Proceedings of the 11th KACC, October 1996

TIME DELAY CONTROL WITH STATE FEEDBACK FOR AZIMUTH MOTION OF THE FRICTIONLESS POSITIONING DEVICE

Ho-Seop Jeong° and Chong-Won Lee

Center for Noise and Vibration Control(NOVIC), Department of Mechanical Engineering
Korea Advanced Institute of Science and Technology, Science Town, Taejon, KOREA
Tel: +82-42-869-3056; Fax:+82-42-869-3095; E-mail: hsjeong@sam.kaist.ac.kr

Abstracts A time delay controller with state feedback is proposed for azimuth motion control of the frictionless positioning device which is subject to the variations of inertia in the presence of measurement noise. The time delay controller, which is combined with a low-pass filter to attenuate the effect of measurement noise, ensures the asymptotic stability of the closed loop system. It is found that the low-pass filter tends to increase the robustness in the design of time delay controller as well as the gain and phase margins of the closed loop system. Numerical and experimental results support that the proposed controller guarantees a good tracking performance irrespective of the variation of inertia and the presence of measurement noise.

Keywords Active magnetic bearings, Position control, Time delay control, State feedback, Steady-state error

1. INTRODUCTION

A frictionless positioning device suspended by cone-shaped AMBs has been developed in the laboratory, which is driven by a brushless DC motor equipped with a resolver[1]. The device is capable of controlling three translational and two rotational motions by using four pairs of cone-shaped electromagnets, when the azimuth motion is controlled by a brushless DC motor. Since such a system may transport a working piece which is attached near the free end of rotating arm during operation, a good tracking performance is not likely to be achieved when we rely on a simple linear controller. In order to attain a desired performance for the azimuth motion with such uncertainty, a linear controller normally requires frequent re-tuning of its controller gains, otherwise a robust control law should be adopted. Several robust control strategies have so far been proposed to control linear and nonlinear systems with uncertain dynamics or unknown parameters. Time delay control (TDC)[4] uses past observations regarding the system response and control input, in order to directly modify the control actions rather than adjusting controller gains or identifying system parameters. Youcef-Toumi and Reddy[5] studied the TDC for high speed and high precision active magnetic bearings. In their work, experimental results indicated that the TDC has impressive static and dynamic stiffness characteristics for the prototype considered. Ro, et al.[2] proposed a state feedback controller with an outer PID loop to eliminate steady-errors to step commands. It is better in the tracking command signals and rejecting disturbance than the PID controlled system. However, it doesn't guarantee the robustness with respect to the parameter variation.

In this paper, a time delay controller with state feedback is proposed for azimuth motion control of the frictionless positioning device to achieve a desired tracking performance. The state feedback is to ensure the stability of the closed loop system and the incorporated time delay controller enables the exact tracking of the command input despite of the uncertainties. An outer loop is used to track the command input so that the system performance is not seriously affected. And a simple first order low-pass filter is included to increase the robustness of TDC to the measurement noise. The stability of the controlled system in the discrete domain is investigated using the Nyquist stability, for digital implementation with the DSP(TMS320C30) and host PC. Finally experimental works are performed to compare the tracking performance of the proposed controller with the conventional optimal controller.

2. MODELING FOR THE AZIMUTH MOTION OF FRICTIONLESS POSITIONING DEVICE

Figure 1 is the schematic of the tested frictionless positioning device. The system consists of a rotor, two radial cone-shaped AMBs, a driving brushless DC motor, a resolver, and five eddy current type proximity probes. The azimuth motion of the frictionless positioning device is controlled by a built in brushless DC motor and a resolver. Assuming that the back emf in the armature can be ignored and the dynamics of power amplifier and low-pass filter may be neglected as the corresponding poles are located far left in the Laplace domain compared with the system poles, the transfer function of the rotational angle, θ, with respect to the control voltage, u, can be well approximated as

$$G_p(s)=\frac{\Theta(s)}{U(s)}=\frac{K_{cm}K_T K_\theta}{J_m s^2+C_m s+K_m} \tag{1}$$

where

$$\tau_{cm}=\frac{L_m}{K_{am}R_{fm}+R_m},\quad K_{cm}=\frac{K_{am}}{K_{am}R_{fm}+R_m}$$

Here $\Theta(s)$ and $U(s)$ are the Laplace transforms of the rotational angle and control voltage of motor, respectively; K_{am} is the voltage gain of power amplifier; R_m is the resistance of motor coil; R_{fm} is the current feedback gain; K_{cm} is the voltage-to-current gain; τ_{cm} is the time constant of motor coil with current feedback; K_T is the torque constant(N-m/A); K_θ is the conversion factor(180/π); J_m, C_m and K_m are the polar moment of inertia, the torsional damping and stiffness of azimuth motion, respectively. The equation of motion can be rewritten, in the state-space form, as

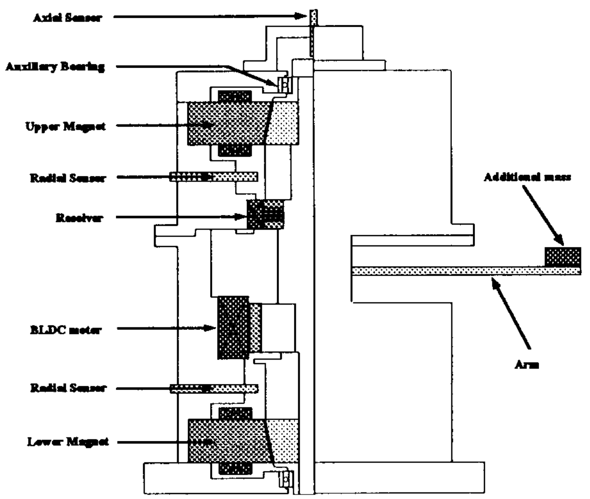

Fig. 1 Schematics of the tested frictionless positioning device

$$\dot{x}_p = A_p x_p + B_p u$$
$$y_p = C_p x_p \tag{2}$$

where

$$\omega_n = \sqrt{\frac{K_m}{J_m}},\ \zeta_n = \frac{C_m}{2J_m\omega_n},\ b = \frac{K_{cm}K_T K_\theta}{J_m},\ x_p = \{\theta\ \dot{\theta}\}^T$$

$$A_p = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix},\ B_p = \begin{Bmatrix} 0 \\ b \end{Bmatrix},\ C_p = [1\ \ 0]$$

Here b is the control input gain.

3. CONTROLLER DESIGN

3.1 Optimal Controller with Integral Action

As the open-loop system has no free integrators, steady-state position error may occur in case of tracking control. To eliminate the steady-state error, an integral action can be added to the existing state feedback controller as shown in Fig. 2. The integrator dynamics can be written as

$$\dot{z}_p = y_p \tag{3}$$

Introducing a new state vector, $x = \{z_p\ \ x_p\}^T$, we can rewrite Eq.(2) as

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{4}$$

where

$$A = \begin{bmatrix} 0 & C_p \\ 0 & A_p \end{bmatrix},\ B = \begin{Bmatrix} 0 \\ B_p \end{Bmatrix},\ C = \{0\ \ C_p\}$$

Now consider the quadratic performance index given by

$$J = \int_0^\infty \left(x^T Qx + u^T Ru\right)dt \tag{5}$$

where Q and R are the positive semi-definite and positive definite weighting matrices, respectively. Then solution to the minimization of J is the optimal control law given by

$$u = -K_{opt}x = -[K_I\ \ K_s]\begin{Bmatrix} z_p \\ x_p \end{Bmatrix}$$
$$K_{opt} = R^{-1}B^T P \tag{6}$$

where P is the solution of the algebraic matrix Riccati equation.

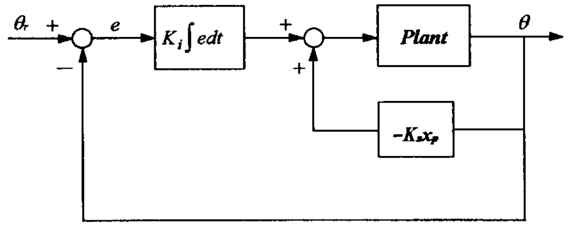

Fig. 2 Optimal state feedback controller with integral action

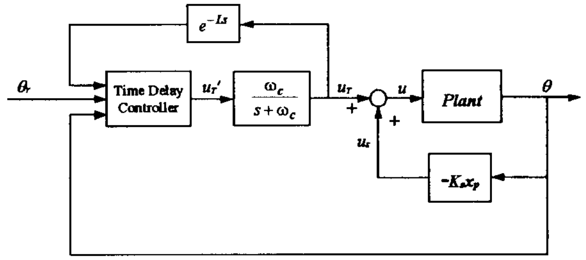

Fig. 3 Time delay controller with state feedback

When the system characteristics are known, the designer can assign poles in the arbitrary locations by choosing matrices Q and R. However, when the azimuth motion has a working piece which is attached at the free end of rotating arm during operation, as shown in Fig. 1, an optimal controller with integral action may not guarantee a satisfactory performance. In that case, a time delay controller may be incorporated with the state feedback, as shown in Fig. 3. Use of an outer loop enables exact tracking of step commands without seriously affecting system performance.

3.2 Time Delay Controller with State Feedback

A second order reference model is chosen as

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\omega_r^2}{s^2 + 2\zeta_r\omega_r s + \omega_r^2} \tag{7}$$

where $\Theta_m(t)$ is the desired trajectory, $\Theta_r(t)$ is the reference input, ω_r and ζ_r are the natural frequency and damping ratio of the second order reference model. The time delay control law was chosen to satisfy the requirement of Eq. (7) for azimuth motion of Eq. (2), i.e.[4]

$$u_T(t) = u_T(t-L) + \frac{1}{\hat{b}}\left[-\hat{\ddot{\theta}}(t-L) - \omega_r^2\theta(t) - 2\zeta_r\omega_r\hat{\dot{\theta}}(t) + \omega_r^2\theta_r(t)\right] \tag{8}$$

where $\hat{\dot{\theta}}$ and $\hat{\ddot{\theta}}$ are the estimates of velocity and acceleration of the rotor, $\hat{b}$ is the estimate of control input gain, $b(\theta, t)$, and L is the delay time. Note that the TDC law does not use any detailed information about the system. Instead, it requires accurate estimation of the velocity and acceleration from the measured displacement signal. On the other hand, signals are easily contaminated by measurement noises, which tend to be amplified in the velocity and acceleration estimation process. Low-pass filter is commonly used to smooth out such high frequency noises, but the filter dynamics should be taken into account in the controller design[6], as shown in Fig. 3. Knowing that the output from the controller is different from the input to the plant and letting the delay time L be equal to one sampling period in the digital implementation, we can modify the time delay controller of Eq.(8), in the discrete domain, as

$$u'_T(k) = u_T(k-1) + \frac{1}{\hat{b}}\left[-\hat{\ddot{\theta}}(k-1) - \omega_r^2\theta(k) - 2\zeta_r\omega_r\hat{\dot{\theta}}(k) + \omega_r^2\theta_r(k)\right] \quad (9)$$

where $u'_T(k)$ is the input to the filter and $u_T(k)$ is the output from the filter. The filter can be approximated by a backward difference model, i.e.,

$$u_T(k) = a_{f0}u_T(k-1) + a_{f1}u'_T(k) \quad (10)$$

where $\omega'_c = \omega_c L$, $a_{f0} = \frac{1}{1+\omega'_c}$, $a_{f1} = \frac{\omega'_c}{1+\omega'_c}$

Here, ω_c is the cutoff frequency of low-pass filter. As a result, the control input, $u(k)$, the sum of the state feedback outputs, $u_s(k)$, and the time delay control output, $u_T(k)$, is written as

$$u(k) = u_T(k) + u_s(k) \quad (11)$$

where

$$u_s(k) = -K_s x_p(k) = -K_p\theta(k) - K_d\hat{\dot{\theta}}(k)$$

Here K_p and K_d are the proportional and derivative gains, respectively.

3.3 Stability Analysis

It will prove convenient to check the stability of the system by using the equivalent discrete system[3]. The plant dynamics can be approximated, by using the zero order hold method, as

$$G_p(z) = \left(1-z^{-1}\right)Z\left[\frac{G_p(s)}{s}\right] = \frac{b}{\omega_n^2}\frac{(b_1 z + b_2)}{\left(z^2 + a_1 z + a_0\right)} \quad (12)$$

where

$$\alpha = \zeta_n\omega_n L,\ \beta = \omega_n L\sqrt{1-\zeta_n^2},\ a_0 = e^{-2\alpha},$$

$$a_1 = -2e^{-\alpha}\cos\beta,\ b_1 = 1 - e^{-\alpha}\cos\beta - \frac{\alpha}{\beta}e^{-\alpha}\cos\beta$$

$$b_2 = e^{-2\alpha} + \frac{\alpha}{\beta}e^{-\alpha}\sin\beta - e^{-\alpha}\cos\beta$$

Referring to Fig. 4, we can rewrite the controlled system, by adding the state feedback, as

$$\frac{\Theta(z)}{U_T(z)} = \frac{b}{\omega_n^2}\frac{b_1 z^2 + b_2 z}{z^3 + a'_2 z^2 + a'_1 z + a'_0} \quad (13)$$

where

$$a'_0 = -\frac{b_2 c K_d}{L},\ a'_1 = a_0 + b_2 c K_p + \frac{b_2 c K_d}{L} - \frac{b_1 c K_d}{L},$$
$$a'_2 = a_1 + b_1 c K_p + \frac{b_1 c K_d}{L}$$

To estimate the velocity and acceleration from the measured displacement, we use the backward difference model. Combining the Eqs. (10), (11), and (13), we can obtain the characteristic equation in the discrete domain as

$$1 + G(z) = 0 \quad (14)$$

where

$$G(z) = \frac{b}{\hat{b}\omega_n^2}\frac{\left(b_1 z^2 + b_2 z\right)}{\left(1/a_{f0}\,z - \left(a_{f0}/a_{f1}+1\right)\right)\left(z^3 + a'_2 z^2 + a'_1 z + a'_0\right)} \times\left[\omega_r^2 z + \frac{(z-1)^2}{zL^2} + 2\zeta_r\omega_r\frac{z-1}{L}\right]$$

Note that the system characteristics are determined by the sampling time, L, the estimation of control input gain, $\hat{b}$, and the cutoff frequency of first-order filter, ω_c. The time delay controller essentially assumes, for stability, that the sampling

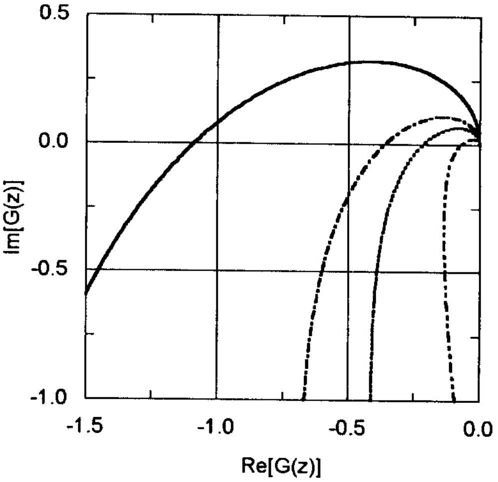

Fig. 4 Nyquist plot with b is varied:
——— w/o filter, – – – – w/filter : b=300
- - - - - - - w/o filter, – - – - - w/filter : b=1500

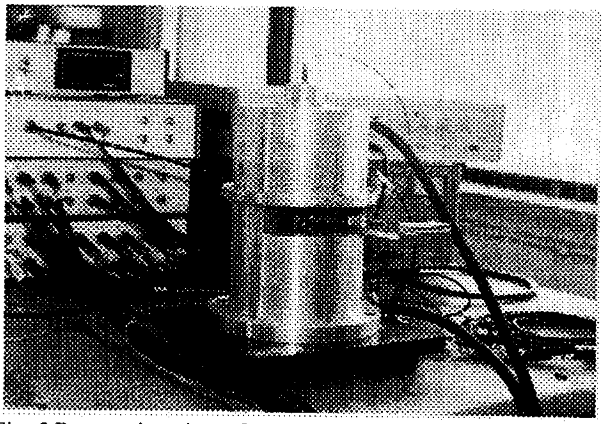

Fig. 5 Perspective view of tested frictionless positioning device

time is very small. Figure 4 shows the Nyquist plot with $\hat{b}$ varied, for ω_c = 500 rad/sec, ω_r = 20 rad/sec, ζ_r = 1, and L = 1 msec. Note that a low value of $\hat{b}$ tends to destabilize the system, and that use of a low-pass filter and a high value of $\hat{b}$ increases the gain and phase margins.

4. EXPERIMENT

Figure 5 is the perspective view of the tested frictionless positioning device. An additional mass is attached at the end of arm, as shown in Fig. 5, to simulate the working piece during operation. The rotor rotational angle is measured by a resolver with resolution of 1 arcmin, converted to 16bit digital signal using the resolver-to-digital converter and fed to the host PC through the digital input/output board. The cone-shaped AMBs are safely suspended by using an optimal controller with integral action. The coupling of AMBs and azimuth motion is negligible. The parameter values throughout experiments are given as: ω_c = 500 rad/sec, ω_r = 20 rad/sec, ζ_r = 1.0, L = 1 msec., and $\hat{b} = 1500$. Figure 6 shows the step responses of the controlled system when an additional mass is attached at the free end of arm. As seen in Fig. 6(a), the step responses of the system with an additional mass by the optimal controller become oscillatory, requiring re-tuning of the controller gains to attain a satisfactory tracking

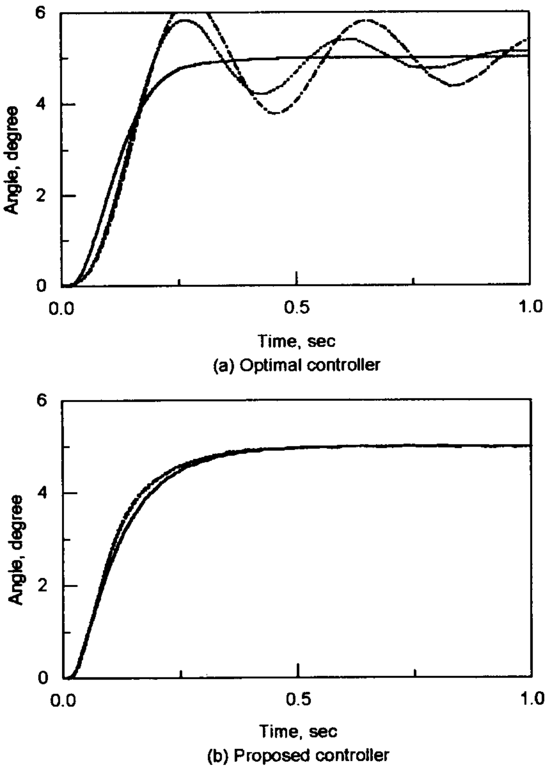

Fig. 6 Step responses with inertia varied(experiment)
——— nominal ; ------- 95g; — - — - — 120g

performance. On the other hand, the time delay controller with state feedback gives the robust responses to the inertia variation, as shown in Fig. 6(b).

The effect of measurement noise on the system performance is important because time delay controller inherently includes the estimation of velocity and acceleration. Figure 7 shows the experimental result for noisy estimations of velocity, acceleration and the control voltage during step tracking. As shown in Fig. 7, the control signal is very noisy due to the numerical differentiation of digital resolver signal. However, as previously shown in the simulation, it does not cause a serious problem. To suppress the undesirable high frequency noise in the control input, an appropriate filter can be implemented for real-time applications.

5. CONCLUSION

A time delay controller with state feedback is proposed for the azimuth motion control of the frictionless position device and proven to be very effective throughout the simulations and experiments. The Nyquist stability analysis in discrete time domain shows that the proposed controller assures the system stability and that use of a low-pass filter tends to increase the gain and phase margins and decrease the destabilization effect due to approximation in the discrete time domain. Finally numerical and experimental results support that the proposed controller guarantees a better tracking performance than the conventional optimal controller irrespective of the variations in inertia, the approximation effect, and the presence of measurement noise.

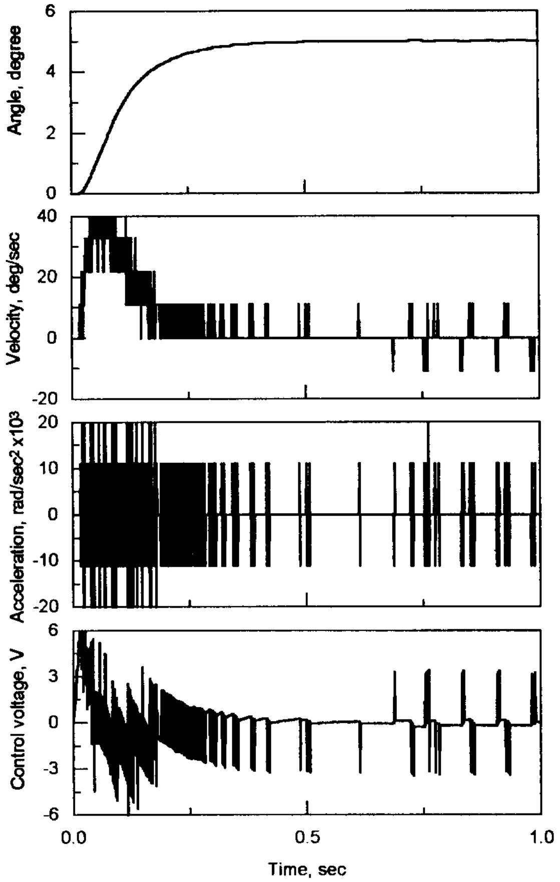

Fig. 7 Step response with noisy estimation

REFERENCES

[1] C. W. Lee and H. S. Jeong, "Dynamic Modeling and Optimal Control of Cone-Shaped Active Magnetic Bearing System," *Control Engineering Practice*, 1996.(submitted)

[2] P. I. Ro, G. Rao, and Q. Ma, "Nanometric Motion Control of A Traction Drive," *ASME Biennial Conference on Dynamic Systems and Control*, DSC-Vol. 55-2, pp879-883, 1994.

[3] K. Youcef-Toumi, J. Bobbett, "Stability of Uncertain Linear Systems with Time Delay," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 113, pp558-567, 1991.

[4] K. Youcef-Toumi, S. Reddy, "Analysis of Linear Time Invariant Systems with Time Delay," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 114, pp544-555, 1992.

[5] K. Youcef-Toumi, S. Reddy, "Dynamic Analysis and Control of High Speed and High Precision Active Magnetic Bearings," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 114, pp623-633, 1992.

[6] K. Youcef-Toumi, S. T., Wu, "Input/Output Linearization Using Time Delay Control," *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 114, pp10-19, 1992.