Proceedings of the 11th
KACC, October 1996

2개의 유연한 링크를 갖는 매니퓰레이터의 설계및 제어
(Design and Control of Two-Link Flexible Manipulators)

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Abstracts In this paper, we propose a design method and control law for planar type two-link flexible manipulator. In designing flexible links, we use Rayleigh's principle. To control flexible manipulator, input distribution controller is used , which is primarily on the basis of nonlinear variable structure control(VSC). The simulation results are also shown.

Keywords Flexible mode, Nonlinear dynamics, Variable structure control, Zero dynamics

1. INTRODUCTION

The light weight, low power consumption and safety are main reasons for the use of flexible manipulator over rigid one. Some areas like space projects use flexible manipulators despite the difficulties of control to utilize the pre-mentioned advantages. Most of the difficulties associated with flexible manipulators come from the insufficient number of control input to cover the motional degree of freedom that consists of rigid motion and vibration due to the link flexibility. Moreover, non-colocated sensing gives the system non-minimum phase property which limits the control performance. Many researches have been done to find out the dynamic structures of flexible manipulators and developed the control techniques and performance. However, the control examples are generally on the single-link flexible manipulators and multi-link flexible manipulators seem to be not so plentiful. Also, the design issue of flexible manipulator has not been fully discussed. The dynamic modelings of multi-link flexible manipulators were performed by Book [1] and by Luca and Siciliano [6]. Cannon and Schmitz [2] performed the initial experiment for the one-link flexible robot with non-colocated sensing using end-point feedback. Cetinkunt and Yu [3] did comparative study for the closed-loop behavior of a feedback controlled flexible arm. In these works, various boundary conditions and various mathematical models are compared and verified. The method of singular perturbation approaches have been also proposed by Siciliano and Book [8] and Lewis and Vandegrift [4]. Luca and et al[5] and W. Yim[9] addressed zero dynamic stability of flexible manipulators and output correction.

In the following section, design issue of two-link flexible manipulator is considered. In section 3, we deal with general control law to stabilize the dynamic systems which have less number of control input than that of motional degree of freedom. In section 4, the simulation results are shown and finally conclusion is given in section 5.

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2. DESIGN OF TWO-LINK FLEXIBLE MANIPULATOR

In the design of two-link flexible manipulator, the cross sectional shape of each link is chosen in such a way that in moving direction, link is sufficiently flexible and in gravity force direction, link is stiff enough. Different from the design of general rigid manipulator, the object of flexible one is that each link should have pre-designed fundamental frequency with an assumption that vibration of each link is dominated by the fundamental mode of vibration. In this section, we will give two different methods, that is previously known analytic formulation which is difficult to produce design parameters [6] and our approximate method, to determine the fundamental mode of vibration of each link. The result of our method is confirmed by substituting to the analytic formulation.

1. *ANALYTIC FORMULATION*

Let's consider the following vibration model for each link [7]

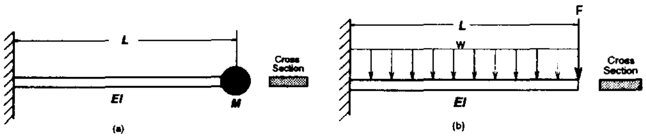
$$EI\frac{\partial^4 u}{\partial x^4} + \bar{\rho}\frac{\partial^2 u}{\partial t^2} = 0, \qquad (1)$$

where EI, u, $\bar{\rho}$ and x are material rigidity, link deflection, mass per unit length and spatial variable, respectively. The appropriate geometric and natural boundary conditions are

$$\begin{aligned} u(0,t) &= 0 \\ \frac{\partial u(0,t)}{\partial t} &= 0 \\ EI\frac{\partial^2 u(L,t)}{\partial x^2} &= -J_P\frac{d^2}{dt^2}\left(\frac{\partial u(L,t)}{\partial x}\right) \\ EI\frac{\partial^3 u(L,t)}{\partial x^3} &= M_P\frac{d^2}{dt^2}u(L,t), \end{aligned}$$

where J_P and M_P are end point moment of inertia and end point mass of nominal configuration. Applying the separation of variables with spatial and time function,

$$u(x,t) = \sum_{i=1}^{n}\phi_i(x)q_i(t),$$



(a) Physical link model (b) Mathematical link model

그림 1: Static deflection and mechanical model with tip mass

$$
\begin{aligned}
q(t) &= e^{j\omega t} \\
\phi'''' - \beta^4 \phi &= 0, \quad \beta^4 = \frac{\bar{\rho}\omega^2}{EI}.
\end{aligned}
\tag{2}
$$

The solution of Eq. (2) has the form such that

$$
\begin{aligned}
\phi(x) = \quad & C_1 \sin\beta x + C_2 \cos\beta x + \\
& C_3 \sinh\beta x + C_4 \cosh\beta x.
\end{aligned}
\tag{3}
$$

If we impose the above boundary conditions, then,

$$
\boldsymbol{F}(\lambda)\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \boldsymbol{0}, \quad \lambda = \beta L, \tag{4}
$$

$$
f_J = J_p \frac{\beta^3}{\bar{\rho}}, \quad f_M = M_p \frac{\beta}{\bar{\rho}}
$$

where F_{11}, F_{12}, F_{21} and F_{22} are, respectively

$$
\begin{aligned}
F_{11} &= \sin\lambda + \sinh\lambda + f_J(\cos\lambda - \cosh\lambda) \\
F_{12} &= \cos\lambda + \cosh\lambda - f_J(\sin\lambda + \sinh\lambda) \\
F_{21} &= \cos\lambda + \cosh\lambda - f_M(\sin\lambda - \sinh\lambda) \\
F_{22} &= -\sin\lambda + \sinh\lambda - f_M(\cos\lambda - \cosh\lambda).
\end{aligned}
$$

The nontrivial solution for Eq. (4) will be the eigenvalues which are directly related with natural frequency of given system. Finally, the homogeneous frequency equation can be obtained by

$$
\det \boldsymbol{F}(\lambda) = 0,
$$

that is,

$$
\begin{aligned}
&(-2 - 2f_J f_M) + (2f_J - 2f_M)\cos\lambda \sinh\lambda + \\
&\quad (-2 + 2f_J f_M)\cos\lambda\cosh\lambda + (2f_J + 2f_M) \\
&\qquad \sin\lambda\cosh\lambda = 0.
\end{aligned}
\tag{5}
$$

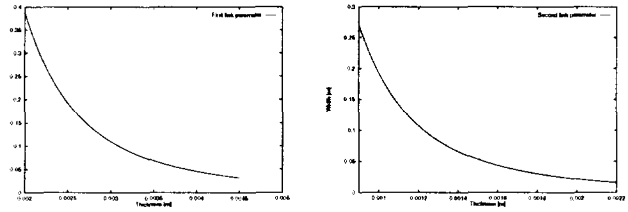
2. *APPROXIMATE METHOD*

Although the above analytic method gives more precise frequency information, it is difficult to solve and find out the feasible set of link cross sectional shape. So we will give approximate method based on the *Rayleigh's principle*. Fig. 1 shows static deflection model of each link and for this, deflection $y(x)$ is given as

$$
\begin{aligned}
y(x) &= \frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2) + \\
&\quad \frac{F}{6EI}(x^3 - 3Lx^2),
\end{aligned}
\tag{6}
$$

where w and F are distributed force due to weight of beam itself and end point force due to concentrated weight respectively. Taking the static deflection $y(x)$ as a trial function, the *Rayleigh quotient* will be given as

$$
\omega^{app2} = \frac{V_{max}}{T^*} = \frac{\int_0^L EIy''^2 dx}{\int_0^L \bar{\rho} y^2 dx + Re}, \tag{7}
$$



(a) Feasible designs for 3Hz First link (b) Feasible designs for 1.5Hz Second link

그림 2: Link design

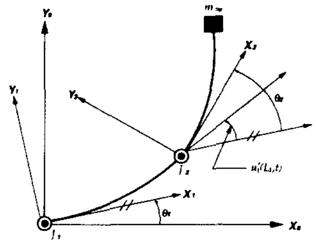


그림 3: Two-link flexible manipulator

where ω^{app} and Re are angular frequency and remaining lumped mass kinetic energy. If we rigorously manipulate Eq. (7), we can obtain the simple algebraic equation such that

$$
\omega^{app2} = \frac{N}{D}, \tag{8}
$$

where N and D are, respectively

$$
\begin{aligned}
N &= EI\left(\frac{3L^5w^2 + 20F^2L^3}{60E^2I^2} - \frac{FL^4w}{4E^2I^2}\right) \\
D &= \frac{M_p(3L^4w - 8FL^3)^2}{576E^2I^2} + \frac{J_p(L^3w - 3FL^2)^2}{36E^2I^2} + \\
&\quad \frac{\bar{\rho}(728L^9w^2 - 3717FL^8w + 4752F^2L^7)}{181440E^2I^2}.
\end{aligned}
$$

For each link of the flexible manipulator, if we apply the above approximate method, we can find out the feasible set of cross sectional shape having the desired link natural frequency. The possible design candidates are shown in Fig. 2. Considering inequality constraints such as static deflection along gravity direction, ultimate strength, and other factors, among the feasible set of design, we can choose appropriate shape. The given link constants and finally chosen cross sectional shapes are summerized in Table 1, 2, respectively. To verify the validity of the approximate method, first mode frequency is calculated for the designed link using analytic formulation in Table 3.

3. CONTROLLER DESIGN FOR TWO-LINK FLEXIBLE MANIPULATOR

1. *PROBLEM FORMULATION*

Fig. 3 shows the schematic diagram of the flexible manipulator and we can derive dynamic equation by evaluating the kinetic and potential energy independently with an assumption that each link is Euler-Bernoulli beam of uniform density and cross section. Truncating higher frequency modes

표 1: Given link constants

Constants	*Link 1*	*Link 2*	*unit*
E	71	71	$[GPa]$
ρ	2710	2710	$[kg/m^3]$
L	0.40	0.45	$[m]$
M_p	1.5	0.1	$[kg]$
J_p	0.05	0.002	$[kgm^2]$
Desired freq.	3	1.5	$[Hz]$

표 2: Designed link parameters

Parameter	*Link 1*	*Link 2*	*unit*
Thickness	0.004	0.0015	$[m]$
Width	0.045	0.0176	$[m]$
Weight	0.196	0.0322	$[kg]$

표 3: Comparison of the design method

Link	*First mode by analytic method*	*Desired value of First mode*
*Link*1	2.955 Hz	3.0 Hz
*Link*2	1.5 Hz	1.5 Hz

of vibration and leaving only lowest frequency mode, the result of Lagrangian dynamics is described by finite order generalized coordinate as follows.

$$M\ddot{q} + C\dot{q} + Kq = Q\tau, \qquad (9)$$

where M, C, $K \in \Re^{4\times4}$, $Q \in \Re^{4\times2}$ and $\tau \in \Re^{2\times1}$. The matrices M, C, K and Q represent inertia matrix, Coriolis and centripetal matrix according to Christoffel's symbol, stiffness matrix and input channel matrix, respectively. The detailed description of elements of each matrix is omitted [1, 6]. When we write this manipulator dynamics into nonlinear input affine form as

$$\dot{x} = f(x) + G(x)u,$$

then,

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0_{4\times1} \\ M^{-1}Q \end{bmatrix} \tau \qquad (10)$$

Finally, we have a nonlinear dynamics which will be plugged in the following controller design.

2. CONTROLLER DESIGN

In this section, we propose a variable structure based controller that faithfully follows desired trajectory. Widely known feedback linearization method divides a nonlinear system into controllable subsystem and zero dynamic subsystem via coordinate transformation. Therefore, the stability of a nonlinear system is determined by zero dynamic subsystem. Unfortunately, it is known that zero dynamics of flexible manipulator is unstable or marginally stable. To modify the eigenvalue characteristics of zero dynamics, some previous works like ?? used feedback linearization with virtual output or output correction. However, the whole characteristics of zero dynamics of multi-link flexible system were not definetely addressed. Let's define a sliding surface as

$$\begin{aligned} s &= \dot{e} + He = (\dot{q} - \dot{q}^d) + H(q - q^d) \\ &= \begin{bmatrix} (\dot{q}_a - \dot{q}_a^d) + H_a(q_a - q_a^d) \\ (\dot{q}_f - \dot{q}_f^d) + H_f(q_f - q_f^d) \end{bmatrix} = 0, \end{aligned} \qquad (11)$$

where H, q_a and q_f represent $\Re^{4\times4}$ diagonal gain matrix, $\Re^2$ actuator position and $\Re^2$ flexible mode, respectively.

PROPOSITION **3..1** *If we take control input as follows, the whole error dynamics of given system asymptotically approaches zero.*

$$u = -\frac{D^T s(s^T b)}{(s^T D)(s^T D)^T} = -\frac{D^T s(s^T b)}{s^T D D^T s}, \qquad (12)$$

where b *and* D *are*

$$W \cdot \frac{\partial s}{\partial t} + W \cdot \frac{\partial s}{\partial x} f(x) + \Lambda s + \frac{1}{2}\dot{W}s$$

and

$$W \cdot \frac{\partial s}{\partial x} G(x)$$

respectively. We call this control as input distribution control.

Proof Consider the following function with a positive definite weighting matrix W.

$$\begin{aligned} V(e, \dot{e}, t) &= \frac{1}{2} s^T W s + e^T H \Lambda e \\ &= \frac{1}{2} \begin{bmatrix} \dot{e}^T e^T \end{bmatrix} J \begin{bmatrix} \dot{e} \\ e \end{bmatrix} \end{aligned} \qquad (13)$$

where J is $\begin{bmatrix} W & WH \\ H^T W & H^T W H + 2H\Lambda \end{bmatrix}$, H and Λ are diagonal positive definite matrices. The leading principal minors of J are

- $\det(W) > 0$ trivially true by assumption
- $\det(J) = \det(W)\det(H^T W H$

$$\begin{aligned} & 2H\Lambda - H^T W W^{-1} W H) \\ =\ & \det(W)\det(2H\Lambda) > 0. \end{aligned}$$

The above function $V(e, \dot{e}, t)$, therefore, can be a Lyapunov function candidate. If we take directional derivative for (13) along the dynamic flow (10),

$$\begin{aligned} \dot{V} &= s^T W \dot{s} + \frac{1}{2} s^T \dot{W} s + 2e^T H \Lambda \dot{e} \\ &= s^T [W\dot{s} + \frac{1}{2}\dot{W}s + \Lambda s - \Lambda s] + 2e^T H \Lambda \dot{e} \\ &= s^T [W \cdot \frac{\partial s}{\partial t} + W \cdot \frac{\partial s}{\partial x}(f + G \cdot u) + \frac{1}{2}\dot{W}s + \\ & \quad \Lambda s] - s^T \Lambda s + 2e^T H \Lambda \dot{e} \\ &= s^T [b + Du] - [\dot{e}^T \Lambda \dot{e} + \dot{e}^T \Lambda H e + \\ & \quad e^T H^T \Lambda \dot{e} + e^T H^T \Lambda H e] + 2e^T H \Lambda \dot{e} \\ &= s^T [b + Du] - [\dot{e}^T \Lambda \dot{e} + e^T H^T \Lambda H e] \end{aligned}$$

Introducing the control input (12) to above equation,

$$\dot{V}(\boldsymbol{e}, \dot{\boldsymbol{e}}, t) = -[\dot{\boldsymbol{e}}^T \boldsymbol{\Lambda} \dot{\boldsymbol{e}} + \boldsymbol{e}^T \boldsymbol{H}^T \boldsymbol{\Lambda} \boldsymbol{H} \boldsymbol{e}] < 0 \quad (14)$$

So, the derivative of Lyapunov function along dynamic flow is strictly negative definite through all time, which means that the above controller moves all the states to zero as time goes to infinity. ■

The role of gain matrix $\boldsymbol{\Lambda}$ is PD control action that contracts state error. The above controller distributes control effort to hold all the state in sliding surface or at least in small neighborhood of sliding surface despite the insufficient number of control input. However, whenever all the state are in the vicinity of sliding surface, the proposed controller is in near singular point. To avoid the singularity in real implementation, we add a small configuration varying positive number in denominator part of the proposed controller as

$$\boldsymbol{u} = \frac{\boldsymbol{D}^T \boldsymbol{s}(\boldsymbol{s}^T \boldsymbol{b})}{\boldsymbol{s}^T \boldsymbol{D}\boldsymbol{D}^T \boldsymbol{s} + \epsilon \exp(-\boldsymbol{s}^T \boldsymbol{D}\boldsymbol{D}^T \boldsymbol{s})}.$$

The slightly modified control shows a property that for large state error, it approaches original input distribution control and for small state error, it deviate from original control for a small amount to avoid singularity.

4. SIMULATION RESULTS

To verify the validity of proposed controller, we conduct a task to follow a desired circle of radius $0.25m$ during $9sec$ for the two-link flexible manipulator. The physical parameters are the same as designed in section 2. in Table 2. The desired output trajectory is generated by fifth order smooth polynomial as

$$c(t) = 20.0\pi(\frac{t}{t_f})^3 - 30.0\pi(\frac{t}{t_f})^4 + 12.0\pi(\frac{t}{t_f})^5$$
$$x^d(t) = x_0 + r\cos(c(t)), \quad y^d(t) = y_0 + r\sin(c(t))$$

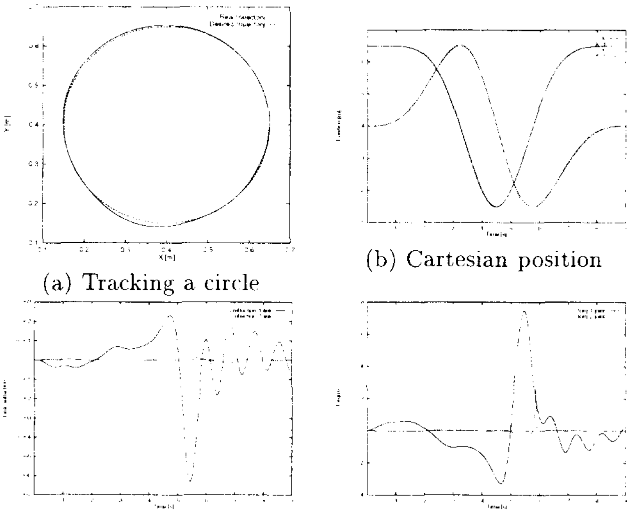
and $\boldsymbol{q}_f^d$ and $\dot{\boldsymbol{q}}_f^d$ are to be zero. If we take the weighting matrix of proposed controller as system inertia matrix, we don't have to calculate of inverse of inertia matrix during the construction of control torque. The gain matrices are chosen as

$$\boldsymbol{\Lambda} = \text{diag}\ \{4.5,\ 3.2,\ 2.8,\ 2.2\}$$
$$\boldsymbol{H} = \text{diag}\ \{7.0,\ 5.0,\ 4.5,\ 3.0\}.$$

The simulation results are shown in Fig. 4. As can be shown in this figure, the tracking performance of the proposed control method is good.

5. CONCLUSION

In this paper, we proposed a design method for two-link flexible manipulator. The result has confirmed using the analytic formulation for flexible body, and the result showed that the proposed method can predict very accurately the first modes of 2 links. To faithfully follow a desired trajectory, we proposed input distribution control for the two-link flexible manipulator which has smaller number of input than that of control variable. The performance of the proposed controller was also verified by simulation. In the near future, we will also verify the proposed design method and controller by experimental study.



(a) Tracking a circle (b) Cartesian position

(c) End def. of each link (d) Input torque

그림 4: Performance of input distribution control

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