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Trajectory Control of the Flexible Manipulator with Time-Varying Arm

°Hidetaka Yamazaki*, Toshiro Ono*, Chang-Yong Park*

*Dep. of Mechanical Systems Engineering, College of Engineering,
Osaka Prefecture University, 1-1 Gakuen-cho, Sakai, Osaka 593, Japan
Tel: +81-722-52-1161(2259); Fax: +81-722-59-3340; E-mail: yamazaki@ctrlab.mecha.osakafu-u.ac.jp.

Abstracts Several papers have already been reported on the flexible manipulator with constant arm length. Some of industrial manipulators, however, have sliding joints. It means that the length of their arm or link varies with time. This paper discusses the trajectory control of such a manipulator model, and shows some of the experimental results.

Keywords Flexible Manipulator, Sliding Joint, Oscillation Analysis, Trajectory Control Experiment, Modeling

1. INTRODUCTION

Recently, the speedy and precise movement is demanded on an industrial manipulator to improve its productive capacity. When an industrial manipulator, however, moves in high speed, it often causes unignorable elastic deflection of arms. In the present situation, we meet these demands by raising the rigidity of manipulator arms and its input power. However, there is a limit to these method. Therefore, it may be reasonable to consider its arm as an elastic body rather than as rigid one. Many papers on flexible manipulators have been reported in recent years. As for the motion control problems of flexible manipulators, Book[1] first reported the formulation and examination concerning the control problem for reducing oscillation of the manipulator with 2 links and 2 rotation joints. Actually, however, there are a lot of industrial manipulators which vary the length of their links by sliding joints. Moreover, the oscillation analysis and the trajectory control on those manipulators still remain almost unestablished. This paper concerns the trajectory control of such a flexible manipulator. Since the flexible arm of that manipulator has a sliding joint, the arm length of elastic part varies with time. Therefore, we can not assume a solution by the method of separation of variables in the strict sense.

First, we derive the expression of elastic deflection model under new assumptions that make it possible to use the method of modal analysis. Second, we built up a hardware model of polar coordinate type with a flexible sliding arm to meet the demands stated above. Then, we conducted a trajectory control experiment with the control input derived by the expression of elastic deflection model. We compared the performance of the control system designed by using the rigid model with that of the control system designed using the elastic one through an open-loop optimal control experiment. As a result, we have shown the validity of the expression of elastic deflection.

2. MODELING OF THE MANIPULATOR OF POLAR COORDINATE TYPE

From the engineering point of view, there are 2 lines of approach for trajectory control of a flexible manipulator. One is to determine the motion of sliding joint's part beforehand and to control the motion of rotation joint's part. The other is to determine the motion of rotation joint's part beforehand and to control the motion of sliding joint's part. The former approach was reported by Nishibayashi[2] and Park[3][4], to show a few simulated results. We take the latter approach.

2.1 Exposition of The Arm System

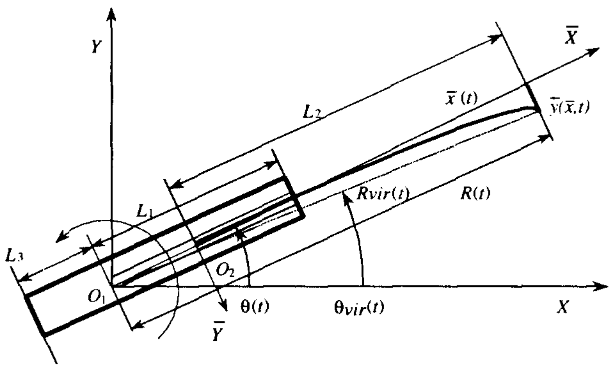

Fig. 1. Manipulator of polar coordinate type.

Fig. 1 shows the whole schematic of the manipulator of polar coordinate type. The first link is the rigid body. The second link is the elastic body, and it connects with the first link by sliding joint. The length of the first link is $L_1 + L_3$, that of the second is L_2 and the total length of manipulator is $R(t)$. $\theta(t)$ is the angle between $\bar{x}$ and X. We denote the origin of the first link by O_1, and that of the second by O_2. We define the line between O_1 and the end-effector of the second link as 'virtual rigid body'. Its length is $R_{vir}(t)$, and its rotation angle is $\theta_{vir}(t)$. The stational coordinates of inertia O_1XY that is named 'the standard coordinates system' and the distance from the origin O_1 is defined as x_0. The coordinates $O_2\bar{X}\bar{Y}$ that is fixed in the second link is named 'the objective coordinates system' and the distance between the end-effector and the origin O_2 is defined as $\bar{x}$. The elastic deflection $\bar{y}(\bar{x}, t)$ of the second link is at $\bar{x}$.

2.2 Modeling of Arm System

The relation between $R_{vir}(t)$ and $\theta_{vir}(t)$ on the objective coordinates system is as follows.

$$\theta_{vir}(t) = \theta(t) - \tan^{-1}\left(\frac{\bar{y}(L_2,t)}{R(t)}\right), \quad (1)$$

$$R_{vir}(t) = \sqrt{R^2(t) + \bar{y}^2(L_2,t)}. \quad (2)$$

Next, we show the relation between the standard coordinates system and the objective coordinate system.

$$X = R(t)\cos\theta(t) + \bar{y}(L_2,t)\sin\theta(t), \quad (3)$$

$$Y = R(t)\sin\theta(t) - \bar{y}(L_2,t)\cos\theta(t), \quad (4)$$

or

$$X = R_{vir}(t)\cos\theta_{vir}(t), \quad (5)$$

$$Y = R_{vir}(t)\sin\theta_{vir}(t). \quad (6)$$

Next we derive the equation of motion of elastic deflection concerning the arm system. We set some assumptions for modeling the flexible manipulator.

[$A-1$] Elastic deflection on the second link can be modeled as Bernoulli-Euler Beam[2].
[$A-2$] Rotation angle $\theta(t)$ is determined beforehand.
[$A-3$] Elastic deflection $\bar{y}(\bar{x},t) = 0$ when $\bar{x} = L_1 + L_2 - R(t)$.
[$A-4$] We ignore the terms more than the second degree with respect to $y(x,t), \dot{y}(x,t), \dot{R}(t)$ and $\dot{\theta}(t)$

We use Hamilton's principle to formulate the complex mechanism like flexible arm easily and neatly. The kinetic energy T, the potential energy U and the virtual work W of the manipulator of polar coordinate type are as follows. " $\cdot$ " means differential with time.

$$\begin{aligned} 2T &= \int_{-L_3}^{L_1} \rho_1 A_1 \dot{r}_1^{\tau} \dot{r}_1 dx_0 \\ &+ \int_0^{L_1+L_2-R(t)} \rho_2 A_2 \dot{r}_2^{\tau} \dot{r}_2 d\bar{x} \\ &+ \int_{L_1+L_2-R(t)}^{L_2} \rho_2 A_2 \dot{r}_3^{\tau} \dot{r}_3 d\bar{x}, \end{aligned} \quad (7)$$

and

$$r_1 = [x_0\cos\theta(t) \;\; x_0\sin\theta(t)]^{\tau} \;\; (0 \le x_0 \le L_1), \quad (8)$$

$$\begin{aligned} r_2 = \; & [(R(t) - L_2 + \bar{x})\cos\theta(t) \\ & (R(t) - L_2 + \bar{x})\sin\theta(t)]^{\tau} \\ & (0 \le \bar{x} \le L_1 + L_2 - R(t)), \end{aligned} \quad (9)$$

$$\begin{aligned} r_3 = \; & [(R(t) - L_2 + \bar{x})\cos\theta(t) + \bar{y}(\bar{x},t)\sin\theta(t) \\ & (R(t) - L_2 + \bar{x})\sin\theta(t) - \bar{y}(\bar{x},t)\cos\theta(t)]]^{\tau} \\ & (L_1 + L_2 - R(t) \le \bar{x} \le L_2), \end{aligned} \quad (10)$$

$$2U = \int_{L_1+L_2-R(t)}^{L_2} EI\left(\frac{\partial^2 \bar{y}(\bar{x},t)}{\partial \bar{x}^2}\right)^2 d\bar{x}$$
$$(L_1 + L_2 - R(t) \le \bar{x} \le L_2), \quad (11)$$

$$W = w\theta(t). \quad (12)$$

Where $\rho_1 A_1$ is the mass per unit length of the first link and $\rho_2 A_2$ is that of the second link. EI is flexural rigidity, and w is the torque to rotate the first link. " τ " means the transposition. Next using the following relationship, we can nondimentionalize the above equations to make the analysis easy.

$$\bar{x} = (R(t) - L_1)x + (L_1 + L_2 - R(t)), \quad (13)$$

$$\bar{y}(\bar{x},t) = (R(t) - L_1)y(x,t). \quad (14)$$

Arrange the Eqs.(7) $\sim$ (14) and substitute them into Eq.(15),

$$\int_{t_1}^{t_2} \delta(T - V + W)dt \equiv 0, \quad (15)$$

where t_1, t_2 express optional time and the symbol δ in the integral signifies the first-order variation. Then we get next nonlinear differential equation and its boundary conditions.

$$\begin{aligned} & \frac{EI}{(R(t) - L_1)^3}\frac{\partial^4 y(x,t)}{\partial x^4} \\ & + \rho_2 A_2[\{\ddot{R}(t)y(x,t) + (R(t) - L_1)\ddot{y}(x,t)\} \\ & - \ddot{\theta}(t)\{(R(t) - L_1)x + L_1\}] = 0, \end{aligned} \quad (16)$$

$$y(0,t) = \frac{\partial y(0,t)}{\partial x} = \frac{\partial^2 y(1,t)}{\partial x^2} = \frac{\partial^3 y(1,t)}{\partial x^3} = 0 \quad (17)$$

2.3 Method of Modal Analysis

Generally, it is difficult to solve the partial differential equation like (16). So we use the method of modal analysis regarding the elastic deflection $y(x,t)$ as the linear combination of the product of proper modal function $\phi_i(x)$ $(i = 1, 2, \cdots)$ and time function $q_i(t)$ $(i = 1, 2, \cdots)$.

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t). \quad (18)$$

This method is focused on the constant length links. However, we cannot separate the variables to the link with time-varying length like this model. Therefore the method of modal analysis cannot be applied to it in the strict sense. Accordingly we derive the numerical model by introducing the following tricky way of thinking to make it possible to use the method of modal analysis.

1.Getting the equation of elastic deflection with constant length at a certain time t under the assumption that the elastic arm oscillates in the same manner even after that time instant. We can use the method of modal analysis because the link's length is presumed

as to be constant.
2.After a tiny time Δt has elapsed, we derive the equation of elastic deflection with constant length at that time instant under the similar presumption that it continues the same oscillation after that time. Then we use the method of modal analysis.
3.Repeat the same processes, and bring a tiny time to tend to zero. Then we get the equation of elastic deflection.

Based on this way of thinking, first we consider the whole length $R(t)$ as constant and apply the method of modal analysis to the Eq.(16) by substitution of the equation like (18), after all calculations finished we consider $R(t)$ as the time-varying function. As a result $\phi_i(x)$ and $q_i(t)$ are obtained as follows.

$$\phi_i(x) = \mu_i\{(\sinh k_i + \sin k_i)(\cosh k_i x - \cos k_i x)$$

$$-(\cosh k_i + \cos k_i)(\sinh k_i x - \sin k_i x)\}, \quad (19)$$

$$q_i(t) = \frac{\beta_i(t)}{\alpha_i(t)} - \frac{\beta_i(0)}{\alpha_i(t)} \cos \sqrt{\alpha_i(t)} t, \quad (20)$$

$$\alpha_i(t) = \frac{EIk_i^4}{\rho_2 A_2 (R(t) - L_1)^4} + \frac{\ddot{R}(t)}{R(t) - L_1}, \quad (21)$$

$$\beta_i(t) = \ddot{\theta}(t) \int_0^1 \left(x + \frac{L_1}{R(t) - L_1} \right) \phi_i(x) dx, \quad (22)$$

where μ_i is normalized constants, and k_i $(i = 1, 2, \cdots)$ is the solution of the following equation.

$$1 + \cos\ k_i \cdot \cosh\ k_i = 0, (0 < k_1 < k_2 < \cdots). \quad (23)$$

2.4 Modeling of Driving Part

On the driving part, the rotational motion are converted into the linear motion by using the ball nut and ball screw mechanism. The relation between the rotation of ball screw and the linear motion of ball nut is as follows.

$$\dot{R}(t) = \frac{l}{2\pi} \dot{\theta}(t), \quad (24)$$

where l is the lead of the ball screw and $\dot{\theta}(t)$ is the rotational angle of the ball screw. From this relation we can get the Eq.(25) as follows.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u(t), \quad (25)$$

where

$$\mathbf{x} = \begin{bmatrix} R(t) \\ \dot{R}(t) \end{bmatrix}, \quad (26)$$

and $u(t)$ is the electric current, $\mathbf{A}, \mathbf{B}$ is the matrix that are determined by the parameters of the driving part.

3. PLANNING OF AN OPTIMAL CONTROL INPUT

We make the end-effector of the flexible arm tracked along the desired trajectory by inputting the electric current $u(t)$ under the pre-determined angular velocity. In this paper, we set up the straight line trajectory as the desired one for the end-effector of manipulator. An input is determined for the end-effector to get along that trajectory by using Fletcher-Reeves method, which is one of the popular algorithms used in the optimal control calculation. When the flexible arm moves, the deflection and oscillation often occurs because of link's elasticity, and it generates an error against the desired trajectory. We derive the control input to minimize that error using the optimization algorithms. We show the functional equation and the process for using algorithms.

Functional equation :

$$J = g(t_f, \mathbf{x}(t_f)) + \int_0^{t_f} L(t, \mathbf{x}(t), u(t)) dt, \quad (27)$$

$$\begin{aligned} g(t_f, \mathbf{x}(t_f)) &= wg_1^2(R_d(t_f) - R_{vir}(t_f))^2 \\ &+ wg_2^2(\dot{R}_d(t_f) - \dot{R}_{vir}(t_f))^2, \quad (28) \end{aligned}$$

$$\begin{aligned} L(t, \mathbf{x}(t), u(t)) &= wl_1^2(R_d(t) - R_{vir}(t))^2 \\ &+ wl_2^2(\dot{R}_d(t) - \dot{R}_{vir}(t))^2 \\ &+ \left(\frac{u(t)}{u_{max}}\right)^2, \quad (29) \end{aligned}$$

where wg_1, wg_2, wl_1 and wl_2 are weighting coefficients, $R_d(t)$ is the desired trajectory and $\dot{R}_d(t)$ is the desired velocity, and t_f is terminal time.

We get the optimal input $u(t)$ that satisfies the differential equation (25) and the initial condition $\mathbf{x}|_{t=0} = [L_1, 0]^T$ to minimize the value of (27).

Calculation process :
(i)Guess the initial input $u(t)$.
(ii)Derive the value of $R(t), \dot{R}(t)$ by using $u(t)$.
(iii)Derive the value of $\bar{y}(\bar{x}, t)$ by using $R(t), \dot{R}(t)$.
(iv)Derive the value of $R_{vir}(t), \dot{R}_{vir}(t)$ by using $\bar{y}(\bar{x}, t)$.
(v)Derive the value of J by using $R_{vir}(t), \dot{R}_{vir}(t)$.
(vi)If J is not sufficiently small, derive the new input $u(t)$ by means of Fletcher-Reeves method, or else finish calculation.

4. EXPERIMENTS

4.1 Experimental Device

Fig. 2 shows the schematic of experimental device. There are 2 motors. One is for the part of the rotation joint. Its torque is conveyed to the reducing gear through belt, and generates the rotation of the first link. The other is for the part of sliding joint, and its rotational motion is changed into the translational motion of the second link through the ball nut and screw mechanism. A fine needle is attached to the end-effector. We measure the tracking error of the end-effector by using the needle with ink. We conduct the experiment of trajectory control by using this device.

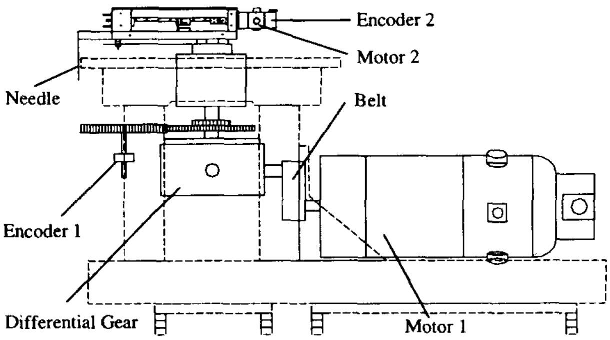

Fig. 2. Schematic of the experimental device.

4.2 *Control Experiment*

As we said before, the optimal input is planned by using the functional equation. But we do not know which weighting coefficients are significant for the trajectory control when we consider the flexibility of arm. Then we simulate 5 cases changing the weighting coefficient in the functional as follows.

Case (1) $wg_1 = 1000, wg_2 = 1000, wl_1 = 1000, wl_2 = 1000$
Case (2) $wg_1 = 0, wg_2 = 1000, wl_1 = 1000, wl_2 = 1000$
Case (3) $wg_1 = 1000, wg_2 = 0, wl_1 = 1000, wl_2 = 1000$
Case (4) $wg_1 = 1000, wg_2 = 1000, wl_1 = 0, wl_2 = 1000$
Case (5) $wg_1 = 1000, wg_2 = 1000, wl_1 = 1000, wl_2 = 0$

We have simulated the all cases, but we show here only the cases (4) and (5) which indicate the clear differences in control performance between by using the flexible manipulator model and by using the rigid manipulator model. So we conduct the control experiment in the cases of (4) and (5). Then we get the experimental results shown in Figs. 3 and 4. The experimental data of error are shown in TABLE 1.

When we compare the trajectory error of the flexible manipulator with that of rigid one, we can see that a better result obtained from the optimal control using the equations of elastic deflection.

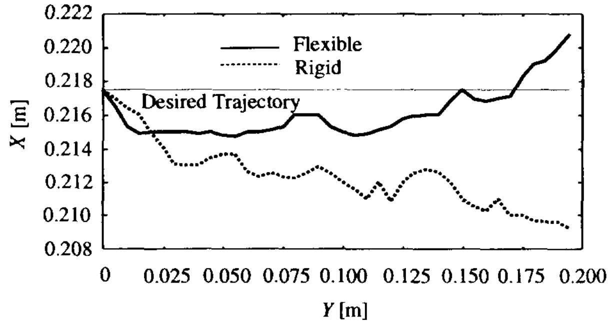

Fig. 3. Experimental results for case (4).

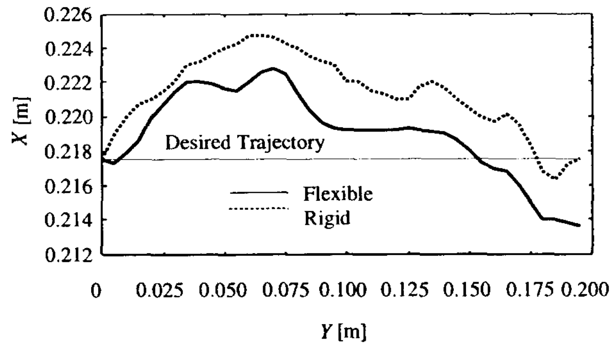

Fig. 4. Experimental results for case (5).

TABLE 1 The Error obtained from the experiment.

	Case (4)		Case (5)	
	Rigid	Flexible	Rigid	Flexible
Maximum Error[m]	0.0083	0.0033	0.0072	0.0053
Mean Error[m]	0.0051	0.0019	0.0039	0.0025

5. CONCLUSIONS

The oscillation analysis and trajectory control on the manipulators with sliding joints are considered in this paper.
(i) As to the oscillation analysis, we modeled the equation of the elastic deflection of time-varying flexible arm under the new assumptions.
(ii)As to the trajectory control, we built up an experimental system and conducted a preliminary experiment to show the validity of the present approach. Although we could get the data of the positioning error through the experiment, the following problems still remain for further study. (i)To consider the measuring method to obtain the elastic deflection as time varying system.
(ii)To conduct the additional experiments with the desired trajectory of other type.

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