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# SELF-TUNING OPTIMAL CONTROL OF AN ACTIVE SUSPENSION USING A NEURAL NETWORK

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Abstracts In this paper, a self-tuning optimal control algorithm is proposed to retain the optimal performance of an active suspension system, when the vehicle has some time varying parameters and parameter uncertainties. We consider a 2 DOF time-varying quarter car model which has the parameter variation of sprung mass, suspension spring constant and suspension damping constant. Instead of solving algebraic riccati equation on line, we propose a neural network approach as an alternative. The optimal feedback gains obtained from the off line computation, according to parameter variations, are used as the neural network training data. When the active suspension system is on, the parameters are identified by the recursive least square method and the trained neural network controller designer finds the proper optimal feedback gains. The simulation results are represented and discussed.

Keywords Active suspension, Self-tuning optimal control, Neural network, Slow time varying

## 1. INTRODUCTION

In recent years, the improvement of vehicle suspension system has been the subject of intense research and development. To improve ride comfort and vehicle maneuverability, a good vehicle suspension system needs to reduce sprung mass acceleration, provide adequate suspension deflection to maintain tire-terrain contact and have small suspension stroke. It is a trade-off for which suspension system must provide.

In order to achieve a good performance of suspension system, several researchers have been studying many kinds of suspension systems. It is only natural that a number of researchers have investigated the use of linear quadratic optimal controllers; typically based on the minimization of some weighted combination of some weighted combination of passenger comport, suspension stroke, and tire deflection[1,2]. But, due to the linear time invariant assumption, where the feedback gains are constant, they cannot cope with the changing operating conditions with time varying characteristics The nonlinear time varying characteristics of the vehicle plant arise from many factors such as uncertainties in vehicle parameters and external disturbances, nonlinear characteristics of the components, signal noise, load variations, etc[3].

Hung Hsu Fred Chen and Dennis A. Guenther proposed a self-tuning optimal control scheme when the weight and the moments of inertia of the sprung mass are varied[4]. But, their simulation results show that the parameter value increment of sprung mass do not have bad effect on the ride comfort. On the contrary, the increment of sprung mass reduce the magnitude of acceleration of sprung mass at the frequency band above the sprung mass resonance frequency. As well as these variations, the slow time varying characteristics of other parameters must be considered. In this paper, the suspension spring constant and damping constant are also considered. And, in order to reduce the sequential computation steps solving an algebraic riccati equation(ARE), the neural network(NN) based controller design scheme is proposed. As an identification method, we use the recursive least square(RLS) method[5].

We assume the range of parameter variations. The nominal parameter values and parameter variations are shown in Table.1. The training data of neural network architecture is based on it. When parameter variations occur, the nominal optimal feedback gain and the feedback gain modified by the proposed algorithm are used respectively in the simulation. And both simulation results are shown and discussed.

## 2. MATHEMATICAL MODEL

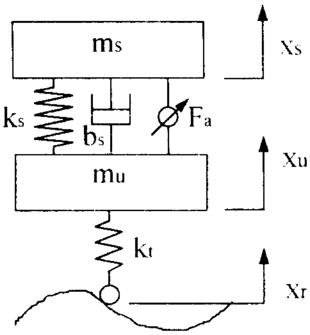



Fig. 1 A quarter car suspension model

The linear model for the 2 DOF quarter-car suspension system illustrated in Fig. 1 can be described by the following equations:

$$
\begin{aligned}
m_s \ddot{x}_s &= -k_s(x_s - x_u) - b_s(\dot{x}_s - \dot{x}_u) + F_a \\
m_u \ddot{x}_u &= k_s(x_s - x_u) + b_s(\dot{x}_s - \dot{x}_u) - k_t(x_u - x_r) - F_a
\end{aligned}
\quad (1)
$$

where

$x_s$ : sprung mass displacement

$x_u$ : unsprung mass displacement

$x_r$ : road profile irregularities

$x_s - x_u$ : suspension deflection

$x_u - x_r$ : tire deflection

We rearrange the above equation into state space equation as follows:

$$\dot{X} = AX + Bu + Ex_r \tag{2}$$

where

$$X = [\dot{x}_s \quad \dot{x}_u \quad x_s - x_u \quad x_u - x_r]^T$$

$$A = \begin{bmatrix} -\frac{b_s}{m_s} & \frac{b_s}{m_s} & -\frac{k_s}{m_s} & 0 \\ \frac{b_s}{m_u} & -\frac{b_s}{m_u} & \frac{k_s}{m_u} & -\frac{k_t}{m_u} \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{m_s} & -\frac{1}{m_u} & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}$$

The nominal values of model parameters and the range of parameter variations are illustrated as Table. 1.

| Parameters | | The nominal values |
|---|---|---|
| Ms | Sprung mass | 280 (kg) |
| Mu | Unsprung mass | 50 (kg) |
| Ks | Suspension spring constant | 23520 (N/m) |
| Bs | Suspension damping constant | 1500 (Nsec/m) |
| Kt | Tire spring constant | 179074 (N/m) |

Table. 1 Quarter car linear model parameters

## 3. SELF-TUNING OPTIMAL CONTROL

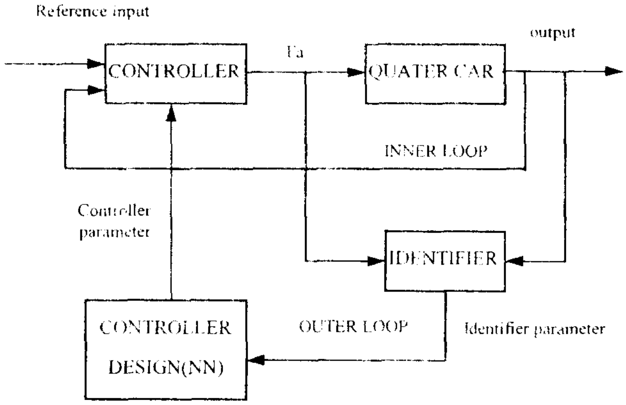



Fig. 2 Self-tuning controller

Fig.2 is a schematic of a self-tuning controller, which is composed of an inner loop and an outer loop. The outer loop is composed of an identifier and controller designer which receives the identified parameters from the identifier and uses them to adjust the optimal feedback gains. And , the inner loop contains the conventional underlying control strategies. In this paper, an optimal control is used. As an identifier, we use the RLS based identifier. And we use a NN architecture as a controller designer.

### 3.1. Identifier

In order to identify the parameters which is Ms,Ks and Bs, the equation (1) can be rearranged as follows:

$$F_a = m_s \ddot{x}_s + k_s (x_s - x_u) + b_s (\dot{x}_s - \dot{x}_u) \tag{3}$$

$$y = \Theta^T \Phi \tag{4}$$

where

$$y = F_a, \Theta^T = \begin{bmatrix} m_s & k_s & b_s \end{bmatrix}, \Phi^T = \begin{bmatrix} \ddot{x}_s & x_s - x_u & \dot{x}_s - \dot{x}_u \end{bmatrix}$$

$\Phi$ and y can be measured, so we can use the RLS method to identify $\Theta$. The RLS algorithm is as follows:

(i) Obtain $\Phi(t)$ using new data

(ii) The covariance matrix P(t) given by:

$$P(t) = (I - K(t)\Phi^T(t))P(t-1)$$
$$K(t) = P(t-1)\Phi^T(t)(I + \Phi^T(t)P(t-1)\Phi(t))^{-1} \tag{5}$$

(iii) The estimated coefficients $\hat{\Theta}(t)$ are updated:

$$\hat{\Theta}(t) = \hat{\Theta}(t-1) + K(t)\left(y(t) - \Phi^T(t)\hat{\Theta}(t-1)\right) \tag{6}$$

### 3.2. Controller Design

In this paper, the full state feedback control is investigated and to provide for the trade off of performance, following optimal control performance index is investigated.

$$PI = \frac{1}{T}\int_0^T (\rho_1 \ddot{x}_s^2 + \rho_2 \dot{x}_u^2 + \rho_3 (x_s - x_u)^2 + \rho_4 (x_u - x_r)^2 + \rho_5 u^2) dt$$
$$= \frac{1}{T}\int_0^T (X^T Q X + u^T R u + 2X^T N u) dt$$
$$= \frac{1}{T}\int_0^T (X^T (Q - NR^{-1}N^T) X + (u + R^{-1}N^T X)^T R (u + R^{-1}S^T X)) dt \tag{7}$$

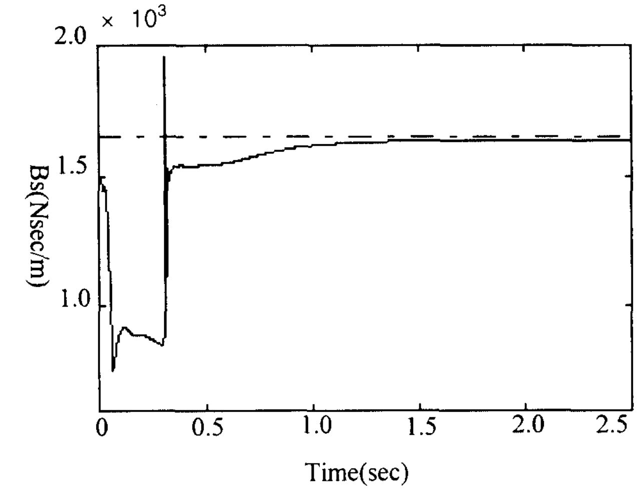
where $(Q - NR^{-1}N^T) \geq 0$. From optimal control theory[6],

$$u(t) = -kX(t) \tag{8}$$

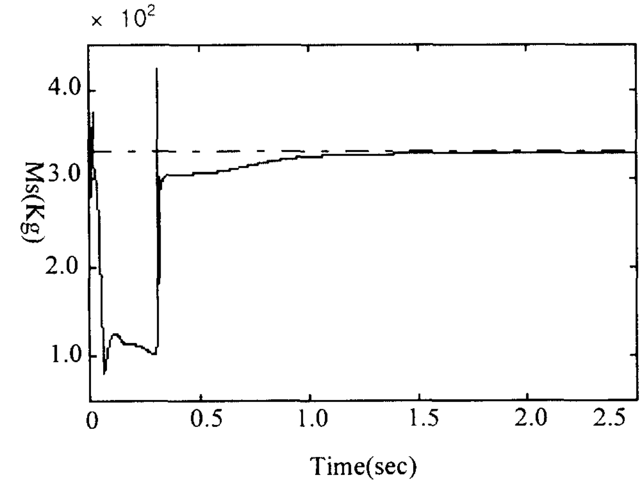
$$k = R^{-1}(B^T \bar{P} + N^T) = [k_1 \; k_2 \; k_3 \; k_4] \tag{9}$$

$$\bar{P}(A - BR^{-1}N^T) + (A^T - NR^{-1}B^T)\bar{P} - \bar{P}BR^{-1}B^T\bar{P} + Q - NR^{-1}N^T = 0 \tag{10}$$

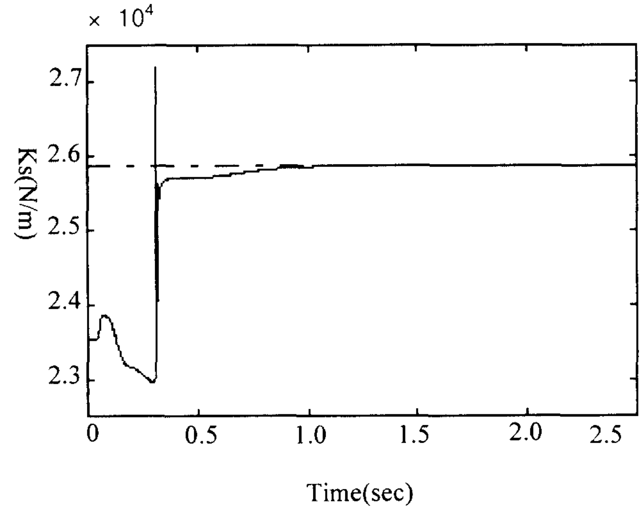
The optimal control gains are computed by (9) and (10) from the data shown in Table.1 with 10% variations. From pairs of the parameters which have variations and respective K gains, the training data of NN is constructed. We use four NN architectures for k1, k2, k3 and k4. The proposed NN has four layers and each layer has 3, 4, 2 and 1 neurons respectively. And as a training method, we use the error backpropagation method[7]. After completing the training of NN, the trained NN outputs the proper optimal feedback gains using the three parameters, which is Ms, Bs and Ks, as the inputs.




(a) Identification of suspension spring constant




(b) Identification of suspension damping constant




(c)Identification of sprung mass

Fig. 3. The Identification results of parameters ( Ks : 25872 N/m, Bs : 1650 Nsec/m, Ms : 330 Kg )

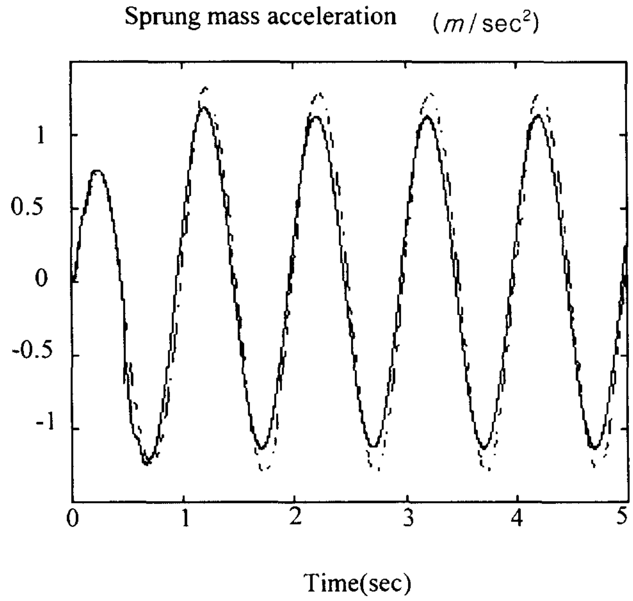



Fig. 4 Sinusoidal road input response

## 4. SIMULATION AND ANALYSIS

The simulation results are represented as follows. For a computer simulation, the MATLAB(the MathWorks, Inc) is used. And the nominal simulation parameters are shown in Table.1.

### 4.1. Identification

As an example, let the Ms(sprung mass) be 330 Kg, Bs(damping constant) be 1650 Nsec/m, and Ks(spring constant) be 25872 N/m.

When the active suspension system is on, the identifier starts to identify the parameters. As illustrated above, the three parameters are identified. The sinusoidal road disturbance input is used for identification. As shown in Fig. 3, the parameters are well identified within about 1sec. The dashdotted lines are the values of the real parameters and the solid lines are the identified values of vehicle. During identification, the trained NN is used. So, the feedback gains are time varying. After the parameters converge to identified values, the feedback gains from the trained NN are fixed.

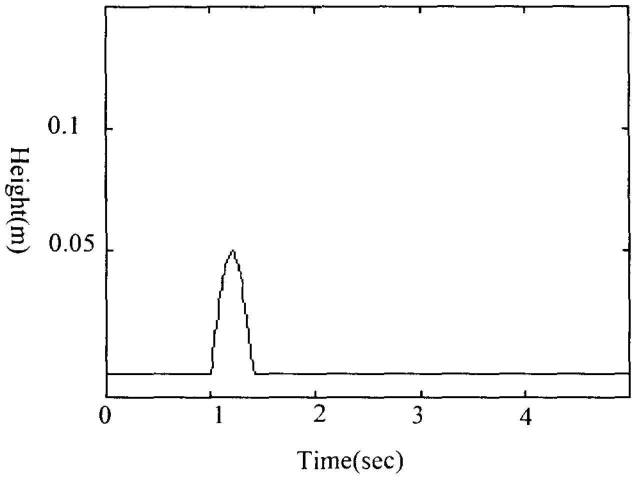
### 4.2. Time response of suspension system

For the ride comfort performance, the sprung mass acceleration is the most important index. Fig. 4 shows the sprung mass acceleration response of sinusoidal road disturbance input, when 1Hz sinusoidal road disturbance input is used and the vehicle has the same parameter variation as above. The dashdotted line is the sprung mass acceleration when the nominal feedback gain is used and the solid line is the response of the self-tuned optimal gain. After 1 sec, the self-tuned optimal gain used response is better than that of nominal feedback gain in the case of

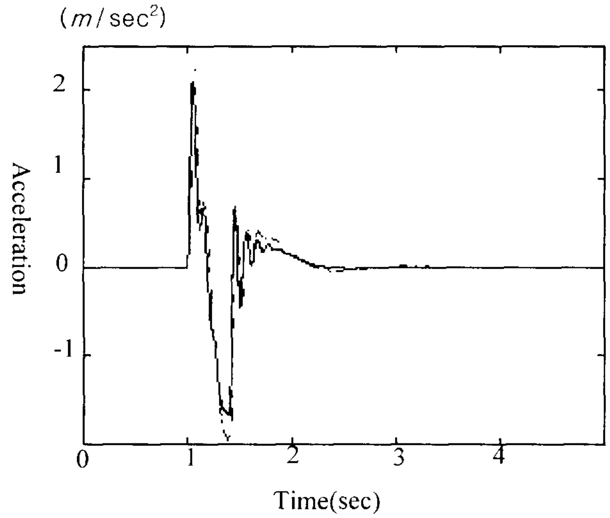
1Hz sinusoidal road disturbance input. Within 1 sec, the parameters which have to be identified is not yet done, it is a transient response.

Fig. 5 shows the response when the bump road input is used. As an example of a bump road input, we let the bump is 2m wide and 5cm high. The shape of bump is assumed to be sinusoidal. And we assume the velocity of vehicle is 18km/hr. At the time domain, the bump is described as shown Fig. 5(a).

In order to see the bump input response of the proposed controller, after the parameters are identified, the bump road input excites the vehicle. Fig. 5 shows the response of bump road input. It can be seen that the self-tuning optimal control has better performance than the nominal feedback control. In the Fig. 5(b), the dashdotted line are the response of nominal feedback gain which is computed from the nominal parameters(Table. 1) and the self-tuning control response is represented as the solid line.




(a) The bump road input




(b) The sprung mass acceleration
( Ms : 290 kg, Bs : 1650 Nsec/m, Ks: 25872N/m )
Fig. 5 The bump road input response

# 5. CONCLUSION

This paper presented an self-tuning optimal control approach using a NN, when the Bs and Ks has the slow time varying characteristics and uncertainties as well as Ms variation. According to parameter variations, instead of solving ARE on line or using gain scheduling approach, which have to store the table of the respective gains, the NN approach is proposed as an alternative. And the simulation results are presented. But, the nonlinearities of vehicle components are not considered. Particularly, the characteristics of tires are not considered in this paper.

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