types of data: periodic data, real-time asynchronous data, and non real-time asynchronous data. Data transfer via a network is carried out in fixed intervals, the sampling periods. The NBCS consist of multiple control loops. Each control loop consists of a controller node, sensor nodes, actuator nodes and event nodes. The event nodes are used in transferring event data such as alarms and programs.

A plant and a controller in a single control loop j can be described in the following state space form :

$$\dot{x}_p^j(t) \;=\; F_p^j x_p^j(t) + G_p^j u_p^j(t)$$

$$y_p^j(t) = H_p^j x_p^j(t), \quad (1)$$
$$\dot{x}_c^j(t) = F_c^j x_c^j(t) + G_c^j u_c^j(t)$$
$$y_c^j(t) = H_c^j x_c^j(t-\tilde{\tau}_c^j) + E_c^j u_c^j(t-\tilde{\tau}_c^j), \quad (2)$$

where $u_p^j(t) \in R^{l_A^j}$, $y_p^j(t) \in R^{l_S^j}$, $x_p^j(t) \in R^{l_{NP}^j}$. l_A^j, l_S^j, and l_{NP}^j are the numbers of the actuators, the numbers of the sensors, and the dimension of the plant system, respectively in the control loop j. $u_c^j(t) \in R^{l_S^j}$, $y_c^j(t) \in R^{l_A^j}$, $x_c^j(t) \in R^{l_{NC}^j}$ and l_{NC}^j is the dimension of the control system in the control loop j. $0 \le \tilde{\tau}_c^j \le \tau_{c,max}^j$, $\tilde{\tau}_c^j$ is the computation delay in the controller j, and $\tau_{c,max}^j$ is the maximum computation delay in the controller j. For conveniences, the computation delay in the controller is treated the same as the output delay. The communication delays in the control loop j are modelled as

$$u_c^j(t) = y_p^j(t-\tilde{\tau}_{sc}^j)$$
$$u_p^j(t) = y_c^j(t-\tilde{\tau}_{ca}^j), \quad (3)$$

where $0 \le \tilde{\tau}_{sc}^j \le \tau_{sc,max}^j$, $0 \le \tilde{\tau}_{ca}^j \le \tau_{ca,max}^j$, $\tilde{\tau}_{sc}^j$ and $\tau_{sc,max}^j$ are the communication delay and the maximum communication delay from the sensors to the controller respectively, and $\tilde{\tau}_{ca}^j$ and $\tau_{ca,max}^j$ are the communication delay and the maximum communication delay from the controller to the actuators respectively.

Using Equations (1), (2), and (3), a control system in the control loop j can be described as

$$\dot{x}^j(t) = F^j x^j(t) + \sum_{i=1}^{3} F_i^j x^j(t-\tilde{\tau}_i^j) \quad (4)$$

where

$$F^j = \begin{bmatrix} F_p^j & 0 \\ 0 & F_c^j \end{bmatrix}, F_1^j = \begin{bmatrix} 0 & 0 \\ G_c^j H_p^j & 0 \end{bmatrix},$$
$$F_2^j = \begin{bmatrix} G_p^j E_c^j H_p^j & 0 \\ 0 & 0 \end{bmatrix}, F_3^j = \begin{bmatrix} 0 & G_p^j H_c^j \\ 0 & 0 \end{bmatrix},$$
$$\tilde{\tau}_1^j = \tilde{\tau}_{sc}^j \le \tau_{sc,max}^j = \tilde{\tau}_{1,max}^j$$
$$\tilde{\tau}_2^j = \tilde{\tau}_{sc}^j + \tilde{\tau}_{ca}^j + \tilde{\tau}_c^j \le \tau_{sc,max}^j + \tau_{ca,max}^j + \tau_{c,max}^j = \tilde{\tau}_{2,max}^j$$
$$\tilde{\tau}_3^j = \tilde{\tau}_{ca}^j + \tilde{\tau}_c^j \le \tau_{ca,max}^j + \tau_{c,max}^j = \tilde{\tau}_{3,max}^j.$$

Each control loop in the NBCS can be described as in Equation (4) using three kinds of delays.

Using the existing results of [4] and [10], the following theorem can be obtained.

THEOREM **1** The total system (4) is asymptotically stable if

$$0 < \delta \le \mu(F^j), \ \sum_{i=1}^{3} \|F_i^j\| < k\delta, \ 0 < k < 1 \quad (5)$$

where $\|\cdot\|$ is the matrix norm induced by the vector norm and $\mu(\cdot)$ is the matrix measure derived from the matrix norm.
This theorem means that the system which satisfies the condition (5) can have an arbitrary large delay bound.

Meanwhile, the following theorem presents the explicit MADB for feedback control systems. Using the existing results of [5] and [6], the following theorem can be obtained.

THEOREM **2** Suppose that $(F^j + \sum_{i=1}^3 F_i^j)$ is asymptotically stable. Then the total system (4) is asymptotically stable if

$$\tau < \frac{\sigma}{\delta \sum_{i=1}^3 \|F_i^j(F^j + \sum_{i=1}^3 F_i^j)\|} \quad (6)$$

where $\tau = \max_i \tilde{\tau}_{i,max}^j$, $\sigma = \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}$, $\delta = \left[\frac{\lambda_{max}(P)}{\lambda_{min}(P)}\right]^{\frac{1}{2}}$. P, Q are the positive-definite symmetric matrices involved in the following Lyapunov equation :

$$(F^j + \sum_{i=1}^{3} F_i^j)^T P + P(F^j + \sum_{i=1}^{3} F_i^j) = -Q. \quad (7)$$

$\lambda_{min}(\cdot)$, $\lambda_{max}(\cdot)$ are the minimum and the maximum eigenvalues of the matrix, respectively.
We will use this delay bound in each control loop as a major parameter in the sampling period determination and the bandwidth allocation.

3. Sampling Period Determination and Bandwidth Allocation in Multiple Control Loops

Many control loops are connected using a single network medium. A single control loop in the total system is analyzed in the previous section emphasizing the characteristics about delays. To consider the total NBCS, some notations should be defined as follows :

- P is the total number of loops that use the same medium.
- l_C^j and l_N are the number of nodes for controllers in the j-th loop and the total number of event nodes, respectively.
- l^j and l_{RS} are the number of nodes that can transmit periodic data in the j-th loop ($l^j = l_C^j + l_S^j$) and number of nodes which transmit periodic data ($l_{RS} = \sum_{j=1}^P l^j$), respectively.
- S_i^j and C^j are the i-th sensor and the controller, respectively in the j-th loop. N is the total number of nodes that transmit data in NBCS ($N = \sum_{j=1}^P l^j + l_N$).
- $D_{S_i}^j$ and D_C^j are the sampling delay of the i-th sensor and the controller delay including the computation time of control values, respectively in the j-th loop.
- $M_{S_i}^j$ and M_C^j are the transmission time from the i-th sensor to a controller and from the controller to actuators, respectively in the j-th loop.
- $T_{S_i}^j$ and T_C^j are the duration time from the instant when data are sampled to the instant when the controller node receives the data and duration time from the instant when computation of control values is finished to the instant when the actuator nodes receive the control values, respectively in the j-th loop.
- T_{RS}, T_{RA}, and T_{NRA} are the periods for real-time periodic data, real-time asynchronous data, and non real-time asynchronous data, respectively in the basic sampling period.
- λ_{RA} and λ_{NRA} are the maximum arrival rate of real-time asynchronous data and arrival rate of non real-time asynchronous data, respectively in a network.
- T_{RA}^M and T_{NRA}^m are the maximum transmission time of one real-time asynchronous data and the mean transmission time of one non real-time asynchronous data, respectively.

- U_{NRA}^{min} and N_E are the minimum utilization of non real-time asynchronous data in a unit time and the number of basic sampling periods in the largest sampling period in the NBCS, respectively.
- T^j : a sampling period of the j-th loop.
- T_O and T_O^x are the maximum overhead time in one node and the maximum overhead time related to nodes which don't take part in transmission during each basic sampling period, respectively.
- T_D^j is the maximum allowable delay time(MADT) in the j-th loop. 1.

The overhead time which is the time duration from the instant when the data is sampled or computed to the instant when the node can transmit the data. This overhead time is time-varying. The MADT is defined as the maximum allowable interval from the instant when the sensor node samples sensor data to the instant when the actuators output the transferred data. For simplicity, let the loop number with the smallest MADT be 1. And loops are renumbered according to the magnitude of the MADT. Note that the minimum sampling period T^1 is considered as a basic sampling period(T_B).

The following assumptions are used in this paper.

- The sampling time of sensors in a loop is synchronized at the starting instant of the basic sampling periods.
- In the network, communications are error-free.
- Sampling periods of sensors in a loop are equal.
- The packets transferred from sensors to controllers have the same length.
- Controller delay time is less than the transmission times of control values and sensor values.

The last assumption is used for the overlapping of controller delay time and node transmission time.

To share a single medium and to choose suitable sampling period for each node in loops, following scheduling rules are suggested.

- The sampling period of each control loop should not be longer than its MADT for the stability of that control loop.
- The nodes with the smallest MADT in the unallocated nodes acquire a medium prior to other nodes.
- Sensor nodes use the medium prior to a controller node in a loop.
- Controller delay times are overlapped with transmission times of other nodes.
- Sampling periods of each loop are adjusted as multiples of the smallest sampling period ($T_{S.P.}$) in the order 2.

To guarantee the utilization of the non real-time asynchronous data, the utilization should be over minimum utilization (U_{NRA}^{min}) of non real-time asynchronous data in a network. That is, the following inequalities,

$$U_{NRA}^{min} \cdot T^1 \leq T_{NRA}, \tag{8}$$

should be satisfied.

To transmit the real-time asynchronous data which arrived during the previous cycle, the following condition,

$$\lambda_{RA} \cdot T^1 \cdot \hat{T}_{RA}^M \leq T_{RA}, \tag{9}$$

must be satisfied, where $\hat{T}_{RA}^M = T_{RA}^M + T_O$. Considering one specific basic sampling period, it can be written as

$$T^1 = \max_{i,\, j \in U_L}[D_{S_i}^j] + \sum_{j \in U_L} \sum_{i=1}^{l_S^j} (M_{S_i}^j + T_O) + \sum_{i \in U_L} (M_C^i + T_O) + \sum_{j \in U_L^*, i \in U_S^*} (M_{S_i}^j + T_O) + T_{RA} + T_{NRA} + T_O^x. \tag{10}$$

where U_L denotes a set of loops in the basic sampling period, U_L^* denotes a set of loops where all nodes are not included in U_L but some of nodes in those loops are partly in the basic sampling period, and U_S^* denotes a set of sensors which are in U_L^*. The number of nodes to transmit the periodic data in one basic sampling period is denoted by N_R. If the transmission times of sensors and controllers are equal as M, then the above equation is changed to

$$N_R \leq \left\| \frac{(1 - \lambda_{RA} \cdot \hat{T}_{RA}^M - U_{NRA}^{min}) \cdot T^1 - \max_{i,\, j \in U_L}[D_{S_i}^j] - T_O^x}{M + T_O} \right\|. \tag{11}$$

where

$$\|x\| = \begin{cases} x & for \quad x > 0 \\ 0 & for \quad x \leq 0 \end{cases}.$$

The maximum value of N_R in Equation (11) are denoted as N_R^M. The total number of transmission of periodic data during the interval $N_E \cdot T^1$ and can be calculated as

$$N_P = \sum_{j=1}^{P} 2^{N^j} \cdot l^j, \tag{12}$$

where $\frac{N_E \cdot T^1}{T^j} = 2^{N^j}$ for $j = 1, \cdots, P$. The schedulability can be checked by comparing $\frac{N_P}{N_E}$ with N_R^M in Equation (11). A basic sampling period can be decided by the following scheduling algorithm.

1. Check the condition of Theorem 1 for each control loop. If the condition of Theorem 1 is satisfied in a control loop j, then $T_D^j = \infty$, else set the MADB of each control loop using Theorem 2.
2. Reorder the control loops according to MADT.
3. Compute N_P using Eq. (12) and the results of the above step.
4. Let $T^1 = T_D^1$.
5. Choose T^j such as $T^j \leq T_D^j$ and $T^j = max(2^k \cdot T^1)$ where k=0,1,2,$\cdots$.
6. Compute N_R^M using Eq. (11).
7. If N_R is adequate, go to the next step, else if N_P is smaller than N_R^M and N_P is smaller than the value of $N_R^M - T_T$, then take the basic sampling period as T_T smaller than the previous basic sampling period, and then go to the step 5, else if N_P is larger than N_R^M, then terminate the algorithm. The tolerant value (T_T) can be set as $min\{M_{S_i}^j - T_O,\ M_C^i - T_O\}$.)
8. For each basic sampling period, $T_B^j (j = 1, \cdots, N_E)$, allocate the medium for sensor nodes and controller nodes using the scheduling rules.

If the scheduling does not satisfy all the MADTs of loops, then other high speed network protocols should be selected or numbers of nodes should be reduced.

4. Examples

To present an example for the presented algorithm, three dc motors connected to a network are presented. Each motor has an armature position controller. If a constant gain (K) is used as a state feedback controller, the equation (4) is changed to

$$\dot{x}_p(t) \quad = \quad F_p x_p(t) + G_p * K x_p(t-\tau) \qquad (13)$$

as a total system with a controller, where $\tau = \tilde{\tau}_c + \tilde{\tau}_{sc} + \tilde{\tau}_{ca}$. Using the notation in the last section, the followings are given :
$l_{RS} = 9,\ l_N = 1,\ P = 3,\ N = \sum_{j=1}^{P}(l_C^j + l_S^j) + l_N = 10$,
$D_{S_i}^j = 0.1msec$ for $i = 1, 2,\ j = 1, 2, 3$,
$D_C^j = 0.15msec$ for $i = 1, 2,\ j = 1, 2, 3$,
$T_D^1 = 3msec, T_D^2 = 6msec, T_D^3 = 7msec$,
$\lambda_{RA} = 0.375, U_{NRA}^m = 0.16$. In this example, the transmission speed is assumed to be 2.5 Mbps regardless of the given network protocols for an equal comparison between the central control and the token control. $M_{S_i}^j$ and M_C^j are 0.16 $msec$. T_O is 0.08 $msec$ in the token control and 0.16 $msec$ in the central control. T_{RA}^M is 0.08 $msec$ and T_{NRA}^M is 0.16 $msec$. $\hat{T}_{RA}^M$ is 0.16 $msec$ in the token control network and 0.24 $msec$ in the central control network. T_O^x is 1.6 $msec$ in the token control and 0 in the central control.

Applying the steps 1 $\sim$ 6 of the scheduling algorithm to the example, N_P is calculated as 12 and N_E is 2. And N_R^M is 2.67 in the token control and 6.72 in the central control. Hence 2 and 6 (nodes) can be scheduled in the token control and the central control network, respectively. Therefore All nodes can not be scheduled using the token control but using the central control from this calculation. Then following the repetition steps of the scheduling algorithm, the sampling time can be reduced in the case of the central control.

The simulation results in the central control case are shown in Fig. 5.. In Fig. 5., the outputs (ω, θ) of the motor position control system in which a controller, sensors, and an actuator are connected directly or connected using a network are displayed. The behaviors of the outputs in the NBCS are similar to those in the directly connected systems from Fig. 5..

5. Conclusions

In this paper, MADB's were obtained for the stability of the NBCS and were used as requirements for a network scheduling method. And a network scheduling method was presented, which guarantees real-time transmissions of real-time synchronous data and periodic data, and possible transmissions of non real-time asynchronous data. In NBCS, the presented method is very useful since it gives a solution to determine the sampling periods of each control loop and it can indicate whether the pre-determined network protocol is possible for the given control systems or not. To show usefulness of the proposed method for NBCS, an example was presented.

참 고 문 헌

[1] Yoram Halevi and Asok Ray, "Integrated Communication and Control Systems : Part I - Analysis," *Journal of Dynamic Systems, Measurement, and Control,* Vol. 110, pp.367-373, Dec., 1988.

[2] Asok Ray and Yoram Halevi, "Integrated Communication and Control Systems : Part II - Design Consideration," *Journal of Dynamic Systems, Measurement, and Control,* Vol. 110, pp.374-381, Dec., 1988.

[3] R. Krtolica, Ü. Özgüner, H. Chan, H. Göktas, J. Winkelman, and M. Liubakka, "Stability of Linear Feedback Systems with Random Communication Delays," *Int. J. Control,* Vol. 59, No. 4, 1994.

[4] Takehiro Mori, Norio Fukuma, and Michiyoshi Kuwahara, "Simple stability criteria for single and composite linear systems with time delays," *International Journal of Control,* Vol.34, No.6, pp.1175-1184, 1981.

[5] Te-Jen Su and Chuan-Guey Huang, "Robust Stability of Delay Dependence for Linear Uncertain Systems," *IEEE Tr. on AC,* Vol.37, No.10, pp.1656-1659, 1992.

[6] Pin-Lin Liu and Te-Jen Su, "On a Sufficient Condition for Robust Stability of Delay Systems," *Proc. of ACC,* pp.328-329, June, 1995.

[7] P.Raja, K.Vijayananda, and J.D.Decotignie, "Polling Algorithms and their Properties for Fieldbus Networks," *IECON'93*, pp.530-534, 1993.

[8] J.P.Beauvais and A-M.Deplanche, "Heuristics for Scheduling Periodic Complex Real-time Tasks in a Distributed System," *IECON'95*, pp.55-60, 1995.

[9] Seung Ho Hong, "Scheduling Algorithm of Data Sampling Times in the Integrated Communication and Control Systems," *IEEE Tr. on CST,* Vol. 3, No. 2, June, 1995.

[10] Jack K. Hale and Sjoerd M. Verduyn Lunel, ***Introduction to Functional Differential Equations***, Springer-Verlag, pp.151-161, 1993.

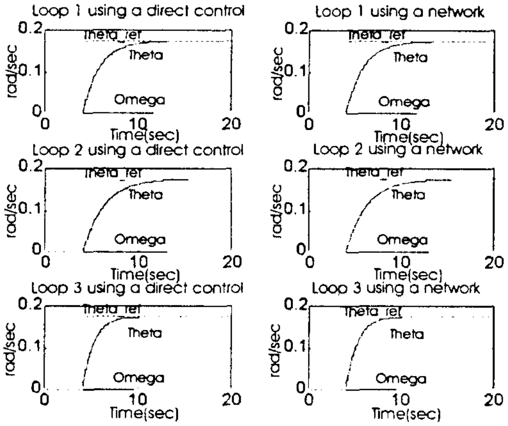

그림 1. 모터 위치 제어 모의실험 결과.

Fig. 1. Outputs of motor position control.