Proceedings of the 11th
KACC, October 1996

Controller of Nonlinear Servo System

Yuzo Yamane   Xiaojun Zhang
Ashikaga Institute of Technology, Ashikaga 326, Japan

Abstract : This paper is dealing with a design of linear controller so that the plant output is regulated to follow a reference model output when the plant equation is described by a class of nonlinear time-varying control systems.
Keywords: model matching, linear controller, state feedback, state feedforward, nonlinear servo system, constant reference input, interactor.

1 INTRODUCTION

This paper is dealing with the design of a linear controller for nonlinear servo systems so that the plant output signal is regulated to follow a prescribed constant command signal when the dynamical plant system is described by a class of nonlinear time varying equations. It is also investigating how the linear controller works for nonlinear time-varying plant systems under the stability conditions.

The command signal is composed of a reference model whose structures are deeply related to the real plant structures from such a requirement that the model matching is achieved for a special reference model input.

The controller is assumed to be of a form of linear plant state feedback and linear reference model state feedforward. The both state gains sought on a basis of the idea of model matching have free parameters contained in an interactor stable polynominal.

To assure a validity of the proposed approach a robot manipulator control problem is chosen as a simulation study.

This paper has the following special features as shown.

1) Formation of linear controller in a sense that the controller has linear plant state feedback and linear model state feedforward gains.
2) Constant assigned objective functions are given by outputs of the reference model
3) Model matching conditions are always satisfied.

2 PROBLEM DESCRIPTION

A plant $\sum_1$ and a reference model $\sum_2$ are shown in n-dimentional canonical controllable forms.

$\sum_J : (J = 1, 2)$

$$\dot{x_J} = A_J x_J + b u_J + f_J(x_J, t) \quad (1)$$

$$y_J = c_J^T x_J \quad (2)$$

where

$$A_J = N_n + b\{a_J^T + \tilde{a}_J{}^T(x_J, t)\}$$

$$b = (0, 0, \cdots 0, 1)^T \in R^n$$

$$N_n = \left(0 \;\middle|\; \begin{array}{c} I_{n-1} \\ \hline 0 \end{array} \right) \in R^{n \times n}$$

Consider how to generate u_2 of $\sum_2$,

When $f_2 \in Range(b)$ the reference model input u_2 is yielded from a following equation

$$(a_2 + \tilde{a}_2)^T x_2 + f_2 + u_2 = 0 \quad (3)$$

Then $\sum_2$ becomes

$$\begin{aligned} \dot{x}_2 &= N_n x_2 \\ y_2 &= c_2^T x_2 \end{aligned} \quad (4)$$

and y_2 is expressed by at most $n-1$ th order polynominals.

The nonlinear function $f_1(x_1, t)$ and the parameter deviation $\tilde{a_1}(x_1, t)$ are assumed to be known.

Transfer functions of Eq (1) with irreducible form are given when $\tilde{a}_J = 0$ and $f_J = 0$. $(w_J \neq 0)$

$$G_J(s) = \frac{r_J(s)}{p_J(s)} \quad (5)$$

Assuming that the degree difference is ν

$$\begin{aligned} c_J^T A_J^d b &= 0, d = 0, \ldots, \nu - 2 \\ c_J^T A_J^d b &= w_J, d = \nu - 1 \end{aligned}$$

are obtained where r_1 and r_2 are of $n - \nu$ degree as follows

$$r_1(s) = s^{n-\nu} + \alpha_1 s^{n-\nu-1} + \ldots + \alpha_{n-\nu} \quad (6)$$

$$r_2(s) = s^{n-\nu} + \beta_1 s^{n-\nu+1} + \ldots + \beta_{n-\nu} \quad (7)$$

where

$$\alpha = (\alpha_1, \ldots, \alpha_{n-\nu})^T \in R^{n-\nu}$$

$$\beta = (\beta_1, \dots, \beta_{n-\nu})^T \in R^{n-\nu}$$

An interactor stable polynominal of plant reduces to be

$$\sigma(s) = s^\nu + \mu_\nu s^{\nu-1} + \dots + \mu_1$$

where

$$\mu = (\mu_1, \cdots, \mu_\nu) \in R^\nu$$

The next assumptions are imposed about infomation of the plant.

1. n and ν are known.
2. plant is a minimum phase system.
3. x_1 is available.
4. f_1 and $\tilde{a_1}$ are known functions.
5. Eq(34) is stable.

A control objective is to design a controller which forces y_1 to follow y_2 for any u_2 under above assumptions.

$$u_1(t) = -k_1^T x_1(t) + k_2^T x_2(t) + m u_2(t) \tag{8}$$

where k_1 and k_2 are feedback and feedforward time constant gains.

3 Controller when $\tilde{a_1} = 0$ and $f_1 = 0$

Combining $\sum_1$ with $\sum_2$ an augmented system is

$$\begin{aligned} \dot\eta(t) &= \Lambda\eta(t) + \gamma u_2(t) \\ \xi(t) &= \pi^T\eta(t) \end{aligned} \tag{9}$$

where

$$\begin{aligned} \eta(t) &= \{x_1^T(t), x_2^T(t)\}^T \\ \xi(t) &= y_2(t) - y_1(t) \\ \Lambda &= \left[\begin{array}{c|c} A_1 - b_1k_1^T & b_1k_2^T \\ \hline 0 & A_2 \end{array}\right] \\ \pi &= [-c_1^T, c_2^T]^T \\ \gamma &= [mb_1^T, b_1^T]^T \end{aligned}$$

A model matching condition to satisfy control objective is required that

$$\frac{y_1(s)}{u_2(s)} = \frac{y_2(s)}{u_2(s)} \tag{10}$$

holds for any u_2 as well as Λ is stable. Eq (10) is equal to

$$\pi^T(sI - \Lambda)^{-1}\gamma = 0 \tag{11}$$

Subsituting π, Λ and r into Eq (10) yields

$$\begin{aligned} & mc_1^T(sI - A_1 + b_1k_1^T)^{-1}b_1 - \\ & c_2^T(sI - A_2 + b_2\frac{k_2^T}{m_1})^{-1}b_2 = 0 \end{aligned} \tag{12}$$

Putting a coefficiet of the highest order of Eq (12) into zero, one obtains

$$m = \frac{w_2}{w_1} \tag{13}$$

Recalling (A_1, b_1) is controllable the feedback gain k_1 is calculated from Eq(12).

$$\begin{aligned} k_1^T &= \frac{1}{w_1}c_1^T\sigma(A_1) \\ &= v_1^T + \frac{1}{w_1}\mu^T\tilde{T}_1 \end{aligned} \tag{14}$$

where

$$v_1^T = \frac{1}{w_1}(c_1^T A_1^\nu)^T = a_1^T + \frac{1}{w_1}c_1^T N_n^\nu$$

and

$$\tilde{T}_1 = [c_1, N_n^T c_1, \cdots, (N_n^T)^{\nu-1}c_1]^T$$

Similarily the feedforward gain is calculated as

$$\begin{aligned} \frac{k_2^T}{m} &= \frac{1}{w_2}(\mu^T, 1)\{\tilde{T}_2^T, (c_2^T A_2^\nu)^T\}^T \\ &= v_2^T + \frac{1}{w_2}\mu^T\tilde{T}_2 \end{aligned} \tag{15}$$

where

$$\begin{aligned} v_2^T &= \frac{1}{w_2}(c_2^T A_2^\nu)^T = a_2^T + \frac{1}{w_2}c_2^T N_n^\nu \\ \tilde{T}_2 &= [c_2, N_n^T c_2, \cdots, (N_n^T)^{\nu-1}c_2]^T \end{aligned}$$

After all the controller becomes

$$\begin{aligned} u_1 &= -v_1^T x_1 + m\{v_2^T + u_2\} \\ &+ \frac{1}{w_1}\mu^T\{\tilde{T}_2 x_2 - \tilde{T}_1 x_1\} \end{aligned} \tag{16}$$

4 Controller when $\tilde{a}_1 \neq 0$ and$f_1 \neq 0$

$\bar{z}_1$ is defined by using an observability matrix of plant (N_n, c_1),

$$z_1 = \tilde{T}_1 x_1 \tag{17}$$

$$\bar{z}_1 = \bar{T}_1 x_1 \tag{18}$$

T_1 satisfies following equations

$$T_1 = \begin{pmatrix} \tilde{T}_1 \\ \bar{T}_1 \end{pmatrix}, \quad \bar{T}_1 = \begin{bmatrix} c_1^T N_n^\nu \\ \vdots \\ c_1^T N_n^{n-1} \end{bmatrix}$$

and

$$T_1 N_n = \begin{bmatrix} N_\nu & \bar{b}c^T \\ 0 & N_{n-\nu} \end{bmatrix} T_1$$

Left multiplying Eq (1) with $\tilde{T}_1$ and $\bar{T}_2$ Eq(1) becomes

$$\dot{z}_1 = N_\nu z_1 + \bar{b}c^T \tilde{T}_1 x_1 + (0,0,\ldots, w_1)^T\{u_1 + (a_1 + \tilde{a}_1)^T x_1\} + \tilde{T}_1 f_1(x_1,t) \quad (19)$$

$$\dot{\bar{z}}_1 = N_{n-\nu}\bar{z}_1 + w_1\alpha\{u_1 + (a_1 + \tilde{a}_1)^T x_1\} + \bar{T}_1 f_1(x_1,t) \quad (20)$$

where $\bar{b} = (0,0,\ldots,0,1)^T \in R^\nu$

$$c = (1,0,\ldots,0,0)^T \in R^{n-\nu}$$

$$N_\nu = \left(\begin{array}{c|c} 0 & \begin{array}{c} I_{\nu-1} \\ \hline 0 \end{array} \end{array}\right) \in R^{\nu\times\nu}$$

$$N_{n-\nu} = \left(\begin{array}{c|c} 0 & \begin{array}{c} I_{n-\nu-1} \\ \hline 0 \end{array} \end{array}\right) \in R^{(n-\nu)\times(n-\nu)}$$

$$T_1 b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ c_1^T N_n^{\nu-1} b \\ \cdots\cdots\cdots \\ c_1^T N_n^\nu b \\ \vdots \\ c_1^T N_n^{n-1} b \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ w_1 \\ \cdots\cdots \\ w_1\alpha \end{bmatrix}$$

Furthermore Eqs (19) and (20) are simplified to be

$$\dot{z}_1 = N_\nu z_1 + w_1\bar{b}\{u_1 + (v_1 + \tilde{a}_1)^T x_1\} + \tilde{T}_1 f_1(x_1,t) \quad (21)$$

$$\dot{\bar{z}}_1 = D_1\bar{z}_1 + w_1\alpha[u_1 + \{(v_1 + \tilde{a}_1)^T x_1] + \bar{T}_1 f_1(x_1,t) \quad (22)$$

where

$$D_1 = \left[\begin{array}{c|c} -\alpha & \begin{array}{c} I_{n-\nu-1} \\ \hline 0 \end{array} \end{array}\right]$$

The plant output becomes

$$\begin{aligned} y_1 &= c_1^T x_1 = (\bar{c}^T, 0) T_1 x_1 \\ &= \bar{c}^T z_1 + 0^T \bar{z}_1 \end{aligned} \quad (23)$$

where $\bar{c} = (1,0,\cdots 0)^T \in R^\nu$

In a same manner the reference model is decomposed into

$$\dot{z}_2 = N_\nu z_2 + w_2\bar{b}\varphi + \tilde{T}_2 f_2(x_2,t) \quad (24)$$

$$\dot{\bar{z}}_2 = D_2\bar{z}_2 + w_2\beta\varphi + \bar{T}_2 f_2(x_2,t) \quad (25)$$

$$y_2 = \bar{c}^T z_2 + 0^T \bar{z}_2 \quad (26)$$

by defining

$$l z_2 = \tilde{T}_2 x_2 \quad (27)$$

$$\bar{z}_2 = \bar{T}_2 x_2 \quad (28)$$

$$\varphi = u_2 + (v_2 + \tilde{a}_2)^T x_2 \quad (29)$$

where

$$\tilde{T}_2 = \begin{bmatrix} c_2^T \\ c_2^T N_n \\ \vdots \\ c_2^T N_n^{\nu-1} \end{bmatrix} \quad \bar{T}_2 = \begin{bmatrix} c_2^T N_n^\nu \\ \vdots \\ c_2^T N_n^{n-1} \end{bmatrix}$$

$$D_2 = \left[\begin{array}{c|c} -\beta & \begin{array}{c} I_{n-\nu-1} \\ \hline 0 \end{array} \end{array}\right]$$

Subsituting Eq(3),Eq(29) is

$$\varphi = \frac{1}{w_2} c_2^T N_n^\nu x_2 - f_2$$

Finally plant and reference model equations combined with reduced state deviation system are got as follows.

$$\begin{pmatrix} \dot{\delta x} \\ \dot{\bar{z}}_1 \\ \dot{\bar{z}}_2 \end{pmatrix} = \begin{bmatrix} N_\nu & 0 & 0 \\ 0 & D_1 & 0 \\ 0 & 0 & D_2 \end{bmatrix} \begin{pmatrix} \delta x \\ \bar{z}_1 \\ \bar{z}_2 \end{pmatrix} + \begin{bmatrix} -\bar{b} \\ \alpha \\ 0 \end{bmatrix} w_1\{u_1 + (v_1 + \tilde{a}_1)^T x_1\} + \begin{bmatrix} w_2\bar{b} \\ 0 \\ w_2\beta \end{bmatrix}\varphi + \begin{bmatrix} \tilde{T}_2 f_2 - \tilde{T}_1 f_1 \\ \bar{T}_1 f_1(x_1,t) \\ \bar{T}_2 f_2(x_2,t) \end{bmatrix} \quad (30)$$

$$\xi = y_2 - y_1 = (\bar{c}^T, 0^T, 0^T)\begin{pmatrix} \delta x \\ \bar{z}_1 \\ \bar{z}_2 \end{pmatrix} \quad (31)$$

where

$$\delta x = z_2 - z_{1o}$$

A model matching condition to satisfy control objective means that

$$y_1 \to y_2 \quad (32)$$

holds for any u_2. When u_1 is implemented by the controller

$$u_1 = -(v_1^T + \tilde{a}_1)x_1 + m\varphi + \frac{1}{w_1}\{\lambda^T\delta x + f\} \quad (33)$$

where λ is a nonliner function of x_1, t and $f = \tilde{T}_1 f_1 - \tilde{T}_2 f_2$, excluding unobservable state $\bar{z}_1$ and $\bar{z}_2$ from Eq (30) one gets

$$\delta\dot{x} = (N_\nu - \bar{b}\lambda^T)\delta x, \quad \xi = \bar{c}^T\delta x \quad (34)$$

Then two gains k_1, k_2 in Eq(8) are expressed by

$$k_1^T = (v_1 + \tilde{a}_1)^T + \frac{\lambda^T}{w_1}\tilde{T}_1 \quad (35)$$

$$\frac{k_2^T}{m} = (v_2 + \tilde{a}_2)^T + \frac{\lambda^T}{w_2}\tilde{T}_2 \quad (36)$$

under conditions that there exists λ so that k_1 and k_2 are constant. λ is considered to be a nonlinear interacter function of the plant.

5 Numerical Examples

Here the effectiveness of proposed controller is shown for reference model output following control presented by robot manipulator simulations. Let a second order plant model be described as follows,

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -2+\tilde{a}_{21} & -1+\tilde{a}_{22} \end{bmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0.06 \end{pmatrix} \sin(x_{11})$$

$$y_1 = (1 \quad 0)\, x_2$$

where

$$\tilde{a}_{21} = e^{-t}\sin t, \quad \tilde{a}_{22} = e^{-(x_{11}+x_{12}\sin t)}$$

and the initial state is

$$x_1(0) = [x_{11}(0), x_{12}(0)]^T = (0,1)^T$$

The second order reference model be described as follows,

$$\dot{x}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_2$$

$$y_2 = (1 \quad 0)\, x_2, \quad x_2(0) = (3,0)^T$$

Here the interactor stable function of plant is chosen as

$$\lambda_1 = 2.013 - e^{-t}\sin t$$

$$\lambda_2 = 1.11 - e^{-(x_{11}+x_{12}\sin t)}$$

Then the controller becomes

$$u_1 = (0.013, 0.110)x_1 + (0.013, -0.890)x_2 + u_2$$

The input of reference model is

$$u_2 = 6 - (e^{-(x_{11}+x_{12}\sin t)}) * 3$$

When λ in Eq(35) is such constant that Eq(34) is stable k_1 in Eq(35) becomes a nonlinear function.

The input of plant is shown in Fig.1 .

The output of plant is shown in Fig.2 .

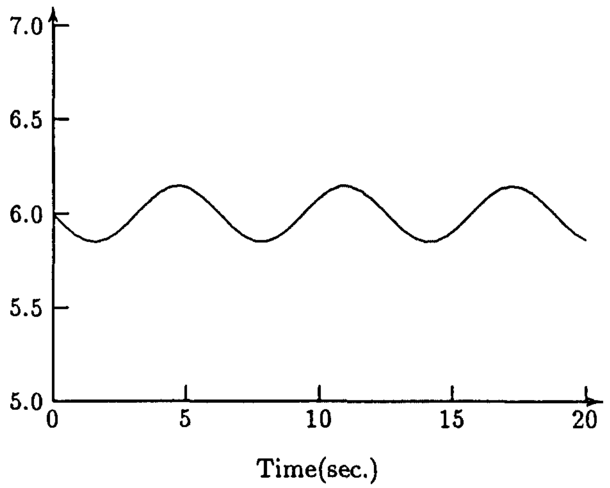

Fig.1 : Plant input signal

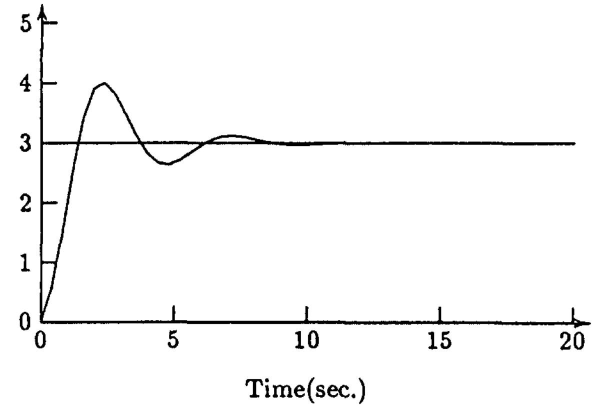

Fig.2: Plant and reference model output signals

References

[1] Yamane,Fujimoto:Model reference robust control design using vss theory, Proc. of COMADEM, 655/664 (1996).