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ROBUST AND ADAPTIVE CONGESTION CONTROL IN PACKET-SWITCHING NETWORKS

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Abstracts In this paper, a feedforward-plus-feedback control scheme is presented to prevent congestion in store-and-forward packet switching networks. The control scheme consists of two algorithms. Specifically, the input traffic adjustment algorithm employs a fairness policy such that the transmission rate of the input traffic is proportional to its offered rate. The control signal computation algorithms to ensure stability of the overall system in the robust sense and to ensure the desired transient behavior in the adaptive, with respect to variations of input traffic, are designed.

Keywords Single Congestion, Feedforward-plus-Feedback Control, Extreme-Point Robust Stability

1. INTRODUCTION

Since the past several decades of years, users of communication networks have increased through a persistent progress in communication techniques. With a rapid increase of users, many problems in network management have been generated, including congestion as a remarkable problem. Generally, congestion can be defined as follows: an undesirable result due to insufficiency of network resources (buffer space, transmission facilities) as compared with users' demands. Congestion, when once it occurs, causes undesirable effects, such as a deterioration of network's throughput and a substantial increase of users's time delay.

Thus, the congestion control problem has been studied as one of the most active areas of research in communication networks. In [1]-[4], adaptive window-based schemes were proposed and investigated. Subsequently, in [5]-[6], rate-based schemes were suggested and analyzed. In some cases, these congestion control schemes may be effective in avoiding or recovering from congestion. However, in these schemes, it was also shown that the overall system is locally unstable in neighborhood of the desired operating point (oscillatory behavior) [6].

In this paper, we provide an analytical design method of a congestion control scheme to ensure asymptotic stability of the overall system in the robust sense, with respect to variations of the input traffic, and to ensure the desired transient behavior of the overall system in the adaptive sense.

This paper is structured as follows: In section 2, the model of store-and-forward packet-switching networks and feedforward-plus-feedback congestion control scheme is constructed respectively. In section 3, we develop the design method of the congestion control scheme in the robust sense and the adaptive sense, respectively. In section 4, the obtained results are confirmed additionally through numerical examples. Finally, in section 5, the conclusions are formulated.

2. THE MODEL

2.1 The Network

The network considered in this paper is assumed to be a deterministic fluid model as follows:

(i) The network uses packet-switching technology and employs a store-and-forward switching service in which users are serviced without prior reservation.

(ii) The network consists of switching nodes, interconnected by transmission links(refer to Fig. 1(a)).

(iii) Each link has a transmission capacity of $c_{tr} = 1/\tau_{tr}$ (packets/sec), where τ_{tr} is the transmission time of a packet.

(iv) Each link has a propagation delay of τ_{pr} sec. It is proportional to the length of the link.

(v) Each node has a switching capacity of $c_{sw} = 1/\tau_{sw}$ (packets/sec), where τ_{sw} is the switching time of a packet. The switching capacity of a node is larger than the total transmission capacity of its incoming links. It means that links rather than switches are overloaded.

(vi) The network traffic is classified into flows corresponding to each source-destination node pair (a, b), denoted by flow (ab).

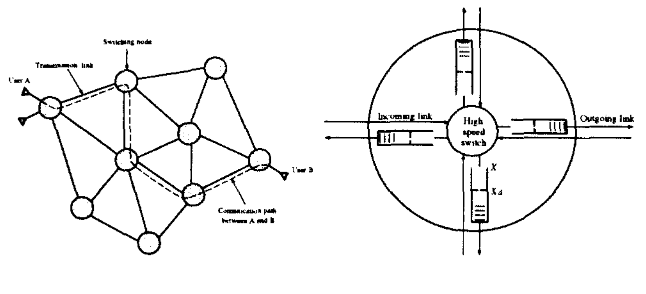
(vii) For each flow (ab), the source node a sends packets to the destination node b through the fixed transmission path of the flow, denoted by $p(ab)$. The routing policy which determines the transmission path of each flow is static. Let $F(i)$ be the set of all flows passing through link i.

(viii) Each node has output-buffering structure(refer to Fig. 1(b)): In a node, there is no need for buffers associated with its incoming links due to high-speed switching, however for each of its outgoing links, however, there is a buffer for storing packets waiting to be transmitted. Let x_i denote the number of packets buffered for transmission through outgoing link i. We assume that each buffer has a finite but large capacity X.

(ix) The input traffic is viewed as a deterministic fluid-flow.

(x) For each flow (ab), let $\lambda_{ab}^0(t)$ denote its offered rate (packets/sec) at time t. Let $f_i^0(t) = \sum_{(ab)\in F(i)} \lambda_{ab}^0(t)$ be the total offered rate (packets/sec) of all flows passing through link i. We assume that the input traffic is such that only one link in a given transmission path is overloaded, i.e. there exists a link i_0 such that $f_{i_0}^0(t) > c_{tr}$ whereas $f_i^0(t) < c_{tr}$ for all $i \neq i_0$.

Under the assumptions (i)-(x), we formulate the dynamics of buffers in the network. First, let $x_i(t)$ denote the occupancy of the buffer associated with link i at time t. Then, the buffer dynamics is represented by the following



(a) Network topology (b) Node configuration

Fig. 1: A configuration of packet-switching networks

differential equation for deterministic fluid-flow:

$$\frac{dx_i(t)}{dt} = \begin{cases} 0 & \text{if } f_i(t) < c_{tr} \text{ and } x_i(t) = 0, \\ & \text{or if } f_i(t) > c_{tr} \text{ and } x_i(t) = X \\ f_i(t) - c_{tr} & \text{otherwise} \end{cases}$$

where $f_i(t)$ is the traffic arrival rate at the buffer of link i at time t. Let $\lambda_{ab}(t)$ denote the rate at which flow (ab) is transmitted to the network at time t. Then, we get

$$f_i(t) = \sum_{(ab)\in F(i)} \lambda_{ab}(t - \tau_{ai}^{ab})$$

where $F(i)$ is the set of all flows passing through link i, and τ_{ai}^{ab} is the forward trip delay of flow (ab) until it arrives at the buffer of link i. The backward trip delay of flow (ab) from link i to source node a, denoted by τ_{ia}^{ab}, is defined in a way similar to the forward trip delay τ_{ai}^{ab}. In fact, the backward path is the reverse of the forward path. Accordingly, as it follows from assumption (x), forward and backward delay for the overload link i_0 are equal, denoted by τ^{ab}:

$$\tau^{ab} = \tau_{ai_0}^{ab} = \tau_{i_0a}^{ab}$$

We assume that τ^{ab} is a multiple of control period T, defined in section 2.2, in other words, τ^{ab}/T is assumed to be an integer for a simplicity, although the non-integer case can be extended directly.

Moreover, it follows from assumption (x) that for some link i except the overloaded link i_0, $f_i(t) - c_{tr}$ must be always negative Therefore, assuming that buffers are initially empty ($x_i(0) = 0, \ \forall i$), the buffers of the non-overloaded links remain continually empty.

$$x_i(t) = 0, \quad t > 0, \quad \forall i \neq i_0$$

As it follows from the above, there is no queuing delay at the buffers of the non-overloaded links.

2.2 The Congestion Control Scheme

The control scheme for congestion prevention is constructed as follows:

(xi) In a node, there is a control signal computation algorithm associated with each of its outgoing links. The algorithm computes periodically a control signal.

(xii) A control signal is newly computed every T sec. Thus, the time is slotted with the slot duration, $[n, n+1)$, $n = 0, 1, \cdots$, equal to T.

(xiii) Each node sends the computed control signal to the sources along a fixed backward path, usually the reverse direction of the traffic transmission path. This data is serviced with high priority.

(xiv) In respond to the control signal, the sources adjust their transmission rates according to the input traffic adjustment algorithm.

As it follows from the above assumptions (xi)-(xiv), the congestion control scheme consists basically of two algorithms: a control signal computation algorithm and an input traffic adjustment algorithm.

Control Signal Computation Algorithm: This algorithm computes periodically a control signal for overloaded link i_0, based on the following equation:

$$q_{i_0}(t) = \Gamma_0^1 \left\{ \frac{c_{tr}}{\lambda^0(t)} - \alpha[x_{i_0}(t) - x_d] + \sum_{k=0}^{K} \beta_k [q_{i_0}(t - kT - T) - \frac{c_{tr}}{\lambda^0(t)}] \right\}$$

where

$$\lambda^0(t) \equiv \sum_{(ab)\in F(i_0)} \lambda_{ab}^0(t),$$

$$\Gamma_x^y\{z\} \equiv \begin{cases} y, & \text{if } z \geq y \\ x, & \text{if } z \leq x \\ z, & \text{otherwise.} \end{cases}$$

In this algorithm, $\lambda^0(t)$ is assumed to be available. Practically, it can be realized through periodical forward transmission of a special data packet including an information on the offered traffic additionally. Note that the above equation consists of a feedforward term $(c_{tr}/\lambda^0(t))$ with intention of convergence of the buffer occupancy to some finite value (oscillation prevention), and two feedback terms: a proportional term $(-\alpha[x_{i_0}(t) - x_d])$ and a compensational term $(\sum_{k=0}^{K} \beta_k[q_{i_0}(t - kT - T) - c_{tr}/\lambda^0(t)])$, where the proportional term is implemented for convergence of the buffer occupancy to the desired value (x_d) at the steady state (asymptotic stability), and the compensational term is implemented for improvement of transient performance.

Input traffic adjustment Algorithm: This algorithm adjust the transmission rate of the offered traffic at the sources, based on the following equation:

$$\lambda_{ab}(t + \Delta) = \lambda_{ab}^0(t) q_{i_0}(t - \tau^{ab}), \quad \forall, \quad (ab) \in F(i_0)$$

where $t = 0, T, 2T, \cdots$, and $0 \leq \Delta < T$.

2.3 The Overall System

Discretizing the buffer dynamics with control period (T) and omitting T, we obtain the following difference equations

$$x_{i_0}(n+1) = \Gamma_0^X \left\{ x_{i_0}(n) + \sum_{(ab)\in F(i_0)} [r_{ab}^0(n - d^{ab}) q_{i_0}(n - 2d^{ab})] - c \right\}, \quad (1)$$

$$q_{i_0}(n) = \Gamma_0^1 \left\{ \frac{c}{r^0(n)} - \alpha[x_{i_0}(n) - x_d] + \sum_{k=0}^{K} \beta_k [q_{i_0}(n - k - 1) - \frac{c}{r^0(n)}] \right\} \quad (2)$$

where $r_{ab}^0(n) \equiv T\lambda_{ab}^0(nT)$, $r^0(n) \equiv T\lambda^0(nT)$, $c = Tc_{tr}$, and $2d^{ab}$ is the normalized round trip delay. Rearranging the arrival flows from (1) and omitting index i_0, we obtain the following equations:

$$x(n+1) = \Gamma_0^X \left\{ x(n) + \sum_{i=0}^{D} r_i(n) q(n-i) - c \right\}, \quad (3)$$

$$q(n) = \Gamma_0^1 \left\{ \frac{c}{r^0(n)} - \alpha[x(n) - x_d] + \sum_{k=0}^{K} \beta_k [q(n-k-1) - \frac{c}{r^0(n)}] \right\} \quad (4)$$

where

$$r_i(n) \equiv \sum_{(ab)\in F(i_0),\ 2d^{ab}=i=2j} r^0_{ab}(n-j),$$

$$D \equiv \max_{(ab)\in F(i_0)} \{2d^{ab}\}.$$

As defined in the above, the $r_i(n)$ can be interpreted as the total arrival rate for the duration $[n, n+1)$ of all flows with the round trip delay (between node a and link i_0) equal to i time slots, and D is the largest normalized round trip delay.

3. THE ANALYSIS

3.1 The Steady State Analysis

Let x_s and q_s denote the steady state values of $x(n)$ and $q(n)$, respectively. Then, we obtain the following steady state solution of eqs. (3)-(4):

$$q_s = \frac{c}{r^0}, \quad x_s = x_d \quad (5)$$

Note that (5) only satisfies the necessary condition at the steady state.

3.2 The Robust Scheme

The main goal of this section is to design the simple-structured control algorithm that guarantees the robust stability of the overall system for variations of the input traffic whose total amount is limited to a certain value. in eq. (4), the compensational term can be removed for only robust stability.

Unfortunately, due to the complex nature of the dynamics (3)-(4), imposed by the saturation function, we are unable to solve the robust stability problem in the global sense. Thus, the robust stability problem in the local sense is only addressed analytically. Removing the saturation nonlinearities in (3) and (4), we get

$$x(n+1) = x(n) + \sum_{i=0}^{D} r_i q(n-i) - c \quad (6)$$

$$q(n) = \frac{c}{r^0} - \alpha[x(n) - x_d]. \quad (7)$$

Combining these two equations, we obtain

$$x(n+1) = x(n) - \sum_{i=0}^{D} r_i \alpha [x(n-i) - x_d] \quad (8)$$

where r_i is a constant value of $r_i(n)$ during the settling time. Subsequently, the characteristic polynomial of eq. (8) can be represented by

$$\Phi(z) = z^{D+1} - z^D + r_0 \alpha z^D + r_1 \alpha z^{D-1} + \cdots + r_n \alpha.$$

To check the robust stability of the above polynomial, we first formulate the following lemma.

Lemma 1 *Given $n > 0$, there exists $\delta > 0$ such that the polynomial $p_e(z) = z^n - z^{n-1} + r$ is stable if and only if $0 < r < \delta$.*

This proof is obtained from Jury's test. Based on Theorem 1 in [7] and Lemma 1, we develop the following theorem and corollary.

Theorem 1 *Given $r > 0$, the set of polynomials $\mathbf{P}(z) = \{z^n - z^{n-1} + a_1 z^{n-1} + \cdots + a_n \mid a_i \geq 0, \forall i,\ a_1 + \cdots + a_n = r\}$ is stable if and only if the extreme polynomial $p_e(z) = z^n - z^{n-1} + r$ is stable.*

Proof: Using Theorem 1 in [7], this can be proved.
It should be noted that Theorem 1 describes that in the coefficient space $(a_1, \cdots, a_n)$, the stability problem of all polynomials corresponding to the special polytope ($a_i \geq 0$, $\forall i$, $0 < a_1 + \ldots + a_n \leq r$) can be reduced to a stability check of the polynomial corresponding to one ($a_i = 0$, $i = 1, \cdots n-1$, $a_n = r$) of the extreme-points of the polytope.

Corollary 1 *Given $r > 0$, the set of polynomials $\mathbf{P}(z) = \{z^n - z^{n-1} + a_1 z^{n-1} + \cdots + a_n \mid a_i \geq 0, \forall i,\ 0 < a_1 + \cdots + a_n \leq r\}$ is stable if and only if $0 < r < 2\sin\left(\frac{\pi}{4n-2}\right)$.*

Proof: This can be easily proved from Theorem 1.
Using Corollary 1, we obtain the condition of the control gain α such that for all input patterns $(r_0(n), r_1(n), \cdots, r_D(n))$ whose total rate is not larger than R^0, the overall system is asymptotically stable.

$$0 < \alpha < \frac{2}{R^0} \sin\left(\frac{\pi}{4D+2}\right) \quad (9)$$

3.3 The Adaptive Scheme

As it follows from the previous section, the robust congestion control scheme can guarantees only the robust stability of the system. In other words, using a control gain α, we are unable to realize the desired transient behavior since the overall system has high-dimensional complicated dynamics due to the various delay factors. The goal of the adaptive scheme is that the overall system achieves a desired transient performance, for a given input traffic pattern. To solve this problem, in the control algorithm, the compensational term is inevitable.

Removing the saturation nonlinearities in (3) and (4), we get

$$x(n+1) = x(n) + \sum_{i=0}^{D} r_i(n) q(n-i) - c \quad (10)$$

$$q(n) = \frac{c}{r^0(n)} - \alpha[x(n) - x_d] + \sum_{k=0}^{K} \beta_k [q(n-k-1) - \frac{c}{r^0(n)}] \quad (11)$$

To adapt control gains to changes in the input traffic pattern, we develop the following theorem.

Theorem 2 *For some given input traffic pattern (r_0, r_1, $\ldots$, r_D) and the desired poles (λ_0, λ_1, $\cdots$, λ_{D+1}) of the above overall system (10)-(11), the poles can be placed at will by the following choice of the control gains $\alpha, \beta_0, \ldots, \beta_D$ ($K = D$):*

$$\alpha = \frac{1}{r^0}\left(1 + \sum_{i=0}^{D+1} a_i\right)$$

$$\beta_k = \frac{1}{r^0}\sum_{i=0}^{k} r_i \left(1+\sum_{j=0}^{D+1} a_j\right) - \left(1+\sum_{j=0}^{k} a_j\right)$$

where $k = 0, 1, \cdots, D$ *and* a_i, $i = 0, 1, \cdots, D$ *are coefficients of the desired characteristic polynomial of closed-loop equations, as defined below.*

$$\Psi(z) = (z-\lambda_0)(z-\lambda_1)\cdots(z-\lambda_{D+1})$$
$$= z^{D+2} + a_0 z^{D+1} + a_1 z^{D} + \cdots + a_{D+1}$$

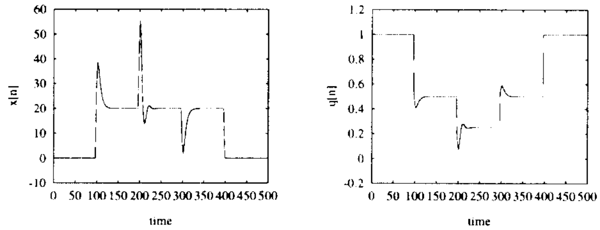
Proof: By matrix representation, this can be proved. Note that using Theorem 2, we are able to obtain adaptation formulas of the control gains to changes of the input traffic pattern $(r_0(n), r_1(n), \ldots, r_D(n))$. Note that the adaptive scheme has the advantage of achieving good performance but at the expense of higher computational requirements as compared with the robust scheme.

4. NUMERICAL EXAMPLES

TABLE 1. Input traffic

n(time slot)	$r_0(n)$	$r_1(n)$	$r_2(n)$	$r_3(n)$	$r_4(n)$
0	1	0	3	0	1
100	2	0	8	0	10
200	2	0	18	0	30
300	2	0	8	0	10
400	1	0	3	0	1

Consider the system with $c = 10$, $x_d = 20$. Fig. 2 shows the network performance of the robust scheme and Figs. 3-4 show that of the adaptive scheme, according to the input traffic such as Table 1. Note that in numerical examples, the adaptive scheme has a fast response as compared with a robust scheme and the overshoot at sudden change of the input traffic is due to the network delay factors, such as the forward trip delay of a data packet and the backward trip delay of a control signal. The overshoot can be reduced by a decrease of control period (T).

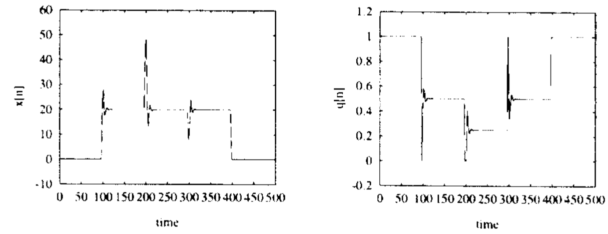


(a) Buffer occupancy (b) Control signal

Fig. 2: *Network response under robust controller with* $\alpha = 0.005$

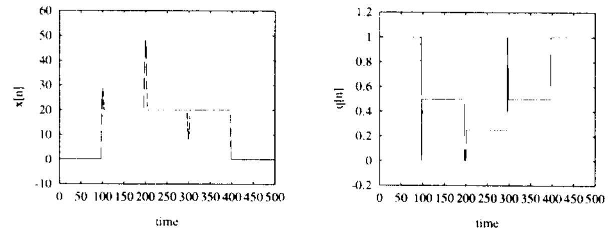
5. CONCLUSIONS

This paper presented and analyzed the feedforward-plus-feedback congestion control scheme to solve a congestion problem in store-and-forward packet switching networks, where congestion control problem is considered as a viewpoint of classical control problem. Specifically, a congestion control scheme in the robust sense was designed to ensure asymptotic stability of the overall system for the offered traffic, and also a congestion control scheme in the adaptive sense was designed to ensure the desired transient performance of the overall system. Specially, a new extreme-point robust-stability result for discrete-time polynomials with special uncertainties in the coefficient space were presented.



(a) Buffer occupancy (b) Control signal

Fig. 3: *Network response under adaptive controller with system poles located at* $(0.5 \pm 0.5i,\ -0.5 \pm 0.5i,\ \pm 0.5i)$



(a) Buffer occupancy (b) Control signal

Fig. 4: *Network response under adaptive controller with all system poles located at 0*

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