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Tracking Control for Multi-Axis System using Two-Degrees-of-Freedom Controller

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Abstracts This paper represents an adaptive position controller with the disturbance observer for multi-axis servo system. The overall control system consists of three parts : the position controller, the disturbance observer with free parameters and cross-coupled controller which enhances contouring performance by reducing errors.

Using two-degrees-of freedom conception, we design the command input response and the closed loop characteristics independently. The servo system can improve the closed loop characteristics without affecting the command input response. The characteristics of the closed loop system is improved by suppressing disturbance torque effectively with the disturbance observer. Moreover, the cross-coupled controller enhances tracking performance. Thus total position control performance is improved. Finally, the performance of the proposed controller shows that it inproves the contouring performance along with the reference trajectory in the XY-table.

Keywords Position feedback loop disturbance observer, Tracking error, Contour error, Cross-Coupled Controller

1. INTRODUCTION

In modern industrial field, automation has been pursued for many years. For example, machine tools or multiaxial robots are equipped in manufacturing plants. When the multiaxial machine tools are tracked by specified trajectories, the trajectories are decomposed by the movement of each axis actuators. When multi-dimensional movement is controlled, the error is occurred by each axis actuator, which causes whole tracking error or contour error. Fundamentally, if the plant input can't tracked well, the tracking error is main factor which occurs contour errors. Therefore, at first, reducing the tracking error contributes to the total performance improvement. Most controllers are designed under the assumption which is the optimal condition. Using this assumption, the chosen controller is applied to the real system. In this way, sometimes desired output can't obtained because of disturbances or modelling errors. Especially, these errors occur big damages to the system performance in the field which require very accurate control. So we need robust tracking controller which can reduce the disturbance of the system and don't influenced by modelling errors. For achieving this purpose, of use the disturbance observer affect robustness in the disturbances and the uncertainties. Therefore, in this paper regulating the free parameters of the disturbance observer improves these problems. Furthermore, in the design of servo system, the command input response and closed-loop characteristics can be improved independently using two-degrees-of freedom conception. It is similar to the disturbance observer structure suggested by Umeno and Hori [1][2][3][4], and a number of investigators have researched [5][6][7][8].

In the multiple axis system, even if each axis is equipped with high performance tracking controller, the error of one axis influences the whole motion of system. If this phenomenon is not considered and each axis is controlled independently, the contour error is occurred. As a result, it depreciates the system performance.

Therefore, reducing the contour error is more necessary than reducing the tracking error in the multiple axis system. So, many investigators have developed techniques for reducing contour error in multiple axis system. Poo et al. showed that low steady-state and transient contour errors are obtained when the different axes were matched with dynamics. So, the dynamics mismatch is one of the main causes of contour error under typical machining conditions. Hence to reduce calculated contour error, the input of a Cross-Coupled Controller (*CCC*)[9][10][11][12] is used .

In this paper, disturbance observer is used to design high performance optimal tracking controller against outer disturbance and modelling error, and basically *CCC* is equipped to reduce contour error.

2. MODELLING OF 2-AXIS SYSTEM (X-Y TABLE)

X-Y table is moved to a straight line and is connected by a right angle. X-Y table, which is using right angle co-ordinates, is two dimensional position system. This X-Y table is simply separated from each independent moving. So general mechanical equation of each axis is represented equation (1),(2).

$$J_x \frac{dv_x(t)}{dt} + B_x w_x = u_x(t) \tag{1}$$

$$J_y \frac{dv_y(t)}{dt} + B_y w_y = u_y(t) \tag{2}$$

where, J_x and J_y are inertia of each axis, $u_x(t)$ and $u_y(t)$ are input torque which is moving motor of each axis, and B_x and B_y are viscous-friction coefficient of each axis.

State equation of total system is described by equation (3).

$$\begin{bmatrix} \dot{\theta}_x \\ \dot{\omega}_x \\ \dot{\theta}_y \\ \dot{\omega}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{B_x}{J_x} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{B_y}{J_y} \end{bmatrix} \begin{bmatrix} \theta_x \\ \omega_x \\ \theta_y \\ \omega_x \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{K_x}{J_x} & 0 \\ 0 & 0 \\ 0 & \frac{K_y}{J_y} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{3}$$

3. DESIGN OF THE DISTURBANCE OBSERVER

The disturbance observer is not limited to dc-disturbances, and bandwidth for disturbance rejection can be adjusted. In motion control systems, the disturbance observer can be included in the velocity feedback loop or position loop. Even though major uncertainties are removed by the disturbance observer in the velocity feedback loop, there are many nonlinear properties which are backlash, dead zone, etc. in the coupling part between motor and X-Y table, so we applied disturbance observer in the position feedback loop. The proposed disturbance observer considers the disturbance torque, and estimates the equivalent disturbance which is applied to the nominal model, and the estimation is also utilized to cancel disturbance signal.

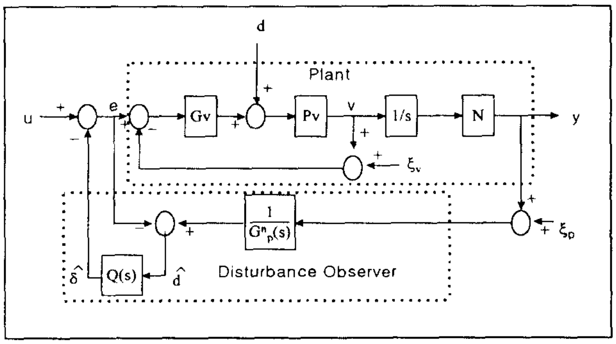

Fig. 1. The formation of position loop disturbance observer

Nominal model of disturbance observer itself, represented in equation (9) can't be implemented. So the relative order of $Q(s)$ which is equal or greater than that of $G^n_p(s)$ is used as a filter. $Q(s)/G^n_p(s)$ can be realizable.

In fig. 1., control input u, disturbance input d, velocity measurement noise ξ_v and position measurement noise ξ_p are considered, the position equation is described by equation (4).

$$p = G_{uv}(s)u + G_{dv}(s)d + G_{\xi v}(s)\xi_v + G_{\xi v}(s)\xi_p \tag{4}$$

where,

$$G_{up} = \frac{G_p G^n_p}{(1+G_v P_v)G^n_p + (G_p - G^n_p)Q}$$

$$G_{dp} = \frac{G^n_p P_v (1-Q)}{(1+G_v P_v)G^n_p + (G_p - G^n_p)Q}$$

$$G_{\xi_v p} = \frac{-G_p G^n_p (1-Q)}{(1+G_v P_v)G^n_p + (G_p - G^n_p)Q}$$

$$G_{\xi_p p} = \frac{-G_p (1-Q)}{(1+G_v P_v)G^n_p + (G_p - G^n_p)Q}$$

$Q(s)$ is very important factor which has a effect on disturbance restrain, optimal performance, and sensor noise rejection. $Q(s)$ is set up to nearly 1 at low frequency region but $Q(s)$ is set up to very small value at high frequency region. Parameter of $Q(s)$ is selected by considering frequency region of sensitivity function and complementary sensitivity function. Sensitivity function of the system is represented by equation (5).

where, $G_{uv}(s)$ is reference input response of total system.

The characteristic equation of closed loop is derived by equation (6)

$$S(s) = (1 - Q(s))(1 - G_{uv}(s)) \tag{5}$$

$$G_{dv}(s) = S(s)\, G^n_{\varepsilon v}(s) \tag{6}$$

where, $S(s)$ is sensitivity function, $G_{uv}(s)$ is command input response of the whole system and $G_{dv}(s)$ is disturbance input response.

The sensitivity function represents the robustness to the system variations. Therefore, for achieving robustness, it is desirable to keep the norm $|S(j\omega)|$ very small. Complementary sensitivity function $T(s)$ which is represented by equation(7) is also considered in the closed loop characteristics.

$$T(s) = 1 - S(s) \tag{7}$$

$T(s)$ is equal to the transfer function from the sensor noise ξ_p to the output p, therefore $|T(j\omega)|$ should be small enough. Meanwhile, it is impossible to keep both $|S(j\omega)|$ and $|T(j\omega)|$ small simultaneously through the whole frequency range in the actual system. At low frequency region, disturbance rejection is more important than noise prevention and in the high frequency region, vice versa is true. So, $|S(j\omega)|$ is made small at low frequency region, and $|T(j\omega)|$ is made small at high frequency region. To achieve these characteristics, we should regulated the parameters. With assumption, the cutoff frequency of $Q(s)$ is much larger than of G_{up}. We can obtain equation (8) and (9).

$$|S(j\omega)| = |1 - Q(j\omega)| \tag{8}$$

$$|T(j\omega)| = |Q(j\omega)| \tag{9}$$

With the above equations, we can insist that $Q(s)$ should be designed so that it behaves like the frequency characteristics of a low pass filter, and $1-Q(s)$ that of high pass filter. $Q(s)$, which satisfies above stated properties, has been suggested by Umeno and Hori is represented as

$$Q(s) = \frac{1 + \sum_{k=1}^{N-r} a_k(\tau s)^k}{1 + \sum_{k=1}^{N} a_k(\tau s)^k} \tag{10}$$

where, r must be equal or greater than the relative order of the transfer function describing the nominal plant. In the experimental system in this paper, r must be equal or greater than 2.

In this paper, Butterworth filter is selected to determine $Q(s)$. The reason is its low resonance peak value. Resonance peak value very much affects on sensitivity to noise.

$$Q(s) = \frac{6(\tau s)^2 + 4(\tau s) + 1}{(\tau s)^4 + 4(\tau s)^3 + 6(\tau s)^2 + 4(\tau s) + 1} \tag{11}$$

where, τ determines the cutoff frequency of the parameter.

4. TRACKING ERROR, CONTOUR ERROR AND CROSS-COUPLED CONTROLLER

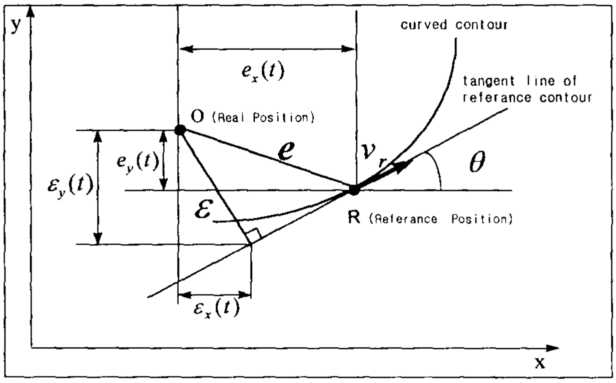

Fig. 2. Definition of tracking error and contour error

4.1 Tracking error

The tracking error is defined to be the difference between the desired location and actual location. In real physical field, inertia, friction and other nonlinear factors are mixed and also exist time delay. So, if reference input is time varying, which is not constant, than output is not follow the reference input and it results in tracking error.

4.2 Contour error

The contour error is defined to be the difference between the desired path and actual path. It is shown in Fig. 2., And it is calculated by equation(12). Generally, contour error is the function of inclination (θ), which is the function between each axis position error and reference trajectory.

$$\varepsilon(t) = e_y(t)\cos\theta - e_x(t)\sin\theta \tag{12}$$

where, $e_x(t)$ and $e_y(t)$ are the position errors along the individual axis. θ is the inclination of the contour with respect to the x-axis and ε is the contour error.

$$\theta = \arctan\left(\frac{v_y}{v_x}\right) \tag{13}$$

where, v_x and v_y are velocity factors of x and y axes for reference trajectory.

4.3 Cross-coupled controller (CCC)

When one axis load or disturbance is sudden changed, to correct this phenomenon, that is, to reduceing contour error not only the one axis but also another axis is compensated simultaneously.

When we control multi-axis system by using cross-coupled controller, we can give compensation input which is calculated with contour error, and can be reduced the contour error.

It is fundamental expression that equation (14) and (15) which generate control input from position error and contour error.

$$u_x = Kp_x e_x + Gp_x \varepsilon_x \tag{14}$$

$$u_y = Kp_y e_y + Gp_y \varepsilon_y \tag{15}$$

where, Kp_x and Kp_y are feedback controller, Gp_x and Gp_y are cross-coupled controller, e_x and e_y are tracking error component, and ε_x and ε_y are contour error component.

The components of the contour error are determined as

$$\varepsilon_x = -m_y\varepsilon = m_y^2 e_x - m_x m_y e_y \tag{16}$$

$$\varepsilon_y = -m_x\varepsilon = m_x^2 e_y - m_x m_y e_x \tag{17}$$

where, $m_x = \cos\theta$, $m_y = \sin\theta$

5. THE WHOLE SYSTEM STRUCTURE AND SIMULATION RESULTS

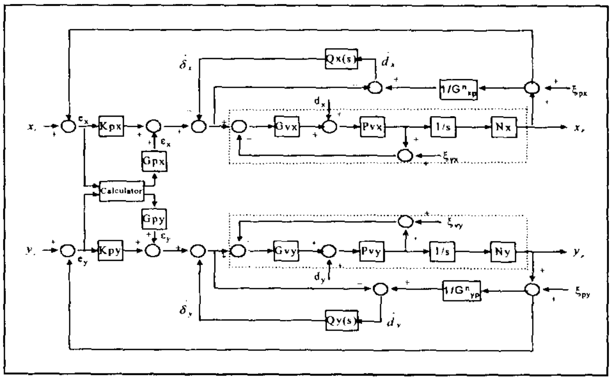

Fig. 3. The whole system structure

Proposed whole system structure is below. To evaluate the proposed controller, we have simulated the case that disturbance observer is equipped or not. *CCC* is equipped simultaneously.

The radial tracking error is defined as

$$e_t = \sqrt{(x_r - x_p)^2 + (y_r - y_p)^2} \quad (18)$$

and the radial contour error is defined as

$$e_c = \sqrt{(x_p - r)^2 + (y_p - r)^2} \quad (19)$$

where, x_r and y_r are desired position, x_p and y_p are actual position, and r is the radius of circle

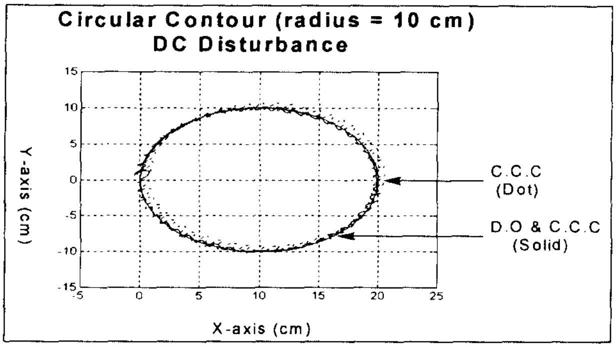

Fig. 4. : Examining tracking ability by simulation of a circular motion.
Conditions: DC disturbance, No modelling error

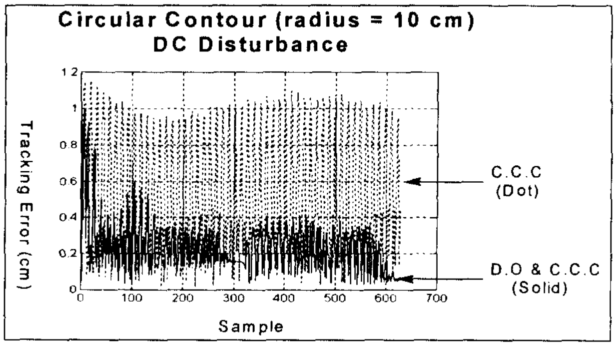

Fig. 5. : Tracking error,
Conditions: DC disturbance, No modelling error

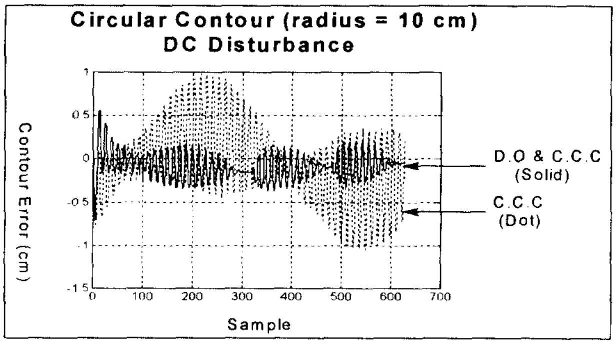

Fig. 6. : Contour error,
Conditions: DC disturbance, No modelling error

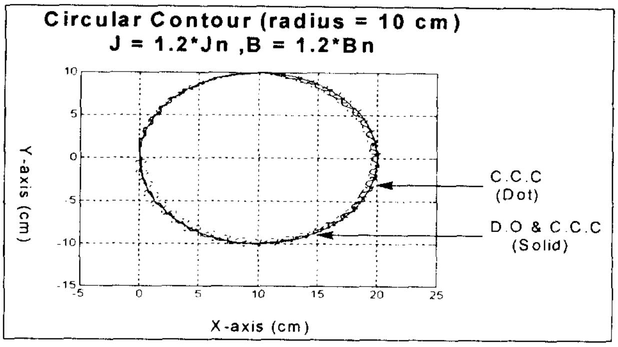

Fig. 7. Circle tracking simulation
Condition: J=Jn*1.2, B=Bn*1.2 No disturbance input

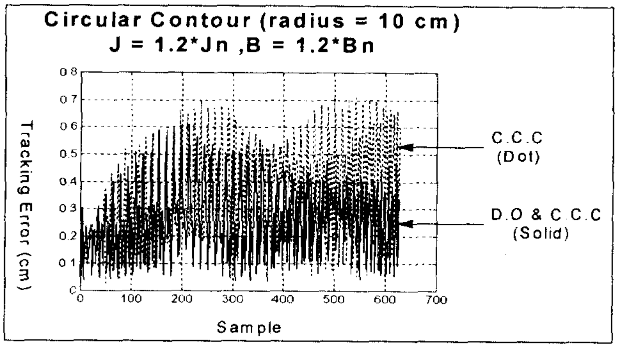

Fig. 8. Tracking error
Condition: J=Jn*1.2, B=Bn*1.2 No disturbance input

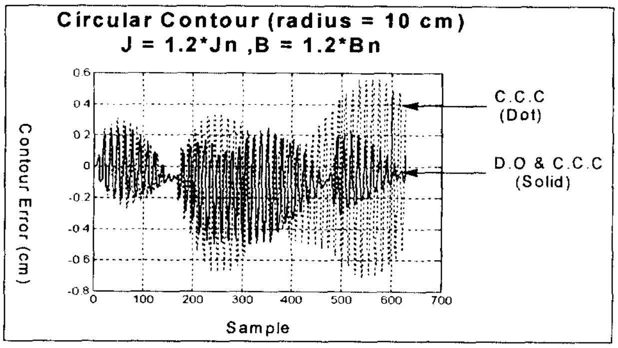

Fig. 9. Contour error
Condition: J=Jn*1.2, B=Bn*1.2 No disturbance input

TABLE 1. The Comparison of Average error

Case	Controller	Tracking error(mm)	Contour error(mm)
DC Disturbance	*CCC*	5.221	3.454
	CCC & D.O	2.332	1.528
	Comparison	46.67 %	44.24 %
J=Jn*1.2 B=Bn*1.2	*CCC*	3.639	2.537
	CCC & D.O	2.625	1.489
	Comparison	72.16 %	58.69 %

6. CONCLUSION

In this paper, to improve tracking performance and contour performance, disturbance observer which influence outer disturbance and uncertainty of modelling and *CCC* which influence contour error are used. We have simulated proposed controller and achieved good performance about disturbance rejection. Since the performance of the disturbance observer is mainly dependent on the choice of the *Q*-filter, it requests more research how to optimal determine the *Q*-filter.

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