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Optimal Buffer Size Control of Serial Production Lines with Quality Inspection Machines

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Abstract In this paper, based on the performance analysis of serial production lines with quality inspection machines, we develope an buffer size optimization method to maximize the production rate. The total sum of buffer sizes are given and a constant, and under this constraint, using the linear approximation method, we suggest a closed form solution for the optimization problem with an acceptable error. Also, we show that the upstream and downstream buffers of the worst performance machine have a significant effect on the production rate. Finally, the suggested methods are validated by simulations.

Keywords Serial production lines, Performance analysis, Quality Inspection, Buffer Size Optimization

1. Introduction

The quality inspection and the buffer size optimization issues in production lines are the important topics. While the buffer size optimization problem has been mentioned by a lot of articles[1][3], the quality inspection issue in production rate seems not available except [5] and [6]. In [5], the 2 machines and 1 buffer system with quality inspection machines is introduced and analyzed. In [6], the general serial production lines with inspection machines are introduced and analyzed.

In this paper, based on [6], we suggest the optimal buffer sizes that maximize the production rate of serial production lines with inspection machines, unlike most of other methods[1][3], in a closed form formula and within $\mathcal{O}(\epsilon^2)$ error bound. Finally, the suggested methods are validated by simulations.

Consider a serial production line defined by the following assumptions[4][6](see Fig. 1).

(i) The system consists of M machines, m_i, $i = 1,\dots,M$, and buffer B_i, $i = 1,\dots,M-1$, separating each consecutive pair of machines, m_i and m_{i+1}.

(ii) The machines have identical cycle time T. The time axis is slotted with the slot duration T. Machines begin their operation at the beginning of each time slot.

(iii) Each buffer is characterized by its capacity, n_i, $i = 1,\dots,M-1$, where n_i is a positive integer.

(iv) Machine m_i is starved during a time slot if buffer B_{i-1} is empty at the beginning of this slot; m_i is blocked during a time slot if at the beginning of this time slot buffer B_i is full and machine m_{i+1} either fails or is blocked.

(v) Machine m_i, being neither blocked nor starved, produces a material during a time slot with probability $1-p_i$ and fails to do so with probability p_i, $i = 1,\dots,M$, where $0 < p_i \ll 1$.

(vi) Machine m_i introduces a defect to its product with probability r_i and doesn't introduce a defect with probability $1-r_i$ for each cycle time T where $0 \leqslant r_i \ll 1$, $i = 1,\dots,M$. And the defects introduced by m_i cannot be removed by $m_{i+1},\dots,m_M$.

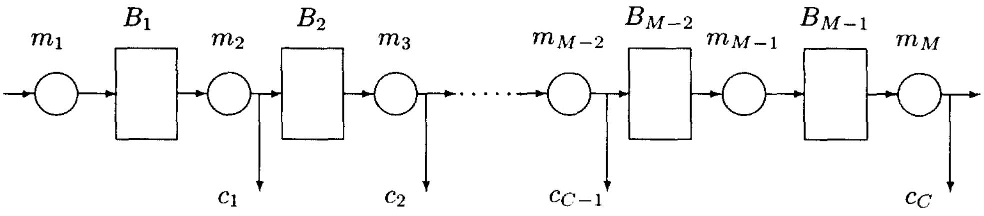

Fig. 1: An open-loop line

(vii) The system has I quality inspection machines, c_i, $i = 1, \ldots, I$ where $I \leqslant M$. A quality inspection machine perfectly detects any defective workpiece. The defective workpiece is removed from the system as soon as it is detected by quality inspection machine.

When the given system is satisfied by (i)–(vii), it is referred to as the *asymptotically reliable serial production line with quality inspection machines* or shortly the *open-loop line.*

2. Buffer size optimization

The steady state average production rate of a general open-loop line is calculated by the following[6].

Theorem 1 *Under assumptions (i)–(vii), suppose $n_i > 1$ for all i. Then the average steady state production rate is*

$$PR(\mathbb{P}, \mathbb{R}, \mathbb{B}) = \sum_{\substack{\xi=1 \\ j>1}}^{j-1} F(p_e(\xi, \xi), n_e(\xi, j-1), p_j) \prod_{w=j}^{M} (1 - r_w) + \sum_{\substack{\xi=j+1 \\ j<M}}^{M} F(p_e(j, \xi-1), n_e(j, \xi-1), p_\xi) \prod_{w=\xi}^{M} (1 - r_w) - (M-2)(1 - p_e(j, M)) + \mathcal{O}(\epsilon^2) \quad (1)$$

where $\mathbb{P} = [p_1, p_2, \ldots, p_M]^T$, $\mathbb{R} = [r_1, r_2, \ldots, r_M]^T$, $\mathbb{B} = [n_1, n_2, \ldots, n_{M-1}]^T$, $j = \{k \in [1, M] \mid p_e(k, M) \geqslant p_e(i, M),\ \forall i \in [1, M]\}$, $p_e(\xi, \nu) = 1 - (1 - p_\xi) \prod_{w=\xi}^{\nu} (1 - r_w)$, $n_e(\zeta, \nu) = \sum_{i=\zeta}^{\nu} n_i + \zeta - \nu$,

$$F(p_1, k, p_2) = \left(1 - \frac{1}{1 + \frac{1}{p_2} \sum_{i=1}^{k} \left(\frac{p_2}{p_1} \frac{1-p_1}{1-p_2}\right)^i}\right)(1 - p_2).$$

The aim of this paper is to optimize the buffer sizes to maximize the steady state average production rate. It is formulated as,

Problem 1 *Under assumptions (i)–(vii), find*

$$\max_{\mathbb{B}} \left(PR(\mathbb{P}, \mathbb{R}, \mathbb{B}) \right),$$

subject to $n_i \geqslant 2$, $i = 1, \ldots, M-1$, *and*

$$\sum_{i=1}^{M-1} n_i = C.$$

It should be mentioned that the following cases are trivial.

(i) $p_i = 0$, $i = 1, \ldots, M$, $r_i = 0$, $i = 1, \ldots, M$,

(ii) $p_j \neq 0$, $r_i = 0$, $i = 1, \ldots, j-1$, $p_i = 0$, $i = 1, \ldots, j-1, j+1, \ldots, M$.

Independent of buffer sizes, the production rate of system (i) and (ii) are 1 and $(1 - p_j) \prod_{w=j}^{M} (1 - r_w)$ respectively. Therefore, we should not consider the trivial cases hereafter.

Theorem 2 *Under assumptions (i)–(vii), let n_i^*, $i = 1, \ldots, M-1$, be the optimal buffer sizes that satisfy the problem 1. Then $n_i' = 2$, $i = 1, \ldots, j-1, j+2, \ldots, M-1$, $n_j' = \sum_{w=1}^{j-1} n_w^* - 2(j-2)$, and $n_{j+1}' = \sum_{w=j}^{M-1} n_w^* - 2(M-j-1)$ are also the optimal solution of problem 1 in $\mathcal{O}(\epsilon^2)$ error bound sense where $j = \{k \in [1, M] \mid p_e(k, M) \geqslant p_e(i, M),\ \forall i \in [1, M]\}$.*

Proof : *Available from the authors upon request.*

It follows from Theorem 2 that we only need to consider the upstream and downstream buffer sizes of the worst performance machine, i.e., m_j, to maximizes the production rate while other buffer sizes remain at 2. Therefore, if the worst machine is m_1, i.e., $j = 1$, then the optimal solution is the following,

$$n_1^* = C - 2M + 4, \qquad n_i^* = 2, \quad i = 2, \ldots, M-1. \quad (2)$$

If the worst machine is m_M then the optimal solution is

$$n_i^* = 2, \quad i = 2, \ldots, M-2, \qquad n_{M-1}^* = C - 2M + 4. \quad (3)$$

Now, consider the following theorem.

Theorem 3 *Under assumptions (i)-(vii),*

$$n_i^* = 2, \qquad i = 1, \ldots, j-1, j+2, \ldots, M-1,$$
$$n_{j+1}^* = \lfloor \frac{C'}{1+\sqrt{\frac{c_f}{c_b}}} \rfloor + 2,$$
$$n_j^* = C' - n_{j+1}^* + 4$$

are the optimal buffer sizes that satisfy the problem 1 in $\mathcal{O}(\epsilon^2)$ error bound sense where

$$C' = C - 2(M-1),$$
$$c_f = \begin{cases} 0, & \text{if } j = 1 \\ \sum_{i=1}^{j-1} c_1(p_j, p_e(i,i), p_j, j-i-1), & \text{otherwise}, \end{cases}$$
$$c_b = \begin{cases} 0, & \text{if } j = M \\ \sum_{i=j+1}^{M-1} c_1(p_j, p_i, p_e(j, i-1), i-j-1), & \text{otherwise}, \end{cases}$$
$$c_1(q_0, q_1, q_2, z) = \frac{q_2(X-1)}{q_0^3 X} \frac{\ln X X^{\frac{1}{q_0}+z}}{(1-X^{\frac{1}{q_0}+z})^2},$$
$$X = \begin{cases} \frac{q_2(1-q_1+q_1^2)}{(q_1-q_1^2)(1-q_2)}, & \text{if } q_1 = q_2 \\ \frac{q_2(1-q_1)}{q_1(1-q_2)}, & \text{otherwise}. \end{cases}$$

Proof : *Available from the authors upon request.*

Theorem 3 gives the optimal solution of problem 1 in $\mathcal{O}(\epsilon^2)$ sense. For eq. 2 and 3, Theorem 3 yields the same results.

3. Simulation

To validate the suggested methods, we simulate 5 machines and 4 buffers systems. We get the optimal buffer sizes by the enumeration based on simulations, and calculate another optimal buffer sizes by the suggested method. Table 1–4 depicts that the error between the production rate of the optimized system by simulations and the production rate of the optimized by the suggested method is in $\mathcal{O}(\epsilon^2)$ error bound where the error is calculated by

$$\text{error} = \frac{PR_{\text{simulation}} - PR_{\text{calculation}}}{PR_{\text{simulation}}} \tag{4}$$

4. Conclusions

We develope the closed form solution for the constrained buffer size optimization problem in serial production line in $\mathcal{O}(\epsilon^2)$ error bound. We find that we only need to consider the upstream and downstream buffer sizes of the worst performance machine while the size of other buffers remains at 2. Also, we suggest the optimal sizes of the downstream buffer and the upstream buffer of the worst performance machine in closed form formula with $\mathcal{O}(\epsilon^2)$ error bound.

Table 1: Case 1

Total Buffer size = 10									
p_1	r_1	p_2	r_2	p_3	r_3	p_4	r_4	p_5	r_5
0.10	0.05	0.15	0.03	0.15	0.05	0.10	0.04	0.10	0.05

method	n_1^*	n_2^*	n_3^*	n_4^*	performance	error
simulation	3	3	2	2	0.659833	
calculation	3	3	2	2	0.652551	0.007282

Table 2: Case 2

Total Buffer size = 14									
p_1	r_1	p_2	r_2	p_3	r_3	p_4	r_4	p_5	r_5
0.10	0.05	0.15	0.03	0.15	0.05	0.10	0.04	0.10	0.05

method	n_1^*	n_2^*	n_3^*	n_4^*	performance	error
simulation	4	5	3	2	0.682700	
calculation	5	5	2	2	0.681864	0.000836

Table 3: Case 3

Total Buffer size = 10									
p_1	r_1	p_2	r_2	p_3	r_3	p_4	r_4	p_5	r_5
0.05	0.05	0.05	0.03	0.15	0.05	0.10	0.04	0.10	0.05

method	n_1^*	n_2^*	n_3^*	n_4^*	performance	error
simulation	2	3	3	2	0.704267	
calculation	2	3	3	2	0.711662	-0.007395

Table 4: Case 4

Total Buffer size = 14									
p_1	r_1	p_2	r_2	p_3	r_3	p_4	r_4	p_5	r_5
0.05	0.05	0.05	0.03	0.15	0.05	0.10	0.04	0.10	0.05

method	n_1^*	n_2^*	n_3^*	n_4^*	performance	error
simulation	3	5	4	2	0.721400	
calculation	2	6	4	2	0.729705	-0.008305

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