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PID CONTROLLER TUNING FOR PROCESSES WITH TIME DELAY

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Abstract

By far the PID controller is most widely used in the process industries. However, current tuning methods yield PID parameters only for a restricted class of process models. There is no general methodology of PID controller tuning for arbitrary process models. In this paper, we generalize the IMC-PID approach and obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series. Further, the PID controller tuned by the proposed PID tuning method gave more closer closed-loop response to the desired response than those tuned by other tuning methods.

Keywords

PID controller tuning, chemical process control, approximation by Maclaurin series

INTRODUCTION

Because the PID controller finds wide spread use in the process industries, a great deal of effort has been directed at finding the best choices for the controller gain, integral and derivative time constants for various process models (Ziegler and Nichols, 1953; Lopez et al., 1967; Smith et al., 1975; Rivera et al., 1986; Chien and Fruehauf, 1990). Among the performance criteria used for PID controller parameter tuning, the criterion to keep the controlled variable response close to the desired closed-loop response has gained widespread acceptance in the chemical process industries because of its simplicity, robustness, and successful practical applications. The IMC-PID tuning method (Rivera, et al., 1986; Morari, et al., 1989) and the direct synthesis method (Smith, et al, 1975) are typical of the tuning methods based on achieving a desired loop response. They obtain the PID controller parameters by first computing the controller which gives the desired closed loop response. Generally, this controller is rather more complicated than a PID controller. However, by clever approximations of the process model, the controller form can be reduced to that of a PID controller, or a PID controller cascaded with a first order lag. An important advantage of such methods is that the closed loop time constant, which is the same as the IMC filter time constant, provides a convenient tuning parameter to adjust the speed and robustness of the closed-loop system. Intuitively, one would expect that as the desired closed loop time constant increases, the PID controller gain and derivative time constants would decrease. The PID controller gain does indeed behave as expected. However, current tuning methods yield derivative and integral time constants that are independent of the closed loop time constant. Also, current tuning methods yield PID parameters only for a restricted class of process models. There is no general methodology for arbitrary process models.

In this paper, we generalize the IMC-PID approach and obtain the PID parameters for general models by approximating the ideal controller with a Maclaurin series in the Laplace variable. It turns out that the PID parameters so obtained provide somewhat better closed loop responses than those obtained previously. Further, all of the PID parameters depend on the desired closed loop time constant in a manner consistent with engineering intuition. Several examples are provided to demonstrate the method and to compare results with alternate tuning methods.

DEVELOPMENT OF THE GENERAL TUNING ALGORITHM FOR PID CONTROLLERS

Single Degree of Freedom Controllers (q_r and G_D = 1 in figure (1))

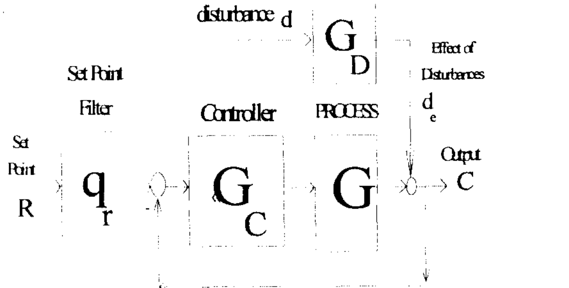

Figure 1. Block diagram of a feedback control system.

Consider stable (i.e. no right half plane poles) process models of the form:

$$G(s) = p_m(s) p_A(s) \qquad (1)$$

where: $p_m(s)$ = The portion of the model inverted by the controller

$p_A(s)$ = The portion of the model not inverted by the controller

$p_A(0) = 1$.

Often, the portion of the model not inverted by the controller is chosen to be all pass (i.e. of the form : $\prod_{i,j}\left(\frac{-\tau_i s + 1}{\tau_i s + 1}\right)\left(\frac{\tau_j^2 s^2 - 2\tau_j \zeta_j s + 1}{\tau_j^2 s^2 + 2\tau_j \zeta_j s + 1}\right) e^{-Ts}$) since this

$\tau_i, \tau_j > 0 \;;\; 0 < \zeta_i < 1$

choice gives the best least squares response. The requirement that $p_A(0) = 1$ is necessary for the controlled variable to track its set point.

Our aim is to choose the controller, G_C, of figure 1 to give the desired closed loop response, C/R of :

$$\frac{C}{R} = \frac{p_A(s)}{(\lambda s + 1)^r} \qquad (2)$$

The term $1/(\lambda s + 1)^r$ Functions as a filter with an adjustable time constant, λ, and an order r chosen so that the controller G_C is realizable.

The controller G_C that yields the desired loop response is given by:

$$G_C(s) = \frac{q}{(1 - Gq)} = \frac{P_m^{-1}(s)}{(\lambda s + 1)^r - P_A(s)} \qquad (3)$$

where q = the IMC controller = $P_m^{-1}(s) / (\lambda s + 1)^r$

The controller G_C can be approximated by a PID controller by first noting that it can be expressed as:

$$G_C \equiv f(s)/s \qquad (4)$$

because $p_A(0)$ is one, and, therefore, at the origin, (i.e. at s = 0), $(1 - P_A(0)) = 0$

Expanding $G_C(s)$ in a Maclaurin series in s gives:

$$G_C(s) = \frac{1}{s}\left(f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + \ldots\right) \qquad (5)$$

The first three terms of the above expansion can be interpreted as the ideal PID controller given by:

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s + \ldots\right) \qquad (6)$$

where $K_C = f'(0)$ (7a)

$\tau_I = f'(0) / f(0)$ (7b)

$\tau_D = f''(0) / 2f'(0)$ (7c)

In order to evaluate the PID controller parameters given by 7a-c, we let

$$D(s) \equiv ((\varepsilon s + 1)^r - p_A(s)) / s \qquad (8a)$$

Then by Maclaurin series expansion we get :

$D(0) = r\varepsilon - p'_A(0)$ (8b)

$D'(0) = (r(r-1)\varepsilon^2 - p_A''(0)) / 2$ (8c)

$D''(0) = (r(r-1)(r-2)\varepsilon^3 - p_A'''(0)) / 3$ (8d)

Using (8), the function f(s) and its first and second derivatives, all evaluated at the origin, are given by:

$$f(0) = \frac{1}{K_p D(0)} \qquad (9a)$$

$$f'(0) = \frac{-(p_m'(0)D(0) + K_p D'(0))}{(K_p D(0))^2} \qquad (9b)$$

$$f''(0) = f'(0)\left[\left(\frac{p_m''(0)D(0) + 2p_m'(0)D'(0) + K_p D''(0)}{p_m'(0)D(0) + K_p D'(0)}\right) + 2f'(0)/f(0)\right] \qquad (9c)$$

where $K_p \equiv p_m(0) = G(0)$

The above formulas can be used to obtain the controller gain, and integral and derivative time constants as analytical functions of the process model parameters and the closed loop time constant, λ, as is done in the next section for several examples.

The derivative and/or integral time constants computed from (7) can be negative for some process models independent of the choice of the closed loop time constant, λ. If negative derivative or integral time constants are encountered, we recommend replacing the simple PID controller with a PID controller cascaded with a first or second order lag of the form $1/(\alpha s + 1)$ or $1/(\beta_2 s^2 + \beta_1 s + 1)$ respectively. To obtain a PID controller cascaded by a first order lag (i.e. $K_C(1+1/\tau_I s + \tau_D s)/(\alpha s+1)$), we rewrite $G_C(s)$ as:

$$G_C(s) \equiv \frac{1}{s} f(s) = \frac{1}{s}\frac{(f(s)\,h(s))}{h(s)} \qquad (10)$$

where $h(s) \equiv 1 + \alpha s$

Now, we expand the quantity $f(s)h(s)$ in a Maclaurin series about the origin and choose the parameter α so that the third order term in the expansion becomes zero. The expansion of (10) then becomes :

$$G_C(s) = (f(0) + (f'(0) + \alpha f(0))s + (f''(0) + 2\alpha f'(0))s^2/2 + (f'''(0) + 3\alpha f''(0))s^3 + \ldots)/s(\alpha s+1) \qquad (11)$$

Selecting the lag parameter, α ,to drop the third order term gives:

$$\alpha = -f'''(0)/3f''(0) \qquad (12a)$$

and the PID parameters are:

$K_C = f'(0) + \alpha f(0)$; $\tau_I = K_C / f(0)$;

$\tau_D = (f''(0) + 2\alpha f'(0))/(2K_C)$; (12b)

Again, the PID•lag controller is: $K_C(1+1/\tau_I s + \tau_D s)/(\alpha s+1)$

To obtain a PID controller cascaded with a second order lag, we write $G_C(s)$ from (3) as:

$$G_C(s) \cong \frac{N(s)}{sD(s)} \qquad (13)$$

where N(s) and D(s) are polynomials obtained by substituting high order (≥ 4) Padé approximations for the exponential terms in $p_m(s)$ and $p_A(s)$. This gives:

$$G_C(s) = \frac{k(\lambda)(\sum_{i=1}^{n} \alpha_i s^i + 1)}{s(\sum_{i=1}^{n} \beta_i(\lambda) s^i + 1)} \qquad (14)$$

where $\alpha_i, \beta_i(\lambda) \geq 0$ and $\beta_i(\lambda)$, $k(\lambda)$ are functions of λ

Dropping terms higher than second order in the numerator and higher than third order in the denominator gives:

$$G_c(s) \cong \frac{k(\lambda)}{s} \frac{(\alpha_2 s^2 + \alpha_1 s + 1)}{(\beta_2(\lambda)s^2 + \beta_1(\lambda)s + 1)} \qquad (15)$$

The controller given by (15) can be viewed as an ideal PID controller cascaded with a second order lag or as a floating integral controller cascaded with a second order lead-lag transfer function. The controller parameters are: $K_C = k(\lambda)\cdot\tau_I$; $\tau_I = \alpha_1$; $\tau_D = \alpha_2 / \tau_I$. The second order lag is given by $(1/(\beta_2 s^2 + \beta_1 s + 1)$. All of the parameters except possibly K_C are positive.

EXAMPLES

First Order plus Dead Time Model

The most commonly used approximate model for chemical processes is the first order plus dead time model given below:

$$G(s) = \frac{K e^{-\theta s}}{\tau s + 1} \qquad (20)$$

Specifying a desired closed loop response of the form $C/R = e^{-\theta s}/(\lambda s+1)$ evaluating the PID parameters from (7), (8), and (9) gives:

$$K_C = \frac{\tau_I}{K(\lambda + \theta)}, \qquad \tau_I = \tau + \frac{\theta^2}{2(\lambda + \theta)},$$
$$\tau_D = \frac{\theta^2}{2(\lambda + \theta)}\left[1 - \frac{\theta}{3\tau_I}\right] \qquad (21)$$

Notice, that as the desired loop time constant, λ, gets large, the controller integral time constant, τ_I, approaches the process model time constant, τ, and the controller gain, K_C, and derivative time constant both approach zero. Thus the PID controller goes smoothly into a PI controller as the desired speed of response decreases.

Figure 2 compares the integral of the squared error for step set point changes using the tuning rule given by (21) with those given by Rivera et al(1986) for varying process dead time to time constant ratios and λ chosen as $\lambda / \theta = 1/3$.

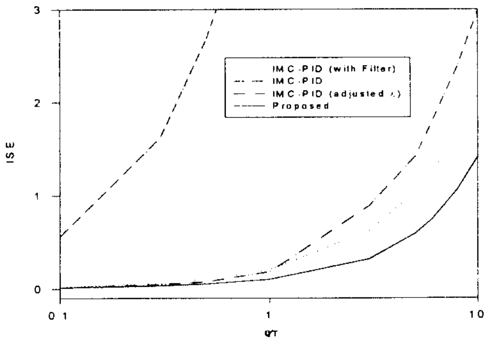

Figure 2. Comparison of the ISE generated by various tuning rules.

To obtain a fair comparison, the same value of λ was used for each tuning rule. The proposed tuning is superior throughout.

Second-Order Plus Dead Time Model

For a process of the form given by (22) below, and a desired closed loop response of $C/R = e^{-\theta s}/(\lambda s+1)^2$, evaluating (7), (8), and (9) gives the PID parameters shown in (23) below (after some tedious algebra).

$$G = \frac{K e^{-\theta s}}{(\tau^2 s^2 + 2\zeta\tau s + 1)}, \quad \tau, \zeta, > 0 \qquad (22)$$

$$K_C = \tau_I/(K(2\lambda + \theta)) \qquad (23a)$$
$$\tau_I = 2\xi\tau - (2\lambda^2 - \theta^2)/(2(2\lambda + \theta)) \qquad (23b)$$
$$\tau_D = \tau_I - 2\xi\tau + (\tau^2 - \theta^3/(6(2\lambda + \theta))/\tau_I \qquad (23c)$$

For process models of the form $Ke^{-\theta s}/((\tau_1 s + 1)(\tau_2 s + 1))$, simply replace $2\xi\tau$ and τ^2 in (23) with $\tau_1 + \tau_2$ and $\tau_1\tau_2$, respectively. For comparison, the tuning rule of Smith et al, (1975) is also shown in Table 1.

Table 1. Various tuning rules to give the desired closed-loop response

Process Model	Tuning Method	K_C	τ_I	τ_D	τ_F
FOPDT	Rivera et al.	$\frac{1}{K}\frac{2\tau+\theta}{2(\lambda+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	
	Rivera et al. (with Filter)	$\frac{2\tau+\theta}{2K(\lambda+\theta)}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$	$\frac{\lambda\theta}{2(\lambda+\theta)}$
	Proposed	$\frac{\tau_I}{K(\lambda+\theta)}$	$\tau + \frac{\theta^2}{2(\lambda+\theta)}$	$\frac{\theta}{6(\lambda+\theta)}\left[3 - \frac{\theta}{\tau_I}\right]$	
FOPDT	Smith	$\frac{\tau}{K(\lambda+\theta)}$	τ		
	Rivera et al. Improved IMC-PI	$\frac{2\tau+\theta}{2K\lambda}$	$\tau + \frac{\theta}{2}$		
	Proposed	$\frac{\tau_I}{K(\lambda+\theta)}$	$\tau + \frac{\theta^2}{2(\lambda+\theta)}$		
SOPDT	Smith	$\frac{\tau_1+\tau_2}{K(\lambda+\theta)}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$	
	Proposed	$\frac{\tau_I}{K(2\lambda+\theta)}$	$2\zeta\tau - \frac{2\lambda^2-\theta^2}{2(2\lambda+\theta)}$	$\tau_I - 2\zeta\tau + (\tau^2 - \frac{\theta}{6(2\lambda+\theta)})/\tau_I$	

Note : Desired Closed-Loop Response $\frac{C}{R} = \frac{e^{-\theta s}}{\lambda s + 1}$, r = 1 or 2

Figure 3 compares the closed-loop responses by several tuning methods for the process given by (22) with $\tau = 10$ and $\zeta = 1$. The resulting PID controller by the proposed method performs better than the controller tuned by the Smith method.

The superior performance of the proposed method is readily apparent.

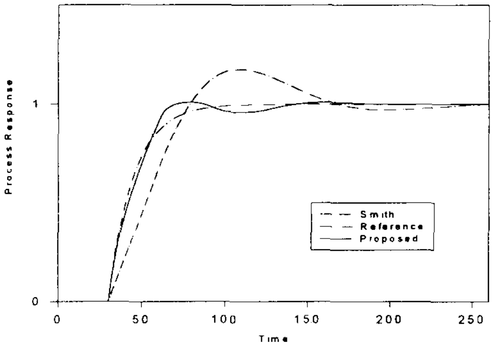

Figure 3. Closed-loop responses to a unit step change in set-point for $G(s)=\dfrac{e^{-30s}}{(10s+1)(10s+1)}$ *;* λ *=15.*

As stated previously, the derivative and/or integral time constants computed (7), (8), and (9) can be negative for some process models independent of the choice of filter time constant. This often occurs when the process model has one or more dominant lead time constants as for example the process given by:

$$\tilde{p}(s)=\frac{s^2+2s+.25}{s^4+6.5s^3+15s^2+14s+4} \quad (24a)$$

$$=\frac{.0625\,(7.46s+1)(.536s+1)}{(2s+1)(.5s+1)^3} \quad (24b)$$

The open loop response of the above process to a unit step change in control effort is given in figure 6. Notice the very large overshoot of the final steady state caused by the strong lead action of the term (7.46s + 1) in the numerator of (24b). Using (7). (8), and (9) to compute PID parameters with a filter time constant of .2 yields a PID controller with τ_I = - 4.60 and τ_D = -7.87. On the other hand, using (12), (8), and (9) gives:

$$\text{PID}\cdot\text{lag}=\frac{40(1.19s^2+2.86s+1)}{s(7.47s+1)} \quad (25)$$

Notice, that the lag time constant in (25) is nearly the same as the large lead time constant in the process model. Indeed, very nearly the same controller would have been obtained by finding the PID controller for the process given by (24b), but with the lead removed. However, to obtain this controller it is necessary to use a second order filter with a time constant of 0.2 to make the controller proper.

The PID controller with a second order lag seems most useful when the process model has a strong second order lead with complex zeros. For example, consider the process given by:

$$\tilde{p}(s)=\frac{.5(16s^2+.4s+1)}{(2s+1)(.5s+1)^3} \quad (26)$$

Using a filter time constant of 0.5 yields an integral time constant of 2.85 and a derivative time constant of -4.98. The lag time constant computed from (12a) is -2.75, and so the controller given by (12c) also can not be used. Finally, the controller given by (15) for a filter time constant of 0.5 is:

$$\text{PID}\cdot\text{lag}=\frac{2(3.75s^2+3.5s+1)}{s(16.1s^2+.65s+1)} \quad (27)$$

Notice that the denominator lag of (27) is very close to the numerator lead in the process model given by (26). Here again, if one eliminates the numerator term, ($16s^2$ + 4s + 1), from the process model, computes the PID controller for the reduced model and then adds back into the controller a lag to cancel the numerator term, the result is:

$$\text{PID}\cdot\text{lag}=\frac{2(2.94s^2+3.25s+1)}{s(16s^2+4s+1)} \quad (28)$$

The control system response using (28) is very similar to that using (27), and both yield excellent approximations to the desired closed loop response of $1/(.5*s+1)^2$.

CONCLUSIONS

A new PID controller tuning method for processes with time delay was proposed in this paper. The PID controller tuned by the proposed method gave more closer closed-loop response to the desired response than those tuned by other tuning methods. Tuning rules for various process models were developed.

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