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Cascade 제어를 위한 실시간 공정 식별법

On-line Process Identification for Cascade Control Systems

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Abstracts In this paper, a new identification method of the cascade control system is proposed which can overcome the weak points of Krishnaswamy and Rangaiah(1987)'s method. This new method consists of two steps. One is on-line process identification using the numerical integration to approximate the two process dynamics with a high order linear transfer function. The other is a model reduction technique to derive out low order transfer function(FOPTD or SOPTD) from the obtained high order linear transfer function to tune the controller using usual tuning rules. While the proposed method preserves the advantages of the Krishnaswamy and Rangaiah(1987)'s method, it has such a simplicity that it requires only measured input and output data and simple least-squares technique. Simulation results show that the proposed method can be a promising alternative in the identification of cascade control systems.

Keywords Cascade Control System, On-line Process Identification, Numerical integration, Model Reduction

1. Introduction

To effectively reject disturbances which are introduced to processes with large time constants or time delays, cascade control systems are widely used based on the secondary measurements and controller.

In cascade control systems as shown in Fig. 1, there are two feedback loops which is known as the primary and secondary loop, respectively. The secondary controller acts to make the secondary output as close as possible to primary controller output. Extensive investigations about cascade control systems can be found in the literature. However, most of the previous researches are based on the assumption that the process model is available or known as first- or second-order plus time delay(FOPTD or SOPTD) model and concerned about the design and analysis of cascade control systems.

In the experimental approach, there are various identification methods which are highly efficient and easy-to-use for controller design in SISO system. For example, step or pulse test, P-controller method(Yuwana and Seborg), relay feedback method(Åström and Hägglund), ATV method(Luyben). However, these identification methods which acquire process information from experimental test may have severe disadvantages when they are applied to the cascade control system. That is, (i) Two tests are required to identify the primary and secondary process, respectively. (ii) The tests should be performed in open-loop manner because the interaction between primary and secondary process can not be avoided with both loops closed.

These issues have been addressed by Krishnaswamy and Rangaiah(1987). And they proposed a method which is based on a single closed-loop dynamic test and has no restriction type of test signal generator(step, pulse, impulse generator with P, PI or PID). In their identification method, a set point change is introduced with both loops closed and two controllers arbitrarily tuned. Then, the primary process is identified using the frequency responses obtained from Fourier transform of transient responses(primary and secondary output) and secondary process using the inverse Nichols chart technique. Even though their method is efficient compared to the conventional methods, it still has several shortcomings. First of all, in the derivation of primary process model, the approximation error can be amplified since it is obtained from the ratio of two transformed responses. Secondly, the modeling error of the obtained model for primary process is included in the derivation of secondary process model and may degrade the model accuracy for secondary process. Thirdly, it needs Fourier transformation and inverse Nichols chart technique. These make their procedure too complicated to be applied to the industrial processes. Finally, the method can not identify systematically model parameters such as steady-state gain, time constant and time delay. Hence, it has several limitations in the application. For example, in the autotuning of the PID controller, many model-based tuning methods such as ITAE, IMC tuning rule can not be used but only Ziegler-Nichols-type tuning rule can be used. If one wants to know the parameters of simple model such as FOPTD or SOPTD, one should read the values from the Bode plot which should be drawn based on the information from experiment. It is time-consuming and can't guarantee the accuracy of model parameters.

Therefore, we propose a new on-line process identification method for the cascade control system which preserves the advantages of the Krishnaswamy and Rangaiah(1987)'s method and can overcome the mentioned disadvantages. This new method consists of two steps. One is on-line process identification using the numerical integration to approximate the two process dynamics with a high order linear transfer function. The other is a model reduction technique to derive low order transfer function(FOPTD or SOPTD) from the high order linear transfer function to tune the controller using usual tuning rules. It has such a simplicity that it requires only measured input and output data and simple least-squares technique. Simulation results show that the proposed method can be a promising alternative in the identification of cascade control systems.

2. Process Identification using numerical integration

Before the discussion of the process identification for cascade control systems, we will consider an identification method using numerical integration in the general SISO system. Assuming that the unknown SISO system is linear time invariant(LTI), the transfer function of process to be identified, $G(s)$, can be generally expressed as follows.

$$G(s)=\frac{y(s)}{u(s)},\quad y(s)=\int_0^\infty y(t)e^{-st}dt,\quad u(s)=\int_0^\infty u(t)e^{-st}dt \qquad (1)$$

$y(s)$ and $u(s)$ is the Laplace transform of process output and input, respectively. The integral in the continuous time domain can be approximated as a sum in the discrete time domain. That is, the increment in the integral of transient response weighted by exponential function can be approximated during the small sampling interval(Δt) at $t=t_j$ as follows.

$$\Delta y(s)\big|_{t=t_j}=\int_{t_j}^{t_j+\Delta t}y(t)e^{-st}dt\cong y(t_j)\cdot\int_{t_j}^{t_j+\Delta t}e^{-st}dt=y(t_j)\cdot\frac{e^{-st_j}(1-e^{-s\Delta t})}{s} \qquad (2)$$

$$\Delta u(s)\big|_{t=t_j}=\int_{t_j}^{t_j+\Delta t}u(t)e^{-st}dt\cong u(t_j)\cdot\int_{t_j}^{t_j+\Delta t}e^{-st}dt=u(t_j)\cdot\frac{e^{-st_j}(1-e^{-s\Delta t})}{s} \qquad (3)$$

where $t_j=t_{j-1}+\Delta t$

Here, $y(t_j)$ and $u(t_j)$ denotes the measurement of the output and input at $t=t_j$, respectively. Then, the approximation of the Laplace transform of output($\tilde{y}$) and input($\tilde{u}$) can be obtained in the form of summation as follows.

$$\tilde{y}(s)=\sum_{j=1}^{n_t}\Delta y(s)\big|_{t=t_j}=\sum_{j=1}^{n_t}\frac{e^{-st_j}(1-e^{-s\Delta t})}{s}y(t_j) \qquad (4)$$

$$\tilde{u}(s)=\sum_{j=1}^{n_t}\Delta u(s)\big|_{t=t_j}=\sum_{j=1}^{n_t}\frac{e^{-st_j}(1-e^{-s\Delta t})}{s}u(t_j) \qquad (5)$$

where n_t is the number of sampling in the identification procedure. Therefore, the approximated transfer function can be obtained by the ratio of input-output approximations(Eq.(4) and (5)), that is, $\tilde{G}(s)=\frac{\tilde{y}(s)}{\tilde{u}(s)}$.

It can be assumed that the process to be identified has the following model which is a general linear transfer function($n\ge m$).

$$G(s)=\frac{b_ms^m+b_{m-1}s^{m-1}+\cdots+b_1s+b_0}{a_ns^n+a_{n-1}s^{n-1}+\cdots+a_1s+1} \qquad (6)$$

The effect of model order(mth/ n th) on the identification performance will be discussed in the later section. The model parameters in Eq.(6) can be obtained through the minimization of sum of squared errors between the approximated($\tilde{G}(s_i)$) and assumed($G(s_i)$) transfer function at the various values of Laplace variable s.

$$\underset{\substack{a_n,a_{n-1},\cdots,a_1\\ b_m,b_{m-1},\cdots,b_0}}{Minimize}\sum_{i=1}^{n_s}e(s_i)^2 \qquad (7)$$

where

$$e(s_i)=\tilde{G}(s_i)-\left(\begin{array}{l}b_ms_i^m+b_{m-1}s_i^{m-1}+\cdots+b_1s_i+b_0\\ -a_ns_i^n\tilde{G}(s_i)-a_{n-1}s_i^{n-1}\tilde{G}(s_i)-\cdots-a_1s_i\tilde{G}(s_i)\end{array}\right)$$

and $$\tilde{G}(s_i)=\frac{\tilde{y}(s_i)}{\tilde{u}(s_i)}=\frac{\sum_{j=1}^{n_t}\frac{e^{-s_it_j}(1-e^{-s_i\Delta t})}{s_i}y(t_j)}{\sum_{j=1}^{n_t}\frac{e^{-s_it_j}(1-e^{-s_i\Delta t})}{s_i}u(t_j)}.$$

It can be noted that Eq.(7) is a linear form so that the model parameters can be easily determined though a simple least-squares technique.

Let's introduce the following vectors such as Y ($n_s\times 1$), θ ($(n+m+1)\times 1$) and matrix Φ ($n_s\times(n+m+1)$) for simplicity.

$$Y=\begin{bmatrix}\tilde{G}(s_1)\\ \tilde{G}(s_2)\\ \vdots\\ \tilde{G}(s_{n_s})\end{bmatrix},\quad \Phi=\begin{bmatrix}\varphi^T(s_1)\\ \varphi^T(s_2)\\ \vdots\\ \varphi^T(s_{n_s})\end{bmatrix},\quad \varphi(s_i)=\begin{bmatrix}1\\ s_i\\ \vdots\\ s_i^m\\ -s_i\tilde{G}(s_i)\\ -s_i^2\tilde{G}(s_i)\\ \vdots\\ -s_i^n\tilde{G}(s_i)\end{bmatrix},\quad \theta=\begin{bmatrix}b_0\\ b_1\\ \vdots\\ b_m\\ a_1\\ a_2\\ \vdots\\ a_n\end{bmatrix} \qquad (8)$$

Then Eq.(7) can be rewritten in a simple form as follows

$$\underset{\theta}{Minimize}\|Y-\Phi\theta\|^2 \qquad (9)$$

The well-known unique minimum of Eq.(9) can be found as follows.

$$\hat{\theta}=(\Phi^T\Phi)^{-1}\Phi^TY=\begin{bmatrix}\hat{b}_0 & \hat{b}_1 & \cdots & \hat{b}_m & \hat{a}_1 & \hat{a}_2 & \cdots & \hat{a}_n\end{bmatrix}^T \qquad (10)$$

This can be thought as finding the coefficients of rational function(Eq.(6)) with the known output measurements($\tilde{G}(s_i)$) at different values of independent variable(s_i). Therefore, the more accurate values of the coefficients could be obtained with the broader range of s and smaller interval in the least squares calculation. However, the following recommendation for the range of variable s_i is found to be sufficient from the extensive simulation studies.

$$s_{\min}\le s_i\le s_{\max} \qquad (11)$$

where

$$s_i=\frac{1}{t_{\min}+i\cdot\Delta t},\quad s_{\min}=\frac{1}{t_{\max}}=\frac{1}{t_{CL,r}} \text{ and } s_{\max}=\frac{1}{t_{\min}}=\frac{1}{(\theta_{CL}/2)}$$

Δt, $t_{CL,r}$, θ_{CL} denotes the sampling time of output measurement, the rise time and the apparent time delay of closed-loop response, respectively. The apparent time delay can be determined from the initial part of time response. The summation in Eq.(4) or (5) should be done up to the time(t_f) at which the following condition is satisfied.

$$e^{-s_{\min}t_f}\le 10^{-5} \qquad (12)$$

The detailed analysis on the effect of the variation of s_i range will be given in the simulation study.

3. On-line Process Identification of cascade control system

The same technique which is introduced in the previous section can be applied to identify the primary and secondary process, respectively, in the cascade control system. The two processes can be identified simultaneously from the transient responses of primary, secondary output and secondary controller output generated by the single closed-loop test. Fig. 2 shows typical time responses of the primary and secondary output to the step change of set-point. It can be noted that the value of $e^{-s_{\min}t}$ goes fast to zero as time passes.

Let the transfer function of primary and secondary process(G_{p1} and G_{p2}) be as follows, respectively.

$$G_{p1}(s)=\frac{y_1(s)}{y_2(s)}=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+1} \quad (13)$$

$$G_{p2}(s)=\frac{y_2(s)}{u_2(s)}=\frac{d_k s^k+d_{k-1}s^{k-1}+\cdots+d_1 s+d_0}{c_l s^l+c_{l-1}s^{l-1}+\cdots+c_1 s+1} \quad (14)$$

In the same way described in the previous section, the both model parameters(a, b and c, d) can be obtained by measuring the transients($y_1(t)$ and $y_2(t)$ for primary process, $y_2(t)$ and $u_2(t)$ for secondary process) and applying least squares technique for each high order model, respectively. The errors for both processes are defined as follows. For primary process, $e_1=Y_1-\Phi_1\theta_1$ with

$$Y_1=\left[\tilde{G}_{p1}(s_1)\ \tilde{G}_{p1}(s_2)\ \cdots\ \tilde{G}_{p1}(s_{n_s})\right]^T,$$

$$\Phi_1=\left[\varphi_1^T(s_1)\ \varphi_1^T(s_2)\ \cdots\ \varphi_1^T(s_{n_s})\right]^T,$$

$$\varphi_1(s_i)=\left[1\ \ s_i\ \cdots\ s_i^m\ \ -s_i\tilde{G}_{p1}(s_i)\ \ -s_i^2\tilde{G}_{p1}(s_i)\ \cdots\ -s_i^n\tilde{G}_{p1}(s_i)\right]^T$$

$$,\theta_1=\left[b_0\ b_1\ \cdots\ b_m\ a_1\ a_2\ \cdots\ a_n\right]^T \quad (15)$$

and for secondary process, $e_2=Y_2-\Phi_2\theta_2$ with

$$Y_2=\left[\tilde{G}_{p2}(s_1)\ \tilde{G}_{p2}(s_2)\ \cdots\ \tilde{G}_{p2}(s_{n_s})\right]^T,$$

$$\Phi_2=\left[\varphi_2^T(s_1)\ \varphi_2^T(s_2)\ \cdots\ \varphi_2^T(s_{n_s})\right]^T,$$

$$\varphi_2(s_i)=\left[1\ \ s_i\ \cdots\ s_i^k\ \ -s_i\tilde{G}_{p2}(s_i)\ \ -s_i^2\tilde{G}_{p2}(s_i)\ \cdots\ -s_i^l\tilde{G}_{p2}(s_i)\right]^T$$

$$,\theta_2=\left[d_0\ d_1\ \cdots\ d_k\ c_1\ c_2\ \cdots\ c_l\right]^T \quad (16)$$

Then, applying least-squares technique to both processes, the model parameters of Eq.(13) and (14) can be obtained in the same way to obtain Eq.(10).

$$\hat{\theta}_1=\left[\hat{b}_0\ \hat{b}_1\ \cdots\ \hat{b}_m\ \hat{a}_1\ \hat{a}_2\ \cdots\ \hat{a}_n\right]^T \quad (17)$$

$$\hat{\theta}_2=\left[\hat{d}_0\ \hat{d}_1\ \cdots\ \hat{d}_k\ \hat{c}_1\ \hat{c}_2\ \cdots\ \hat{c}_l\right]^T \quad (18)$$

It should be noted that the identification procedure for each process is conducted independently so that the accuracy of one model is not affected by that of the other in the proposed method.

4. Simple Model Reduction

We present a simple model reduction technique based on the frequency data sets. As mentioned earlier, the simple model structure such as FOPTD or SOPTD model is often used to represent the process dynamics. In this approach, the SOPTD model is adopted because the secondary output usually shows the oscillatory response to the step outer-loop controller output. The model reduction technique will be applied to both high order approximations independently.

Let's consider the following reduced second-order plus time delay model.

$$G_m(s)=\frac{K_m e^{-\theta_m s}}{\tau_m^2 s^2+2\xi_m\tau_m s+1} \quad (19)$$

Here, the static gain of the model, K_m is determined from the high order approximation($\tilde{G}(s)$) with $s=0$.

$$\hat{K}_m=\tilde{G}(0) \quad (20)$$

That is, for the primary process, $\hat{K}_{m1}=\tilde{G}_{p1}(0)=\hat{b}_0$ and for secondary process, $\hat{K}_{m2}=\tilde{G}_{p2}(0)=\hat{d}_0$.

The amplitude ratio of the model is given as follows.

$$|G_m(j\omega)|=\left|\frac{\hat{K}_m e^{-\theta_m s}}{\tau_m^2 s^2+2\xi_m\tau_m s+1}\right|_{s=j\omega}=\frac{\hat{K}_m}{\sqrt{(1-\tau_m^2\omega^2)^2+(2\xi_m\tau_m\omega)^2}} \quad (21)$$

Using Eq.(21), the difference between the squares of the amplitude ratio of the approximated process transfer function and that of the second-order plus time delay model transfer function can be given

$$|G_m(j\omega)|^2-|\tilde{G}(j\omega)|^2=\frac{\hat{K}_m^2-|\tilde{G}(j\omega)|^2\{(1-\tau_m^2\omega^2)^2+(2\xi_m\tau_m\omega)^2\}}{(1-\tau_m^2\omega^2)^2+(2\xi_m\tau_m\omega)^2} \quad (22)$$

where $|\tilde{G}(j\omega)|$ denotes the amplitude ratio of the high order approximatd transfer function.

Let's define the numerator of Eq.(22) as an error, $e(\omega)$ at an arbitrary frequency(ω) as follows.

$$\begin{aligned}e(\omega)&=\{\hat{K}_m^2-|\tilde{G}(j\omega)|^2\}\\&-2(2\xi_m^2-1)\tau_m^2\cdot\{|\tilde{G}(j\omega)|^2\omega^2\}-\tau_m^4\cdot\{|\tilde{G}(j\omega)|^2\omega^4\}\end{aligned} \quad (23)$$

Note that values in the parenthesis in Eq.(23) are known. It is noteworthy that we have a linear equation for unknown model parameters(τ_m and ξ_m) which can be solved by a simple least squares technique since amplitude ratio is not affected by the time delay term.

$$\underset{\tau_m,\xi_m}{Minimize}\sum_{i=1}^{M}(e(\omega_i))^2 \quad (24)$$

where ω_i denotes the i th frequency which can be generated in the identification algorithm. Defining Y,Φ,θ again as follows, we can use the previous least squares formulation(Eq.(9)).

$$Y=\begin{bmatrix}\hat{K}_m^2-|\tilde{G}(j\omega_1)|^2\\ \hat{K}_m^2-|\tilde{G}(j\omega_2)|^2\\ \vdots\\ \hat{K}_m^2-|\tilde{G}(j\omega_M)|^2\end{bmatrix},\quad \Phi=\begin{bmatrix}\varphi^T(\omega_1)\\ \varphi^T(\omega_2)\\ \vdots\\ \varphi^T(\omega_M)\end{bmatrix},$$

$$\varphi(\omega)=\begin{bmatrix}2|\tilde{G}(j\omega)|^2\omega^2\\ |\tilde{G}(j\omega)|^2\omega^4\end{bmatrix},\quad \theta=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}=\begin{bmatrix}(2\xi_m^2-1)\tau_m^2\\ \tau_m^4\end{bmatrix} \quad (25)$$

The frequency range taken into account in the least squares calculation is recommended from low frequency up to the ultimate frequency(ω_u) because the controller is usually operated under the ultimate frequency. Therefore, ω_M in Eq.(25) is set to ω_u.

Using the result($\hat{\theta}^T=\left[\hat{\theta}_1\ \ \hat{\theta}_2\right]$) of least squares technique in Eq.(10), we can easily estimate the time constant($\hat{\tau}_m$) and damping factor($\hat{\xi}_m$) as follows.

$$\hat{\tau}_m=\sqrt[4]{\hat{\theta}_2}\ \text{ and }\ \hat{\xi}_m=\sqrt{\frac{1}{2}\left(1+\frac{\hat{\theta}_1}{\sqrt{\hat{\theta}_2}}\right)} \quad (26)$$

Using the obtained time constant and damping factor, the time delay($\hat{\theta}_m$) of the reduced second-order plus time delay model can be calculated by the following simple phase angle relation at the ultimate frequency(ω_u).

$$\hat{\theta}_m=\frac{\pi+\arctan 2(-2\hat{\xi}_m\hat{\tau}_m\omega_u,1-\hat{\tau}_m^2\omega_u^2)}{\omega_u} \quad (27)$$

As a result, using Eq.(20), (26), (27), we can obtain the second-order plus time delay model from the high order process model.

5. Simulation study and Results

In this section, the performances of the proposed method are illustrated through several simulations. The process transfer functions are considered in all the following simulations.

$$G_{p1} = \frac{1}{(30s+1)(3s+1)} \text{ and } G_{p2} = \frac{1}{(s+1)^2(10s+1)}$$

The PI and P controller is respectively used as the primary and secondary controller and the controller parameters are as follows.

$$G_{c1} = 6(1+\frac{1}{8s}) \text{ and } G_{c2} = 8$$

The considered processes and controllers are adopted from the Krishnaswamy and Rangaiah(1987). As a first step, a small step change in set point is introduced and then measure the transient responses of both process outputs(y_{p1} and y_{p2}) and secondary controller output(u_2). Specify the primary and secondary high order linear model structures, that is, choose the orders of numerator and denominator in the models. Apply least squares technique and determine the coefficients of high order primary model using the measurements of y_{p1} and y_{p2}. In the same manner, secondary model can be obtained by using the measurements of y_{p2} and u_2. Finally, obtain the parameters of SOPTD models(primary and secondary) using the model reduction technique. In this simulation, 4th/5th linear models are applied as the primary and secondary model, respectively. In the AR(amplitude ratio) and PA(phase angle) plots, good agreements among the three curves(process, high order model and SOPTD model) can be noted in the low and middle frequency region.

It should be noted that the broader range of s_i value is considered in the least squares calculation, the more accurate high order approximation can be obtained. Therefore, the simulations are conducted to investigate whether the uniform identification performance can be obtained or not for the choice of s_{min} and s_{max} according to the recommended rule. It can be noted from the simulation results that the processes can be identified with a high accuracy irrespective of s_{min} and s_{max} value use in the least squares calculation.

And simulation results show that almost similar SOPTD models can be obtained and the proposed method is little affected by measurement noises and unmeasured input disturbances. Moreover, the proposed identification method can approximate the primary and secondary processes without regard to the secondary controller saturation.

6. Conculsions

In this paper, on-line process identification method of cascade control system is proposed. This approach is based on the numerical integration and simple model reduction using least squares technique. The advantages of the proposed method is that it needs only input-output transient response data from the single closed-loop test. There are no limitations in the choice of the test signal generator. Also, the shortcomings of Krishnaswamy and Rangaiah(1987)'s method can be tackled because no dependency between the obtained primary and secondary models exists in the proposed identification procedure. Simulation study shows that the uniform performance can be obtained in the face of the variations in the identification parameter and model structure and it has the robustness to measurement noises and unmeasured input disturbances. Moreover, good performances can be obtained even in the practical situations such as the saturation of the final control element in the secondary loop.

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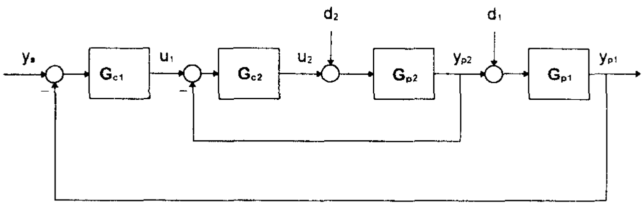

Fig. 1 The block diagram of cascade control system

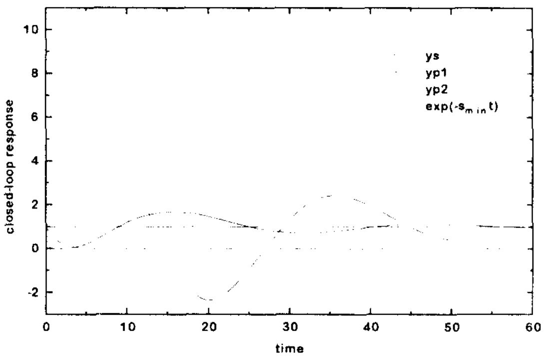

Fig. 2 Transient responses to the step change in the set-point