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Extracting the Axis of Potential Axial Symmetry Employing Variance Minimization

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Abstract Symmetry is one of the important structural properties of shapes both in perceptual psychology and in computer vision. Recently, a number of automatic symmetry finding algorithms have been reported. Among them, the algorithm based on the use of principal axes of objects is the most general and practical. It is, however, of no use when shapes concerned have some asymmetry. Asymmetric shapes which make us associate with certain kinds of symmetry are practically important and they are called shapes with potential symmetry in this paper.

The algorithm we have already proposed can cope with those shapes having potential axial symmetry. The algorithm employs a reflected image of the original and a certain evaluation function. In the former paper, areal minimization was employed for the evaluation function and it yielded satisfactory experimental results. However, it could not cope with those shapes which have larger asymmetry. In this paper, we propose the employment of variance as an alternative evaluation index with respect to the difference image between the reflected and the original shape. The technique is examined its performance by real video images as well as synthetic data. Experimental results are shown and discussion is given.

Keywords potential symmetry, reflection image

1. Introduction

Symmetry is one of the important structural properties of shapes in computer visions well as in perceptual psychology. In recent years, a number of automatic symmetry finding algorithms have been reported[1,2,3]. Among them, the algorithm based on the use of principal axes of objects offers the most general and practical way. The algorithm, however, gives no power to those shapes which have some amount of asymmetry. Asymmetric shapes which let us associate with certain kinds of symmetry are practically important and they are called shapes with potential symmetry in this paper.

The algorithm we have already proposed[4,5,6] can cope with those shapes having potential axial symmetry. The algorithm employs a reflected image of the original image: The reflected image is superposed on the original image by rotation and parallel translation so that a certain evaluation function takes the minimum value. In the former paper, areal minimization was employed for the evaluation function. Although it yielded satisfactory experimental results, it had some difficulty when applied to those shapes with larger asymmetry.

In this paper, we propose an alternative technique which employs variance of the difference image between the reflected and the original shape as an evaluation index. The technique is examined its performance by real video images as well as synthetic data. Some experimental results are shown and discussion is given.

The correspondence is organized as follows. In section 2, after an introduction to the definition of potential symmetry. And in section 3, we explained the proposed algorithm. In section 4, we describe method of superposing the two objects. The results are presented and discussed in section 5. Finally, the conclusion and some ideas about further work are contained in section 6.

2. Definition of potential symmetry

Various forms of potential symmetry exist in nature. Though a face of man, for example, is not exactly symmetric, it almost has bilateral symmetry. We define potential symmetry as follows:

a) It is formed with almost symmetric shape(e.g., a human face).

b) It is formed with symmetric shape which includes asymmetric part(e.g., a cup with a gripper).

c) Deformed shape that was previously symmetric shape(e.g., a deformed can).

3. Extracting the axis of potential axial symmetry

In this section, we present the method of extracting the axis of potential axial symmetry. Figure 1 shows the flow chart of the extracting method.

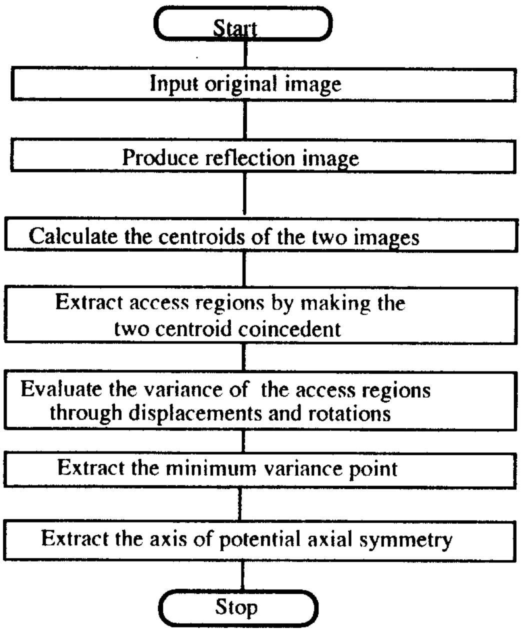


Figure 1 Flow chart of the extracting potential axial symmetry

Firstly, the reflection image is produced from the original input image. The centroids of the original image and the reflection image are then calculated.

After the two centroids were made coincident, difference regions of the two images are extracted. Through displacements and rotations of the reflection image, variance of the difference region is evaluated according to Eq.(1).

Finally, the axis of potential axial symmetry is extracted from the difference region which has the minimum variance in the sense of Eq.(2).

$$\left.\begin{aligned} \mu_x &= \frac{1}{N}\sum_{i=1}^{k} f_i x_i \,, \quad \mu_y = \frac{1}{N}\sum_{j=1}^{l} f_j y_j \\ \sigma_x{}^2 &= \frac{1}{N}\sum_{i=1}^{k} f_i (x_i - \mu_x)^2 = \frac{1}{N}\sum_{i=1}^{k} f_i x_i{}^2 - \mu_x{}^2 \\ \sigma_y{}^2 &= \frac{1}{N}\sum_{j=1}^{l} f_j (y_j - \mu_y)^2 = \frac{1}{N}\sum_{j=1}^{l} f_j y_j{}^2 - \mu_y{}^2 \end{aligned}\right\} \quad (1)$$

$$S = \min(\sigma_x{}^2 + \sigma_y{}^2) \qquad (2)$$

In the aforementioned equation;

k is the number of pixels on the shape concerned,

x_i is the x coordinate which ranges from x_1 to x_k,

y_j is the y coordinate which ranges from y_1 to y_l,

f_i is the frequency of x_i, f_j is the frequency of y_j.

4. Method of superposing two objects

In this section, we describe the method of superposing two objects by rotation and parallel translation to detect the best match position between the original and the reflected image.

Assume that the original image is described by the function F(x,y),

$$F(x, y), \quad (x, y) \in R.$$

The reflection image with respect to the y axis is presented by $F^r(x,y)$ where

$$F^r(x, y) = F(-x, y), \quad (x, y) \in R.$$

F and F^r are defined on region R.

Rotation and displacement of the reflection image are evaluated. The resulting rotation matrix is given as follows;

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Here θ is the angle of rotation.

If the displacement of the centroid in the X and Y directions are u and v, respectively, then the final coordinates resulting from the rotation are

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} X' \\ Y' \end{pmatrix} + \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} X'+u \\ Y'+v \end{pmatrix}$$
$$= \begin{pmatrix} x\cos\theta + y\sin + u \\ -x\sin + y\cos + v \end{pmatrix}$$

The displacement u is related to v by the equation

$$u = v\tan\theta/2 \, .$$

From the aforementioned discussion, the general form of the transformation matrix of the reflection image can be written as

$$T = \begin{pmatrix} \cos\theta & \sin\theta & u \\ -\sin\theta & \cos\theta & v \\ 0 & 0 & 1 \end{pmatrix}$$

By superposing the original and the reflection images, a difference region is extracted. This region is expressed by the following equation

$$\Delta(x,y) = \left| F(x,y) - TF^{r}(x,y) \right|$$
$$(x,y) \in R \cup R^{r}$$

in which R^r is the region of TF^r. This region $\Delta(x,y)$ is evaluated using Eqs.(1),(2).

5. Experimental results

Figure 2 shows configuration of the employed hardware system. In this system, pictures obtained from a camera is scanned by an image scanner and fed into an EWS through a personal computer. In case of video camera, images are directly passed into a PC via a video capture board. The program are written in C language. The average processing time is about 18 seconds.

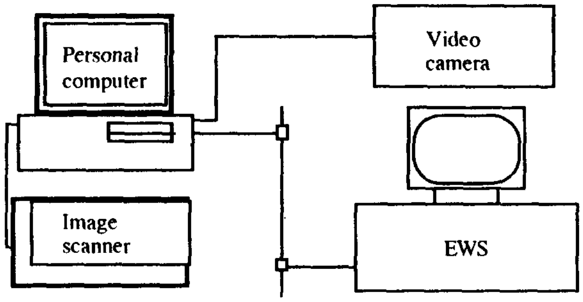
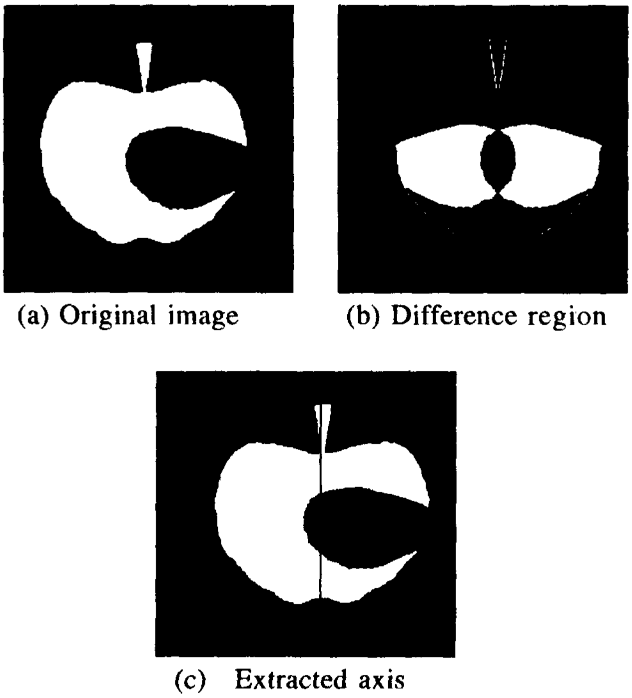


Fig. 2 Employed hardware system

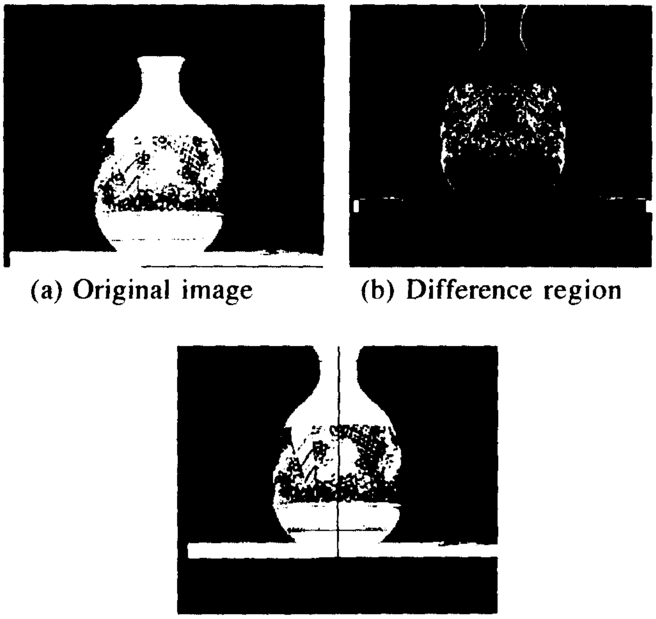
Experimental results are shown in Fig.3, Fig.4 and Fig.5. In Fig.3, the original image of an apple taken an asymmetric bite is given in (a), the difference image of the minimum variance is in (b), and the extracted axis is shown in (c) which coincides with the desired axis. In Fig.4, the original image of bottle which includes asymmetric texture is shown in (a), the difference image of the minimum variance is in (b), and the extracted axis shown in (c). Though the image includes asymmetric texture and noise, the desired axis is extracted a middle line. Fig.5(a) is a sitting human statue and the detected axis shown in Fig.5(b) well approximates a human middle line.



(a) Original image (b) Difference region

(c) Extracted axis

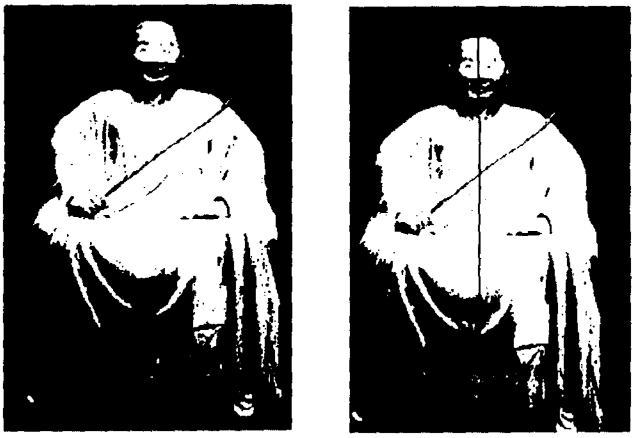
Fig.3 Experimental result 1



(a) Original image (b) Difference region

(c) Extracted axis

Fig.4 Experimental result 2



(a) Original image (b) Extracted axis

Fig.5 Experimental result 3

6. Concluding remarks

A technique was proposed for extracting the axis of potential axial symmetry from a 2D image employing variance minimization. The technique can cope with those shapes having larger asymmetry compared with the former technique which utilizes minimization of areal difference. In practice, the formed technique fails to find the desirable axis(See Fig.3(c)) on the image given in Fig.3(a). The technique proposed in this paper was implemented on a workstation and its performance was examined successfully by real as well as synthetic images. Stability of the axis which this technique extracts should be related to the degree of asymmetry of the shape interested.

Acknowledgment

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