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AN INVERSE LQG/LTR PROBLEM APPLIED TO THE VEHICLE STEERING SYSTEM

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Abstracts This paper describes the robust controller design methods applied to the problem of an automatic steering system for tow-vehicle/trailer combinations. This study followed an inverse Linear Quadratic Regulator(LQR) approach which combines pole assignment methods with conventional LOR methods. It overcomes two concerns associated with these separate methods. It overcomes the robustness problems associated with pole placement methods and trial and error required in the application of the LQR problem. Moreover, a Kalman filter is used as the observer, but is modified by using the loop transfer recovery (LTR) technique with modified transmission zero assignment.

The proposed inverse LQG/LTR controllers enhances the forward motion stability and maneuverability of the combination vehicles. At high speeds, where the inherent yaw damping of the vehicle system decreases, the controller operates to maintain an adequate level of yaw damping. At backward moton, both 4WS (2WS tow-vehicle, 2WS trailer) and 6WS (4WS tow-vehicle, 2WS trailer) control laws are proposed by using inverse LQG/LTR method. To evaluate the stability and robustness of the proposed controllers, simulations for both forward and backward motion were conducted using a detailed nonlinear model. The proposed controllers are significantly more robust than the previous controllers and continues to operate effectively in spite of parameter perturbations that would cause previous controllers to enters limit cycles or to loose stability.

Keywords Steering Control, LQG/LTR, Robustness, 4WS, 6WS

1. INTRODUCTION

The linear quadratic regulator (LQR) is an optimal design technique that yields a robust system with a gain margin of (1/2, ∞) and a phase margin of (-60,60) degree. Although these margins are guaranteed, it is difficult to place the closed loop poles precisely by choosing a weighting matrix. In the LQR method the designer must reduce all of his varied performance requirements to a criterion that is restricted to be quadratic in states and controls. There is no direct connection between such criteria and more specific time domain specifications. Therefore the designer must resort to trial and error iterations. Some methods have been devised to reduce the number of iterations. Kawasaki [1] used a recursive procedure to place the poles of the optimal regulator i open hyperbola. A transformation approach, which places the entire closed-loop eigenspectrum in a specified disc, has been used by Furuta[2]. A second class of problems is that of assigning the closed-loop poles to specific locations while minimizing the performance index. Tan-Jan[3] and Shieh[4] are the ones that consider the problems of assigning the closed loop poles to specific locations. Tan-Jan presented two systematic procedures that can relax damping ratio constraints compared with Shieh's method. However, the problem in Tan-Jan's method is that the real parts of all of the eigenvalues should be shifted together. Therefore, we need some algorithms to place any poles near its desired locations with minimum control action as well as providing margins of LQR problem.

In this paper a method for obtaining the robustness benefits of the LQR regulators together with the specificity of pole placement regulators is derived. The concept is called inverse LQR. The basic concept is to place poles but through the LQR framework. The range of permissible pole location is restricted by the LQR framework to robust locations that have enough margins.

One may ask "what is the difference between the proposed method and simple pole placement method?" The answer may be that it is very difficult to know the region of LQR in high order system, if we simply apply pole assignment method we have to rely on the trial and error. Moreover, in multi-input and multi-output system, the gains generated by pole placement method is not unique. This method is also effective when we schedule the gain of the time varying system with more smooth gains. By programming approach we can also put more weighting on the dominant poles. This will contribute in managing the dominant poles.

Based on the *separation theorem* of estimation and control, LQG compensator can be separated into two subsystems. The first subsystem is a Kalman filter which estimates the state of plant. The second subsystem is regulator generated by the LQR method. If we combine the LQR and Kalman filter the guaranteed margins of LQR are no longer achieved. The eigenstructure of the system with an observer not generally those of the LQR. To solve this problem, a method to tune the filter so that it recovers the full state feedback properties has been presented by Doyle, Stein and Athans[5,6]. This method is known as LQG/LTR(Linear Quadratic Gaussian with Loop Transfer Recovery) which recovers the loop properties of full state feedback(LQR), and still does the Kalman filtering mission but with less accuracy. Actually, this method is a trade-off between the loop recovery and accuracy of the filter.

In this paper we used LQG/LTR method for the steering control of combination vehicles. This method has been successfully applied to the steering system to enhance forward motion stability and maneuverability of combination vehicles. The regulator is derived by the inverse LQR method so that the closed loop poles can assigned but will be restricted to regions yielding robustness. The observer is based on Kalman filtering but is supplemented with LTR techniques. In adition, we modified non-square system matrix to transform non-minimum phase square system to apply LTR techniques. However, if we tighten the filter loop by using LTR method, some of the eigenvalues of the filter loop will shift to the left in the left s-plane. In order to follow the fast dynamics of these filter poles the sampling rate must be increased. This will increase the calculation load of Kalman filter. Therefore, we need a trade-off between LTR performance and the required sampling rate.

The proposed control method, called inverse LQG/LTR here, was succesfully applied to the combination vehicles, especially to control the automatic steering system. First the method was applied to the forard motion when vehicle is running at high speed with the trailer wheel controls. Second, this method also applied to the backward motion which has inherent stability problem. In addition, this design approach was applied to 6WS system which includes the 4WS tow-vehicle.

2. CONTROL METHODOLOGY

2.1 Motivation and problem statement of inverse LQR

An advantage of using LQR design is that a time-invariant system can always be made asymptotically stable if the system is stabilizable and detectable. There are two problems associated with LQR design as discussed. To overcome these problems, we can use pole assignment technique. Then, there would be direct connection to time response characteristics. But, there would be no quarantee of robustness if the assigned poles are not in the LQR region. The problem by using LQR is that we cannot actually move the coupled yaw damping poles to desired region by selecting the weighting matrix diagonally. Thus, we can consider a combination of LQR and pole assignment. An algorithm is proposed which finds appropriates LQR weighting matrices to allow the closed loop poles to approach a set of desired poles. If the desired poles are outside the allowable LQR region, the algorithm finds the poles inside the LQR region that are closest to the desired poles. Although this algorithm may not move all of the poles to the desired position, it gives the designer the ability to move toward the desired pole region if the desired pole is in the LQR region.

From State Equation derived in Appendix, and neglecting state noise and driver's command, the closed loop system with full state feedback gain K_c can be written as

$$\dot{X} = (A - B_t K_c) X \tag{1}$$

where $K_c = R^{-1} B_t^T P$

and P is the steady-state solution of the control algebric Riccati equation (CARE) as

$$PA + A^T P + Q - PB_t R^{-1} B_t^T P = 0 \tag{2}$$

By combining Eq.(1), Eq.(2) can be expressed as

$$\dot{X} = (A - B_t R^{-1} B_t^T P) X = A^* X \tag{3}$$

and the closed loop poles are the eigenvalues of A^*. From Eq.(3), because P is a function of Q and R. The closed loop eigenvalues depend only on Q (state weighting) and R (input weighting).

The purpose of the algorithm can be expressed which minimize the difference between desired pole and achievable pole. This can be written as the cost function

$$J = \sum_{i=1}^{n} W_i (\lambda_i^d - \lambda_i^a)^2 \tag{4}$$

where W_i is the importance weighting of each poles, and λ_i^d and λ_i^a are eigenvalues of the desired and the achievable, respectively. In the LQR problem, the state weighting matrix is required to be positive semidefinite $(Q \geq 0)$ and the control weighting matrix positive definite $(R > 0)$. Q can be positive semidefinite by defining $Q = N^T N$. Likewise, R can be made positive definite by defining $R = M^T M$ and requiring M to be square or more rows and have full column rank. If Q and R are varied by giving small perturbations to N and M, the response of the pole difference cost function J can be determined by $\partial J / \partial M$ and $\partial J / \partial N$. It was possible to find the minimum value of J when $\partial J / \partial M$ and $\partial J / \partial N$ is smaller than a preset tolerance by applying simplex method.

2.2 Algorithm for inverse LQR and design results

A computer program is written by using Matlab software on the personal computer. Some of the internal functions in Matlab like Riccati equation solver(lqe) and optimization function(fmins) were used in the program. Based on the poles determined from LQG method which was designed by trail and error, the desired poles,achievable poles and achieved poles by using this algorithm are shown in Fig. 1 when applied to the forward motion case. The achievable poles are very similar to the desired poles. This means the desired poles are in the LQR region.

2.3 Loop Transfer Recovery(LTR)

The standard LQG design procedure is modified by using fictitious state noise at the input. From Eq.(A.4) we can express the state space representation with white noise as

$$\dot{X} = AX + B_c d_c + B_t d_t + F\gamma \tag{5}$$

$$Y = CX + Dw \tag{6}$$

where F and D are the plant disturbance and measurement noise matrix respectively, γ and w are white noise vectors with unit covariance, and C is observation matrix. The optimal state estimator was applied to estimate the full state. R_1 and R_2 are the covariance of the fictitious plant disturbance and sensor noises, respectively, and expressed as

$$R_1 = E\{F\gamma\gamma^T F\} + q^2 B_t V B_t^T \tag{7}$$

$$R_2 = E\{Dww^T D^T\} \tag{8}$$

where q is a scalar parameter that can be used to increase or decrease the intensity of fictitious plant noise and V is any positive definite symmetric matrix. The Doyle-Stein[5] condition for robustness recovery at input is

$$K_f(I + C\Phi K_f)^{-1} = B_t(C\Phi B_t)^{-1} \quad (9)$$

where $\Phi = (sI - A)^{-1}$.

Note that this condition explicitly requires that $C\Phi B_t$ be invertible; hence it is necessary to "square" a non-square system if the trailer wheels are only controlled. The system we are considering has 1-input(d_t) and 2- output($\underline{r}, \theta$). This is accomplished by augmenting B_t by $4 \times (2-1)$ matrix B_{dummy} such that

$$rank[B_t \;\; B_{dummy}] = 2 \quad (10)$$

One of the systematic procedure for selecting matrix B_{dummy} that satisfies the rank and the non-minimum phase zero condition is using inverse LQR approaches shown in previous section by assigning some poles. This can be obtained by assigning transmission zeros instead of regulator poles.

3. DESIGN RESULTS

The inverse LQG/LTR problem is applied to both forward and backward motion control of the combination vehicles with the parameters and structure as shown in Appendix A. The phase margin and filter poles with respect to q^2 values are shown in Table 1 for the forward motion case. These results clearly shows that the phase margin are recovered as q^2 is ncreased from 0.00001 to 1.0.

The two of four filter poles approach to the transmission zeros of the plant (-3.870$\pm$3.532i) if we increase q^2 value.And the other two approach infinite.

The Bode plot is given in Fig.2 which shows the recovery results for several q^2 value. For the backward motion case we controlled the trailer wheels the same way as the forward motion case. The recovery results are shown in Fig.3. In addition, the phase margin and filter poles for forward motion cases is shown in Table 1. The backward motion results are also improved when compared with the PD control law presented in[8]. The time domain simulation also shows that the design using the inverse LQG/LTR improved the stability and maneuverability for forward motion at high speed. And the robustness performance with respect to major parameter perturbations including the moment of inertia, CG and mass was also improved. The one of the simulation results with the lane change maneuver are shown in Fig.3. This results shows that the inverse LQG/LTR approches the regulator problem and improved the stability to lane change maneuvering.

Although inverse LQG/LTR improves the robustness of the steering control system compared with the previous control law, the main problem of a trailer backward steering system is that the trailer cannot move to the predicted center line of the tow-vehicle when initial off-tracking or the amplitude of driver command is very high. To overcome this problem, we are proposing a trailer steering control combined with 4WS tow-vehicle. The main idea is to command a open loop negative signal to the rear wheel of tow-vehicle if front steering command is positive and hitch angle is negative. This will make more sharp cornering. The same design approach combined with open loop control, which depends on the polarity of the input and hitch angle and the magnitude of the steering command, improved the performance of the backward motion stability. When the initial hitch hitch angle is -25° and parameters are perturbed with the vehicle speed -3.3m/sec, the step command (slope 35°/1.5sec, 35° amplitude) is injected to driver's command, the trajectory is shown in Fig. 4. The plot shows that 4WS(B) with open loop control makes more sharp cornering compared with 4WS(A).

Table 1. Phase Margin and filter poles (forward)

Control Law(q^2)	PM	Filter Poles
Inverse LQR	97.26	Not available
Inverse LQG	79.46	-3.510$\pm$3.476i, -14.195$\pm$12.090i
Inverse LQG/LTR (0.00001)	83.71	-3.514$\pm$3.480i, -18.366$\pm$7.667i
Inverse LQG/LTR (0.001)	93.98	-2.113e2, -10.684, -3.616$\pm$3.529i
Inverse LQG/LTR (0.1)	96.85	-2.112e3, -57.118, -3.862$\pm$3.534i
Inverse LQG/LTR (1.0)	97.13	-6.679e3, -1.787e2, -3.869$\pm$3.532i

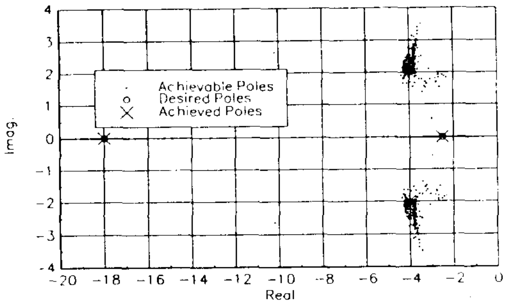

Fig.1 Achievable and achieved pole comparisons

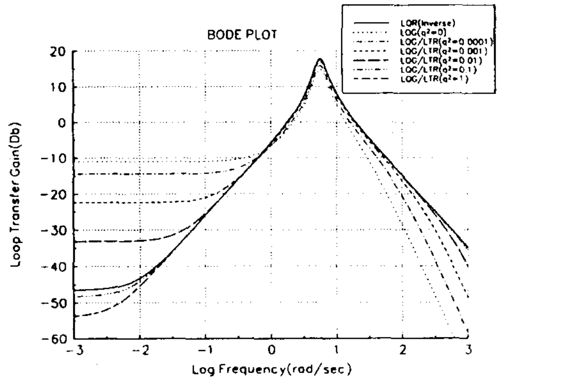

Fig.2 Bode plot(gain) of the loop transfer function with respect to q values (forward motion)

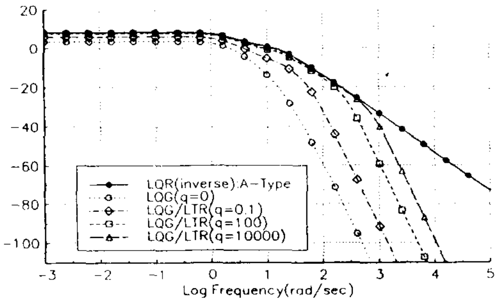

Fig.2 Bode plot(gain) of the loop transfer function with respect to q values (backward motion)

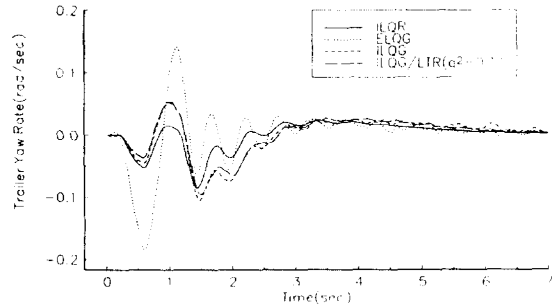

Fig. 3. Yaw rate of the trailer for lane change with perturbed model

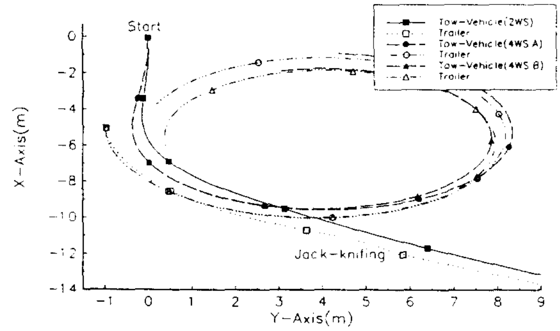

Fig. 4 Trajectory of each control for step input (Backward motion with speed -3.3m/sec)

4. CONCLUSIONS

A robust control methodology called with the inverse LQG/LTR was developed and applied to the forward motion of combination vehicle which has stability problem at high speed, and also applied to the backward motion which is inherent unstable system. The Loop transfer recovery technique is also applied to the combination vehicles which have tansmission zeros at zero point. In the simulation inverse LQG/LTR control was able to operate effectively with large parameter perturbations. For examples, when the mass and moment of inertia are increased three time with respect to the nominal parameters and CG is moved 0.2 meter backward, the controller provides stability for a lane change simulation with 4 meter offset at vehicle speed 25m/sec.

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APPENDIX

The tow-vehicle model used in this paper is Ford minitruck(1.4 ton). The trailer is actually semitrailer. Although the equation of motion is nonlinear, we used linear model in order to design the controller by introducing the conditions that the slip angle is small and the contact-patch of the tire is neglected. The tire lateral force can be written as:

$$F_{2i} = C_i \alpha_i \qquad \text{(A.1)}$$

where C_i is the cornering stiffness for each wheels (i=1, 4). The slip angle of the tow-vehicle and trailer can be as follows:

$$\alpha_i = d_i - \frac{v + r l_i}{u} \qquad \text{(A.2)}$$

$$\underline{\alpha}_i = \underline{d}_i - \theta - \frac{v + hr + (\underline{l}_i - \underline{h})\underline{r}}{u} \qquad \text{(A.3)}$$

In order to design the controller we assumed that the hitch angle is small and combining the EQ.(A.1), (A.2) and (A.3), then the dynamic equations become

$$L\dot{X} = RX + B_c d_c + B_t d_t \qquad \text{(A.4)}$$

where L, R, R_c and R_t become

$$L = \begin{bmatrix} m+\underline{m} & \underline{m}h & -\underline{m}\underline{h} & 0 \\ \underline{m}h & I+\underline{m}h^2 & -\underline{m}h\underline{h} & 0 \\ -\underline{m}\underline{h} & -\underline{m}h\underline{h} & \underline{I}+\underline{m}\underline{h}^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{-(\Sigma C_i + \Sigma \underline{C}_i)}{u} & \frac{-(\Sigma C_i l_i + h\Sigma \underline{C}_i)}{u} - (m+\underline{m})u & \frac{\Sigma \underline{C}_i(\underline{h}-\underline{l}_i)}{u} & -\Sigma(\underline{C}_i + \underline{F}_{1i}) \\ \frac{-(\Sigma C_i l_i + h\Sigma \underline{C}_i)}{u} & \frac{-(C_i l_i^2 + h^2 \Sigma \underline{C}_i)}{u} - \underline{m}hu & \frac{h\Sigma \underline{C}_i(\underline{h}-\underline{l}_i)}{u} & -h\Sigma(\underline{C}_i + \underline{F}_{1i}) \\ \frac{\Sigma \underline{C}_i(\underline{h}-\underline{l}_i)}{u} & \frac{h\Sigma \underline{C}_i(\underline{h}-\underline{l}_i)}{u} + \underline{m}\underline{h}u & \frac{-\Sigma \underline{C}_i(\underline{h}_i-\underline{l}_i)^2}{u} & \Sigma \underline{C}_i(\underline{h}-\underline{l}_i) \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$R_c = \begin{bmatrix} F_{11}+C_1 & F_{12}+C_2 & F_{13}+C_3 & F_{14}+C_4 \\ (F_{11}+C_1)l_1 & (F_{12}+C_2)l_2 & (F_{13}+C_3)l_3 & (F_{14}+C_4)l_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_t = \begin{bmatrix} \underline{F}_{11}+\underline{C}_1 & \underline{F}_{12}+\underline{C}_{12} \\ h(\underline{F}_{11}+\underline{C}_1) & h(\underline{F}_{12}+\underline{C}_2) \\ (\underline{F}_{11}+\underline{C}_1)(\underline{l}_1-\underline{h}) & (\underline{F}_{12}+\underline{C}_2)(\underline{l}_2-\underline{h}) \\ 0 & 0 \end{bmatrix}$$

The model of the tow-vehicle and trailer are shown in Fig. A.1

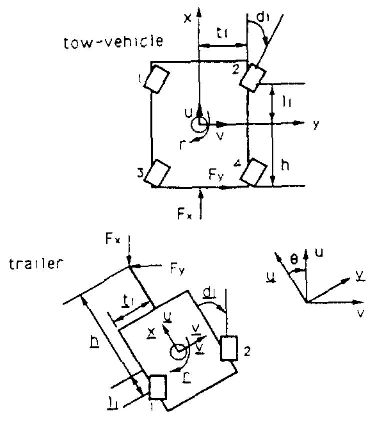

Fig. A.1 The model of the tow-vehicle and trailer