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RECOVERY OF THE CONNECTION RELATIONSHIP AMONG PLANAR OBJECTS

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Abstracts *The shape of an object plays a very important role in pattern analysis and classification. Roughly, the researches on this topic can be classified into three fields, i.e. (i) edge detection, (ii) dominant points extraction, and (iii) shape recognition and classification. Many works have been done in these three fields. However, it is very seldom to see the research that discusses the connection relationship of objects. This problem is very important in robot assembly systems. Therefore, here we focus on this problem and discuss how to recover the connection relationship of planar objects. Our method is based on the partial curve identification algorithm. The experiment results show the efficiency and validity of this method.*

Keywords: contours, dominant points, curve segmentation, partial curve matching, connection relationship

1. INTRODUCTION

The shape of the object plays a very important role in object recognition, analysis and classification. Researches in this field can be roughly classified into (1) edge detection; (2) dominant points detection of the boundary curve; and (3) shape recognition.

The researches about the edge detection focus on edges or contours[1][2]. Those about dominant points detection focus on the points of high curvature[3][4]. And those about the shape recognition pay attention to the entire shape of the boundary curve and identify the objects[5][6]. These researches seldom involve the problem of object connection relationship, i.e., to determine whether a part of an object can connect with that of another one. This problem is very important in robot assembly system. This problem can be thought of as the problem of the partial curve identification. The authors proposed an algorithm to deal with this kind of problem[7]. This paper firstly gives a short overview of this algorithm, and then discuss the recovery of connection relationship among planar objects.

In the following, section 2 gives an overview of the partial curve identification algorithm; Section 3 relates the recovery of the connection relationship of planar objects; Section 4 shows conclusions and future works.

2. AN OVERVIEW OF THE ALGORITHM

This section gives an overview of the algorithm which partly identifies curves in 2-D space. Details are referred to [7]. Here, let us use two curves, α and β, in Fig.1, to show this algorithm. It is clear that curve β matches curve α partly but not wholly, after a certain translation and rotation are performed. The following shows how to detect these partly matched curves.

(i) Dominant points detection

Dominant points of a curve correspond to the points of high curvature. Let us assume that the curve is expressed parametrically by its coordinate functions $x(s)$ and $y(s)$, where s is a path length variable along the curve. The curvature C can be defined in terms of the derivatives of $x(s)$ and $y(s)$. To obtain the dominant points in which we are interested, we employed the scale-based dominant points detector. The coordinates function $x(s)$ and $y(s)$ are convoluted with the Gaussian function. So the Gaussian-smoothed coordinate functions $X(s, \sigma)$ and $Y(s, \sigma)$ are obtained, where σ is the standard deviation of Gaussian distribution. The scale-based curvature C can be determined by functions $X(s, \sigma)$ and $Y(s, \sigma)$. The next step is to detect the local maximum of absolute curvature within the region of support given by the sequence $\{|C_l|, \ldots, |C_{i-1}|, |C_i|, |C_{i+1}|, \ldots, |C_r|\}$, where C_i is the curvature of the point in question, and C_l and C_r are the leftmost and rightmost points of the local region of support, respectively. The region of support for each point i is the largest possible window containing i in which $|C|$ to both the left and right of i is strictly decreasing. The points with local maximal absolute curvature are considered as the dominant points. The dominant points of curves α and β in Fig. 1 are detected and are marked by "○" in Fig.2.

(ii) Curve segmentation

Dominant points as shown in Fig.2 are numbered clockwise and they are considered as the separate points. Therefore, the curves α and β can be segmented as $\{\alpha_{0,1}, \alpha_{1,2}, \ldots, \alpha_{M-1,M}\}$ and $\{\beta_{0,1}, \beta_{1,2}, \ldots, \beta_{N-1,N}\}$, respectively, where $\alpha_{i,j}$ and $\beta_{u,v}$ are partial curves of curves α and β, starting from the dominant point i and u and terminating at the dominant point j and v, correspondingly, and M and N are the numbers of the dominant points of the curves α and β, successively. Here and after, the partial curve segments obtained from curves α and β in Fig.2 are denoted as $S_\alpha = \{\alpha_{0,1}, \alpha_{1,2}, \ldots, \alpha_{M-1,M}\}$ and $S_\beta = \{\beta_{0,1}, \beta_{1,2}, \ldots, \beta_{N-1,N}\}$.

(iii) Partial curve matching

For the two consecutive clockwise partial curves $\alpha_{i-1,i}, \alpha_{i,i+1} \in S_\alpha$, we take two consecutive counterclockwise partial curves $\beta_{j+1,j}, \beta_{j,j-1} \in S_\beta$, then translate $\beta_{j+1,j}, \beta_{j,j-1}$ in order that its dominant point j overlaps the dominant point i of the curve α, completely. The matching error is digitally computed by

$$E(\alpha_{i-1,i},\ \beta_{j+1,j}) = \sum_{\substack{p=0\\q=0}}^{max\{U,V\}} (S_{\triangle(p,p+1,q)}+S_{\triangle(q,q+1,p+1)}) + \sum_{\substack{u=0\\v=0}}^{max\{P,Q\}} (S_{\triangle(u,u+1,v)}+S_{\triangle(v,v+1,u+1)}) \quad (1)$$

where P, Q, U and V are the numbers of digital points of the curve segments $\beta_{j+1,j}$, $\alpha_{i-1,i}$, $\beta_{j,j-1}$ and $\alpha_{i,i+1}$, respectively. As shown in Fig.3, $S_{\triangle(p,p+1,q)}$ is the area of the triangle formed by the points p, $p+1$ and q. Similarly, $S_{\triangle(q,q+1,p+1)}$, $S_{\triangle(u,u+1,v)}$ and $S_{\triangle(v,v+1,u+1)}$ have the same meanings. Here, it is worth to note that if the number of digital points included in a curve segment is less than that of another curve segment in question, its start point or terminal point will be employed to correspond the rest points of the curve segment in question to continue the calculation of equation (1). Which of them will be used is decided by the tracing direction along the curve segment (clockwise or counterclockwise).

The partial curve $\beta_{j-1,j+1}$ is rotated from to 0° to 360° by 1° per step. After each rotation, the matching error is computed. The minimal value of these matching errors is called the matching error between these two partial curves, or simply matching error. The rotation angle corresponding to this minimal matching error is digitally calculated by

$$\theta(\alpha_{i-1,i},\ \beta_{j+1,j}) = arctan(\frac{y_{j-1}-y_{j+1}}{x_{j-1}-x_{j+1}}) - arctan(\frac{y_{i+1}-y_{i-1}}{x_{i+1}-x_{i-1}}) \quad (2)$$

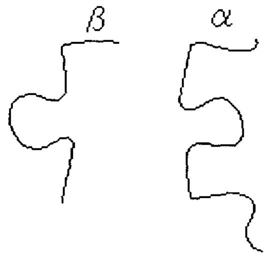
where (x_{j-1}, y_{j-1}) and (x_{j+1}, y_{j+1}) are the coordinates of the dominant points $j-1$ and $j+1$ of the partial curve $\beta_{j+1,j-1}$, respectively, after being translated and rotated. (x_{i-1}, y_{i-1}) and (x_{i+1}, y_{i+1}) are the coordinates of the dominant points $i-1$ and $i+1$ of the partial curve $\alpha_{i-1,i+1}$, successively.

(iv) Longest matched curve

For the partial curves $\alpha_{i-1,i} \in S_\alpha$ and $\beta_{j+1,j} \in S_\beta$, if the following requirements are satisfied, i.e.,

(1) The matching error between them is less than the threshold value E_{thres};
(2) The matching error between the partial curve next to $\alpha_{i-1,i}$ (its clockwise neighbor, i.e., $\alpha_{i,i+1}$) and the partial curve next to $\beta_{j+1,j}$ (its counterclockwise neighbor, i.e., $\beta_{j,j-1}$) is also less than E_{thres};
(3) The absolute value of the difference of the rotation angles of the partial curves $\beta_{j+1,j}$ and $\beta_{j,j-1}$ is less than the threshold θ_{thres}.

$\alpha_{i-1,i}$ and $\beta_{j+1,j}$ are considered as matched partial curves. Then, the focus is shifted to $\alpha_{i,i+1}$ and $\beta_{j,j-1}$, the above operation is done again. This is repeated until all consecutive matched partial curves are obtained. This generates one matched curve which includes multiple partial curves. The processing is over when all possible matched curves are found. The number of the partial curves included in a matched curve is called the length of the matched curve. The matched curve whose length is greater than L_{thres} and whose matching error is the smallest is called the longest matched curve where two curves can be matched perfectly. Fig.4 shows the matched result after the curve β is translated 102 dots along X-axis and 12 dots along Y-axis, and is rotated 182° counterclockwise.



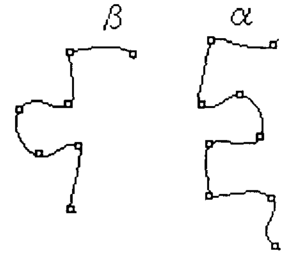


Fig.1 Examples of two curves. Fig.2 Dominant points detected.

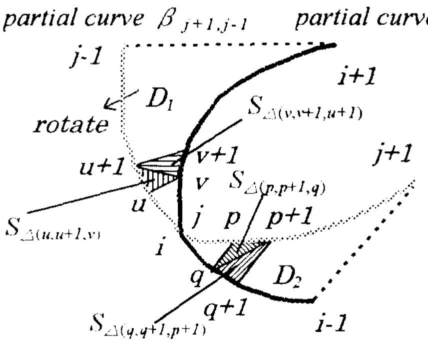


Fig.3 Partial curve $\beta_{j+1,j-1}$ is translated so that dominant point i and j overlap.

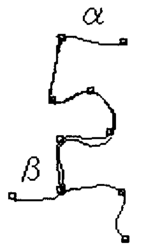


Fig.4 Matched result after the curve β is translated and rotated.

3. RECOVERY OF THE CONNECTION RELATIONSHIP AMONG PLANAR OBJECTS

The algorithm shown above is applied to recover the connection relationship. Fig.5 gives an example image after binarization, which includes four objects. The following shows how to determine their connection relationship.

3.1 Dominant points detection and filtering

The contour of each object is extracted by the contour-tracing algorithm. And then the object is represented by the its corresponding contour. Here and after, the contour is simply called curve in case of no confusion. Because all contours are closed, the curves are also closed. The object is numbered by the order of searching when applying contour-tracing algorithm. In this example, the objects are numbered 0, 1, 2 and 3, sequentially, as shown in Fig.5. The dominant points of each curve are detected according to the method as related in 2.(i) by selecting a proper σ.

For the object i ($i = 0, 1, \ldots, N-1$), let the set DP_i denote its all dominants points, i.e., $DP_i = \{d_{i0}, d_{i1}, \ldots, d_{i,M-1}\}$, where N is the number of objects, and M of dominant points of object i. The interior angle at the dominant point d_{ij} of the object i is denoted by $\angle d_{i,j-1}d_{ij}d_{i,j+1}$ (modulo M). Then the filtering operations are performed as follows:

(1) If $\angle d_{i,j-1}d_{ij}d_{i,j+1}$ is greater than T_{IA1}, and the distance from dominant points d_{ij} to the straight line $d_{i,j-1}\ d_{i,j+1}$ is smaller than H_{thres}, the dominant point d_{ij} is discarded;

(2) If the interior angles of three consecutive dominant points, i.e., $\angle d_{i,j-2}\ d_{i,j-1}\ d_{ij}$, $\angle d_{i,j-1}\ d_{ij}\ d_{i,j+1}$ and $\angle d_{i,j}\ d_{i,j+1}\ d_{i,j+2}$, are all greater than T_{IA2}, the dominant point d_{ij} is discarded.

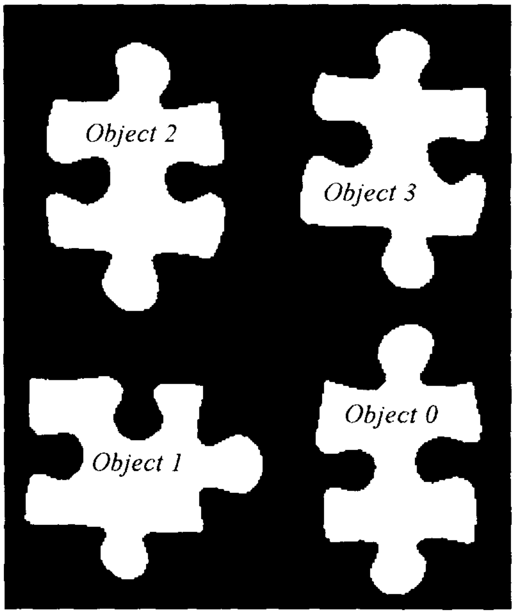


Fig.5 An example image after binarization.

After above filtering operation by setting T_{IA1}, T_{IA2} and H_{thres} to 170°, 100° and 5 dots, respectively, the dominant points of each object are shown in Fig.6 (marked by "o"), and are numbered clockwise.

3.2 Curve segmentation and partial curve matching

According to the method of curve segmentation as related in 2.(ii), each curve is segmented. Curve segments of object i are denoted by the set CS_i, ($i=0, 1, ..., N-1$). All consecutive curve segment pairs in CS_i and CS_j are matched each other, where $i \neq j$, and $i, j=0, 1, ..., N-1$, by the method as shown in 2.(iii). Therefore, the matching errors between the two curve segments which belong to different objects are obtained.

3.3 Determination of longest matched curves

From the matching errors obtained above, the longest matched curves are detected in the manner described in 2.(iv) by setting E_{thres}, θ_{thres} and L_{thres} to 145, 45° and 4, respectively. The longest matched curves obtained in this example are summarized in Table 1. The first column shows the numbers of two objects to which the partial curve matching algorithm is applied. The second and third column present the two partial curves where the two objects can be connected optimally. The fourth column gives the minimal matching errors between the two partial curves. The fifth column displays the rotation angles of another object in question. The positive values mean the clockwise rotation, and the negative values counter-clockwise rotation.

It is worth to note that not all matched partial curves listed in Table 1 are correct. For example, the matched partial curves in the second row and fourth row are contradictory because a partial curve of an object cannot be connected with two partial curves of two different objects. The next section will discuss how to determine the contradictory matched curves, and how to use the true matched curves to recover the connection relationship.

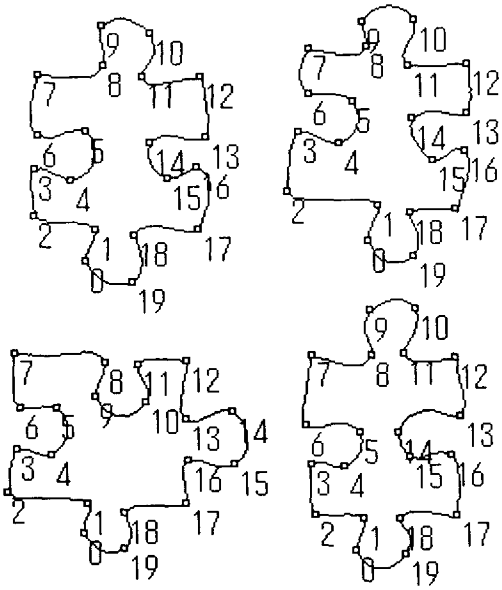


Fig.6 Detected dominant points and curve segments.

Table 1: Longest matched curves obtained.

i-j	Matched partial curve of object *i*	Matched partial curve of object *j*	Min. matching error	Rotation angle of object *j*
0-1	17-18-19-0-1	6-5-4-3-2	131.15	-272°
0-2	12-13-14-15-16	11-10-9-8-7	178.62	268°
0-3	17-18-19-0-1	16-15-14-13-12	211.55	-93°
1-2	12-13-14-15-16	6-5-4-3-2	211.00	0°
1-3	7-8-9-10-11	1-0-19-18-17	161.93	-2°
2-3	17-18-19-0-1	16-15-14-13-12	283.78	-89°

3.4 Recovery of connection relationship

For simplicity, let PC_{ijk} (modulo N_k) denote the partial curve of object k, starting from the dominant point i and terminating at j, where N_k is the number of dominant points of object k. The contradictory matched curves are determined according to the following rules.

[*Rule 1*]

If PC_{ijk} matches PC_{mno}, PC_{pqr}, ..., PC_{uvw} at the same time, where $o \neq r, ..., \neq w$, all these matched curves are of contradiction;

[*Rule 2*]

If PC_{ijk}, PC_{mnk}, ..., PC_{pqk}, *matches* PC_{rst}, PC_{uvw}, ..., PC_{xyz}, *respectively, where* $t \neq w, \ldots, \neq z$, *there does not exist matched curves among the object t, w, ..., and z.*

By applying Rule 1, the matched curves shown in row 2 and 4 of Table 1 are determined as contradictory matched curves because $PC_{17,1,0}$ matches both $PC_{6,2,1}$ and $PC_{16,12,3}$. And by applying Rule 2, the matched curve shown in row 7 of the same table is detected as the contradictory matched curve because $PC_{12,16,1}$ and $PC_{7,11,1}$ matches $PC_{6,2,2}$ and $PC_{1,17,3}$, respectively.

Next, the number of matched curves of each object is calculated. The object with the maximal matched curve number is selected as the base object to which the recovery is performed. In this example, the matched curve number of object *1* is two, and is maximal, therefore object *1* is selected as the base object. The recovery of the connection relationship is performed by two steps.

At the first step, the objects which can be connected to the base object directly are processed. The translation in X-axis is the difference between the x-coordinates of the center points of the two matched partial curves, and that in Y-axis the difference between the y-coordinates. The rotation angle is the value shown in Table 1. It is worth to note that the coordinates of an object will be changed correspondingly if it is translated or rotated. In this example, the connection relationship between object *1* and *2*, *1* and *3*, are recovered at this step.

At the second step, the objects which can be connected to the composite object obtained above are processed. In this example, object *0* can be connected to the composite object obtained in the first step because it can be connected to object *2* which has been attached to base object at the first step. The translations can be obtained in the same way as shown in the first step. The rotation angle is the sum of the rotation angle of the object in question and that of the object which can be connected to the base object.

Fig.7 shows the final recovery result of this example. The translations of the object *2* along X- and Y-axis are *79* dots and *-150* dots, and the rotation angle is *0* degree. These three parameters of the object *3* are *-141* dots, *-83* dots and *-2* degrees, correspondingly. And those of the object *0* are *-63* dots, *70* dots and -268 degrees (which is -(268° $+0^\circ$)), sequentially.

4. CONCLUSIONS AND FUTURE WORKS

This paper discussed the recovery of the connection relationship among planar objects by using the partial curve identification algorithm. The experiments are performed with the actual images. The experiment results show that our method to recover the connection relationship is of validity. This method can be applied in the intelligent robot assembly system.

This experiment employed the images of objects without texture. If the objects have some textures, the boundary curve detection will become more difficult. Moreover, the partial curve matching is performed between every two pairs of curve segments belonging to different objects. This is very time-consuming. Moreover, in this experiment, if a partial curve of an object matches the partial curves of multiple objects, these matched curves are not used. In fact, it is necessary, in this case, to employ the image values near the matched curves to decide the optimally matched partial curve. These are left to be solved in the future.

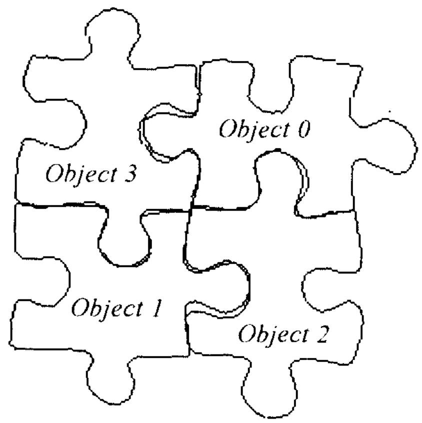


Fig.7 Recovery result of the connection relationship.

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