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Robust Motion Control of a Flexible Micro-Actuator using $\mathcal{H}_\infty$ Control Method

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Abstracts In this paper, robust motion control of a flexible micro-actuator is presented. The actuator is made of a bimorph piezoelectric high-polymer material (PVDF). No mathematical model system can exactly model a physical system such a flexible micro-actuator. For this reason we must be aware of how modeling errors might adversely affect the performance of a control system for such a model. The $\mathcal{H}_\infty$ method addresses a wide range of the control problems, combining the frequency and time domain approaches. The design is an optimal one in the sense of minimization of the maximum of the closed-loop transfer function. It includes colored measurement and process noise. It also addresses the issues of robustness due to model uncertainties, and is applicable to the flexible micro-actuator control problem. Therefore, we adopt the $\mathcal{H}_\infty$ control problem to the robust motion control of the flexible micro-actuator. Theoretical and experimental results demonstrate the satisfactory performance and the effectiveness of the designed controller.

Keywords Robust Motion Control, Piezoelectric High-polymer Material, Flexible Micro-Actuator, Robust Servo System, $\mathcal{H}_\infty$ Control Method.

1. INTRODUCTION

In this paper, robust motion controller design of a flexible micro-actuator is presented. The flexible micro-actuator is made of a bimorph piezoelectric high-polymer material (PVDF:*Poly Vinylidene Fluoride*). The $\mathcal{H}_\infty$ control problem is adopted to the robust motion control of the flexible micro-actuator.

In simple terms, a piezoelectric high-polymer bimorph micro-actuator consists of two PVDF films cemented with a metal shim. When an electric voltage with proper polarity is applied to the terminals of the actuator, one film extends and the other contracts. As a result, the actuator generates bending moment. PVDF bimorphs provide answers to many problems associated with implementing lightweight, compact, and simple electrically energised motional devices.

The major problems of a robust motion controller design for the flexible micro-actuator are guarantee of stability for the controllable space by using a feedback controller, and improving response properties of the system, so that the motion of the actuator achieves high performance. Actually, robustness is also one of the performance indexes for assessing uncertainty and uncontrollability of the whole object system as opposed to assessing the properties of the transient response and tracking characteristics at the steady state. Existence of uncertainty in the design parameters of a controller due to unconsidered higher modes, variations in the environment and external disturbances causes the controller to be out of tune and this consequently worsens the robust performance of the controller in motion control applications a flexible micro-actuator. The mathematical plant model with uncertainty needs a robust controller in order to achieve disturbance rejection even though the plant has different states from that of the nominal plant. Positive inclusion of some uncertainty in the mathematical model, the controller achieves the robust motion control.

The $\mathcal{H}_\infty$ control problem can deal systematically with the robust stability problem and the characteristics of the frequency domain loop shaping[7]. This approach gives the controller design method a guarantee for the robustness to discretionary disturbances which the $\mathcal{L}_2$ norm is limited. Therefore, the $\mathcal{H}_\infty$ control problem can guarantee the robustness of the controller in the face of plant uncertainty; the modeling error, the parameter variations and unexpected disturbances[3].

On the other hand, the tracking control problem differs from the regulator problem, because the controller performance depends not only on the plant parameters, but also on the tracking command profile. Although the $\mathcal{H}_\infty$ control problem is flexible enough to formulate the tracking problem to meet requirements. One important requirement for tracking system is to maintain zero steady-state error. This requirement can be satisfied by up grading the plant with an integrator. This approach was already discussed in the LQG design procedure. The $\mathcal{H}_\infty$ tracking controller design approach is similar to the LQG tracking controller design presented earlier[5][6]. Therefore, we adopt the $\mathcal{H}_\infty$ control problem to the robust servo controller design of the flexible micro-actuator. Finally, theoretical and experimental results demonstrate the satisfactory performance and the effectiveness of the designed controller.

2. SYSTEM DESCRIPTION OF THE FLEXIBLE MICRO-ACTUATOR

2.1. Piezoelectric High-Polymer Bimorph Actuator Model

Fig. 1 shows the piezoelectric high-polymer bimorph flexible micro-actuator model. The piezoelectric high-polymer bimorph micro-actuator consists of two PVDF films cemented with a metal shim in proper polarity. Each layer is connected with an electrode. Applying a control voltage $V(t)$ results in an internal strain which is induced in the PVDF layers by the piezoelectric effect. This strain results from stresses on each of the PVDF layers, when one film extends, while the other shrinks producing a resultant spatially uniform bending moment. Table 1 summarizes a geometric and physical properties of the actuator[1].

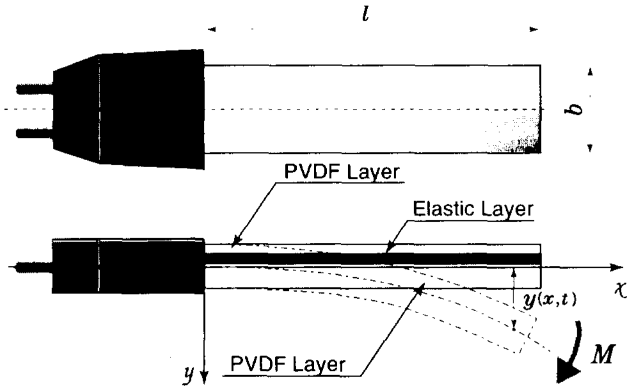


Fig. 1 Construction of Flexible Micro-actuator.

2.2. System Identification of the Nominal Model

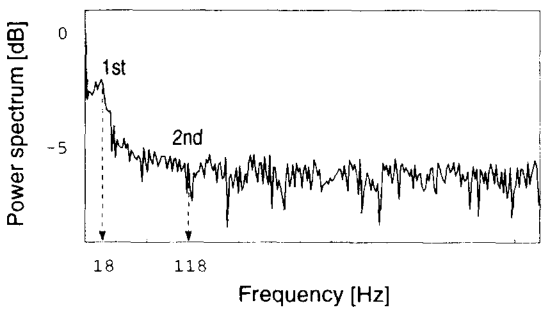


Fig. 2 Power spectrum of the object system under random input. (Experimental result)

The system identification problm is estimate a model of a system besed on observed input/output data for designing

Table 1 Geometric and physical properties of the actuator.

PROPERTY	VALUES	UNIT
PVDF Young's modulus	2×10^{9}	[N/m^2]
Metal Young's modulus	6.56×10^{10}	[N/m^2]
PVDF density	1.78×10^{9}	[kg/m^3]
Metal density	4.45×10^{9}	[kg/m^3]
Actuator length	2.9×10^{-2}	[m]
Actuator width	1.64×10^{-2}	[m]
PVDF thickness	2.8×10^{-5}	[m]
Metal thickness	1.4×10^{-5}	[m]
Piezo strain constant	23×10^{-12}	[m/V]

robust motion controller of the flexible micro-actuator. As shown in fig. 2 the power spectrum of the response of the actuator after subjecting its input to a random signal, it can be seen that the 1st and 2nd mode are well pronounced. A method of estimating the parameters is the prediction error approach, were simply the parameters of the ARMAX model are chosen so that the defference between the models output and the measured output (as shown in Fig. 3) is minimized.

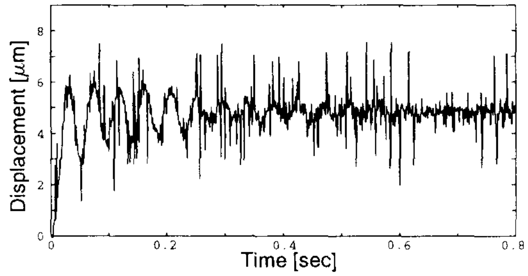


Fig. 3 Input/output data of the object system. (Experimental result)

Finally, the identified state equation is

$$\dot{x}(t) = Ax(t) + bV(t) \tag{1a}$$

and the output equation is given by

$$y(t) = cx(t) \tag{1b}$$

where $V(t)$ is the applied voltage and y is the bending displacement at the end point of the actuator,

$$A^T = \begin{bmatrix} -133.73 & 1.00 & 0 & 0 & 0 \\ -5.45e+5 & 0 & 1.00 & 0 & 0 \\ -3.77e+7 & 0 & 0 & 1.00 & 0 \\ -1.06e+10 & 0 & 0 & 0 & 1.00 \\ -5.64e+11 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$b^T = \begin{bmatrix} 1.00 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$c = \begin{bmatrix} 0.021 & 36.79 & 3.52e+4 & 1.64e+7 & 2.73e+9 \end{bmatrix}.$$

Step response of the identified system model are shown in Fig. 4.

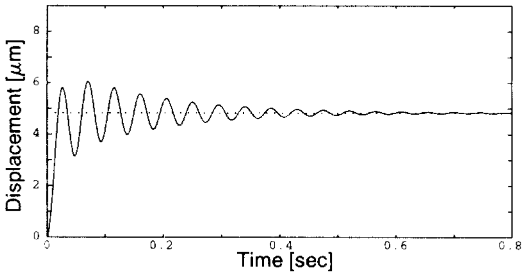


Fig. 4 Step response of the identified system model. (Simulation result)

3. ROBUST MOTION CONTROL BASED $\mathcal{H}_\infty$ SYNTHESIS METHOD

3.1. Output Feedback $\mathcal{H}_\infty$ Control

A feedback system is denoted by

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix} \tag{2}$$

$$u = K(s)y \tag{3}$$

A generalized system $G(s)$ having exogenous input w, controlled output z, control input $u(:= V)$, and measured output y, is described by the state space expression

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{4a}$$

$$z(t) = C_1 x(t) + D_{12} u(t) \tag{4b}$$

$$y(t) = C_2 x(t) + D_{21} u(t) \tag{4c}$$

$$G(s) := \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \triangleq \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]. \tag{5}$$

The transfer function G_{zw} is expressed as

$$G_{zw} = G_{11} + G_{12} K (sI - G_{22} K)^{-1} G_{21} \tag{6}$$

$\mathcal{H}_\infty$ control problem is to find a controller $K(s)$ that makes the closed-loop system internally stable and such that

$$\|G_{zw}\|_\infty < \gamma^2; \quad \gamma > 0 \tag{7}$$

There exists an admissible controller such that equation (7) if and only if the following three conditions hold:

(i) $H \in dom(Ric)$ and $X = Ric(H) \geq 0$;

(ii) $N \in dom(Ric)$ and $Y = Ric(N) \geq 0$;

(iii) $\lambda_{\max}(XY) < \gamma^2$;

where $dom(Ric)$ is the domain of Ric, H and N are Hamiltonian matrices

$$H := \begin{bmatrix} A & B_1 B_1^*/\gamma^2 - B_2 B_2^* \\ -C_1^* C_1 & -A^* \end{bmatrix} \tag{8}$$

$$N := \begin{bmatrix} A & C_1 C_1^*/\gamma^2 - C_2^* C_2 \\ -B_1 B_1^* & -A^* \end{bmatrix}, \tag{9}$$

$Ric(*)$ is the stabilizing solution of an algebraic Riccati equation, $\lambda_{\max}(*)$ is maximum of eigenvalue and $()^*$ is complex conjugate transpose of $()$.

Then, the central solution is given by

$$\dot{\hat{x}} = A\hat{x} + B_1 \hat{w}^\# + B_2 u + ZL(C_2\hat{x} - y) \tag{10a}$$

$$u = F\hat{x} \tag{10b}$$

$$\hat{w}^\# = B_1^* X/\gamma^2 \hat{x} \tag{10c}$$

where

$$F := -B_2^* X, \quad L := -YC_2^*, \quad Z := (I - YX/\gamma^2)^{-1}.$$

3.2. Robust Servo problem using $\mathcal{H}_\infty$ Control Synthesis

Consider a feedback control system with robustness, the purpose of the controller design is to find a feedback compensator which satisfies the following three specifications under some uncertainties: (1) a closed loop internal stability, (2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance rejection, (3) robust servo ability without steady-state error.

It is well known that the $\mathcal{H}_\infty$ control synthesis method is useful for the above first two specifications. The $\mathcal{H}_\infty$ control problem is an optimal control problem subjected to a worst-case disturbance. In the LQG and $\mathcal{H}_2$ control problem we assume that the exogenous inputs are gaussian noise or impulse signal.

This paper addresses the robust motion control of the flexible micro-actuator using the $\mathcal{H}_\infty$ controller design method. The integral type controller is required to be satisfied with the last specification[4]. So we adopt the $\mathcal{H}_\infty$ controller design method with a generalized plant $G(s)$ shown in Fig. 5. This approach enables the loop shaping on frequency domain to operate weighting function Q, R, W, N and E.

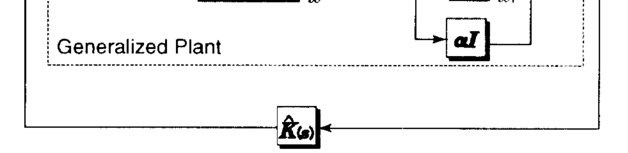
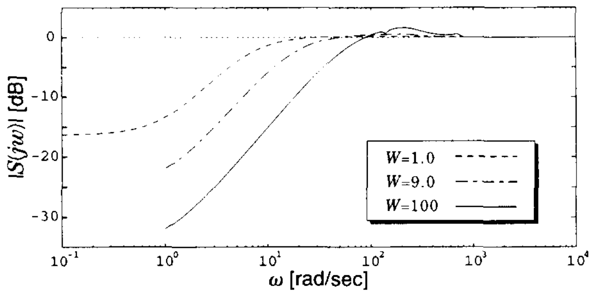


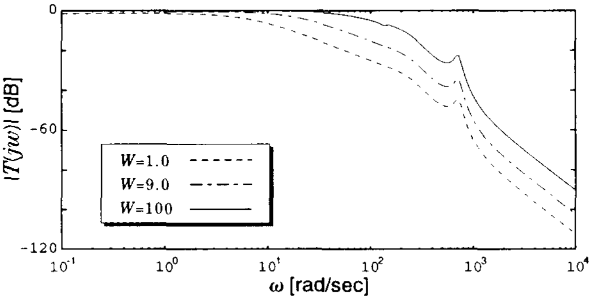
Fig. 5 Robust servo system using $\mathcal{H}_\infty$ control method.

In this case, a realization for the generalized plant is given by

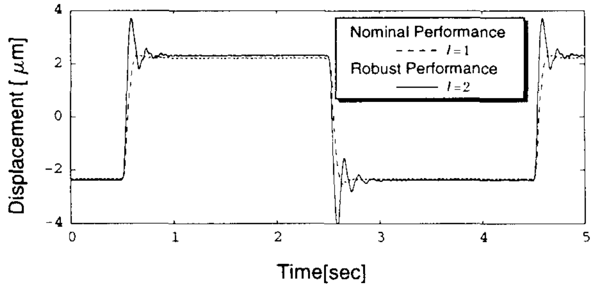
$$G(s) = \left[\begin{array}{cc|ccc} A & 0 & E & 0 & B \\ C & 0 & 0 & N & 0 \\ \hline Q^{1/2} & 0 & 0 & 0 & 0 \\ 0 & W^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R^{1/2} \\ \alpha C & I & 0 & \alpha N & 0 \end{array}\right] \tag{11}$$



(a) $|S(j\omega)|$ Sensitivity function.



(b) $|T(j\omega)|$ Complementary sensitivity function.



(c) Nominal and robust performance of the robust servo system using $\mathcal{H}_\infty$ control method. (Experimental results $W = 9.0$)

Fig. 6 Frequency and time response of the robust servo system.

where

$$K(s) = \beta^{-1}(s)\hat{K}(s) \tag{12}$$

$$\beta(s) := \frac{s}{s+\alpha}; \quad \alpha > 0 \tag{13}$$

4. RESULTS AND DISCUSSION

Recently, stimulated by micro-processor technology, there is an increasing interest in issues related to digital control implementation. For the implementation of the designed robust servo $\mathcal{H}_\infty$ controller, we routinely use the TMS320C31-based digital signal processing system DSP-CIT, along with a set of design and implementation software tools, including an automatic code generator. The DSP-CIT combines the TMS320C31's tremendous computing performance of up to 40[MFlops] with a versatile set of on-board I/O: four analog input channels (16bit, 10[μsec] and 12bit 3[μsec]), four analog output channels (12bit), two incremental encoder channels and a complete subsystem for digital I/O.

We carried out the design in the analogue domain using *MATLAB*[2], and discretized the controller, after checking for the discretization, computational delays, and A/D- and D/A-quantization, the signal processor code was generated and downloaded. The sampling period was about 1 [msec]. A reference signal is a square wave of 2.5[μm] height and a 4 second period.

Fig. 6(a) and (b) shows the sigma plots of the sensitivity function $|S(j\omega)|$ and the complementary sensitivity function $|T(j\omega)|$. The sensitivity gain is the order of 0[dB] in the region above 70[rad/sec] with $W = 9.0$. Thus the control system is capable of suppressing disturbances in several frequency, bounds have been improved resulting in precision following-up of a control signal.

Suppose suspected multiplicative uncertainty

$$B = lB; \quad l > 0 \tag{14}$$

to the input side of the plant, theoretical and experimental results was performed to evaluate the robust performance of the designed controller. Fig. 6(c) shows the nominal and robust performance of the step response of the system using the robust servo $\mathcal{H}_\infty$ controller with $W = 9.0$, $R = 1$, $E = I$, $Q = I$, $N = 0.1$, $\alpha = 10$ and $\gamma^2 = 1.0217$. The $\mathcal{H}_\infty$ control design method increases the system robustness and push down the sensitivity.

5. CONCLUSION

We proposed the robust motion control using $\mathcal{H}_\infty$ controller design method for a flexible micro-manipulator.

The designed $\mathcal{H}_\infty$ robust servo controller were evaluated by the frequency analysis and the experimental results with suspected parameter changing. Both the sensitivity function and the complementary sensitivity function are satisfied with loop shaping. The time response results demonstrates desired robust tracking performance of the system. The effectiveness of the robust motion control verified experimentally using the proposed design method for the flexible micro-actuator motion control.

REFERENCES

[1] "*KYNAR Piezo Film Technical Manual*," Pennwalt Corp., 1987.

[2] G.J.Balas, J.C.doyle, K.Glover, A.Pckard, R.Smith, "*μ-Analysis amd Synthesis Toolbox User's Guide*," The Math Works Inc., 1994.

[3] J.C.Doyle, K.Glover, P.P.Khargonekar and B.A.Fransis, "*State-space solusions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems*," IEEE Trans., AC-34, No.8, pp.831-847, 1989.

[4] S.Hara, "*Synthesis of Robust Servo System*," in summer seminor '92 text, pp.55-67, SICE, 1992. (In Japanese.)

[5] M.Okugawa and M.Sasaki, "*Motion Control of a Flexible Micro-Actuator with Robustness*," Proc. JSME D & D'96, Fukuoka, Japan, No.96-5(1), Vol.B, pp.717-720, 1996.

[6] M.Sasaki and M.Okugawa, "*Motion Control of a Piezopolymer Bimorph Flexible Microactuator*," J. Robotics and Mechatronics, Vol.7, No.6, pp.467-473, 1995.

[7] K.Zhou, J.C.Doyle and K.Glover, "*Robust and Optimal Control*," Prentice-Hall, New Jersey, 1996.