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A DESIGN OF DISCRETE TIME NONLINEAR CONTROL SYSTEM WITH DISTURBANCES USING MODEL FOLLOWING METHOD

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Abstract A model following control system (MFCS) can give general output signals following desired ones. In previous studies ,a method of nonlinear MFCS was proposed by S.Okubo[1]. In this paper, the method of nonlinear MFCS will be extended to discrete time nonlinear systems. Nonlinear systems which are dealt in this paper have the property of norm constraints $\|\mathbf{f}(\mathbf{v}(k)\| \le \alpha+\beta\|\mathbf{v}(k)\|^{\gamma}$,where, $\alpha \ge 0,\ \beta \ge 0,\ 0 \le \gamma \le 1$. When $0 \le \gamma < 1$. It is easy to extend the method to discrete time systems. But in the case $\gamma = 1$ discrete time systems, the proof becomes difficult, because the transfer function from $f(v(k))$ to $v(k)$ can't be a positive real function. In this case, to ensure that internal states are stable, a new criterion is proposed.

Keywords Discrete time systems, Nonlinear control systems, Model following control systems

1. THE EXPRESSION OF PROBLEMS

A controlled object is described in (1),(2),(3) and the accordant model is given in (4),(5).

$$\mathbf{x}(k+1) = A\mathbf{x}(k)+B\mathbf{u}(k)+B_f\mathbf{f}(\mathbf{v}(k))+\mathbf{d}(k) \quad (1)$$

$$\mathbf{v}(k)=C_f\mathbf{x}(k) \quad (2)$$

$$\mathbf{y}(k) = C\mathbf{x}(k)+\mathbf{d}_0(k) \quad (3)$$

$$\mathbf{x}_m(k+1) = A_m\mathbf{x}_m(k)+B_m\mathbf{r}_m(k) \quad (4)$$

$$\mathbf{y}_m(k)=C_m\mathbf{x}_m(k) \quad (5)$$

Where $\mathbf{x}(k), \in R^n$, $\mathbf{u}(k) \in R^l$, $\mathbf{v}(k) \in R^{l_f}$,$\mathbf{f}(\mathbf{v}(k)) \in R^{l_f}$, $\mathbf{y}(k) \in R^l$,$\mathbf{d}(k) \in R^n$, $\mathbf{d}_0(k) \in R^l$, $\mathbf{x}_m(k) \in R^{n_m}$, $\mathbf{r}_m(k) \in R^{l_m}$, $\mathbf{y}_m(k) \in R^l$ .The available states are output y(k) and measurement output v(k). The nonlinear function $\mathbf{f}(\mathbf{v}(k))$ is available and satisfies the condition (6). The (C, A, B) is controllable and observable. The Zeros of $C(zI-A)^{-1}B$ are stable.

$$\|\mathbf{f}(\mathbf{v}(k)\| \le \alpha+\beta\|\mathbf{v}(k)\|^{\gamma} \quad (6)$$

Where $\alpha \ge 0,\ \beta \ge 0,\ 0 \le \gamma \le 1$. $\|\cdot\|$ is Euclidean norm. Disturbance $\mathbf{d}(k), \mathbf{d}_0(k)$ are bounded and satisfy (7),

$$D_d(z)\mathbf{d}(k)=0, \quad D_d(z)\mathbf{d}_0(k)=0 \quad (7)$$

$D_d(z)$ is a scalar characteristic polynomial of disturbances. Output error is given as below.

$$\mathbf{e}(k)=\mathbf{y}(k)-\mathbf{y}_m(k) \quad (8)$$

The aim of control system design is to obtain a control law which makes output error zero and keeps internal states bounded.

2. DESIGN OF NONLINEAR MODEL FOLLOWING CONTROL SYSTEMS

Assuming z is shift operator, (C_m , A_m , B_m) is controllable and observable. Hence the followings are available.

$$C(zI-A)^{-1}B = N(z)/D(z) \quad (9)$$

$$C(zI-A)^{-1}B_f = N_f(z)/D(z) \quad (10)$$

$$C_m(zI-A_m)^{-1}B_m = N_m(z)/D_m(z) \quad (11)$$

Where $D(z)=|zI-A|$, $D_m(z)=|zI-A_m|$. Then, the representations of input-output equation are described as followings.

$$D(z)\mathbf{y}(k)=N(z)\mathbf{u}(k)+N_f(z)\mathbf{f}(\mathbf{v}(k))+\mathbf{w}(k) \quad (12)$$

$$D_m(z)\mathbf{y}_m(k)=N_m(z)\mathbf{r}_m(k) \quad (13)$$

$$\mathbf{w}(k)=C\,adj(zI-A)\mathbf{d}(k)+D(z)\mathbf{d}_0(k) \quad (14)$$

Where, $\partial_{ri}(N(z))=\sigma_i$, $\partial_{ri}(N_f(z))=\sigma_{fi}$, $\partial_{ri}(N_m(z))=\sigma_{mi}$,and $\Gamma_r(N(z))=N_r$ ($\Gamma_r(\cdot)$ is the coefficient matrix of the element with maximum of row degree[2]), as well as $N_r \ne 0$. Since the disturbances satisfy (7),then,

$$D_d(z)\mathbf{w}(k)=0 \quad (15)$$

The first step of designing , a monic and stable polynomial $T(z)$ which has the degree of $\rho(\rho \ge n_d+2n-n_m-1-\sigma_i)$ is chosen .

Then, $R(z)$, $S(z)$ can be derived from

$$T(z)D_m(z)=D_d(z)D(z)R(z)+S(z) \quad (16)$$

Here, the degree of each polynomial is: $\partial T(z)=\rho$, $\partial D_m(z)=n_m$ $\partial D_d(z)=n_d$, $\partial D(z)=n$, $\partial R(z)=\rho+n_m-n_d-n$. and $\partial S(z) \le n_d+n-1$.

The output error $\mathbf{e}(k)$ is represented as below.

$$T(z)D_m(z)\mathbf{e}(k) = D_d(z)R(z)N(z)\mathbf{u}(k) + D_d(z)R(z)N_f(z)\mathbf{f}(\mathbf{v}(k)) + S(z)\mathbf{y}(k) - T(z)N_m(z)\mathbf{r}_m(k) \quad (17)$$

Let the right hand side of above equation equal zero, that $\mathbf{u}(k)$ will be described as following.

$$\mathbf{u}(k) = -N_r^{-1}Q^{-1}(z)[D_d(z)R(z)N(z) - Q(z)N_r]\mathbf{u}(k)$$
$$- N_r^{-1}Q^{-1}(z)D_d(z)R(z)N_f(z)\mathbf{f}(\mathbf{v}(k))$$
$$- N_r^{-1}Q(z)S(z)\mathbf{y}(k) + N_r^{-1}(z)Q^{-1}(z)T(z)N_m(z)\mathbf{r}_m(k) \quad (18)$$

Where Q(z)=diag $\{z^{\delta_i}\}$, $\delta_i = \rho + n_m - n + \sigma_i$ (i=1,2,...n). Then the state space expression of $\mathbf{u}(k)$ is shown as following:

$$\mathbf{u}(k) = -H_1\xi_1(k) - E_2\mathbf{y}(k) - H_2\xi_2(k) - E_3\mathbf{f}(\mathbf{v}(k))$$
$$-H_3\xi_3(k) + E_4\mathbf{r}_m(k) + H_4\xi_4(k) \quad (19)$$

where

$$Q^{-1}(z)[D_d(z)R(z)N(z) - Q(z)N_r] = N_rH_1(zI - F_1)^{-1}G_1 \quad (20)$$
$$Q^{-1}(z)S(z) = N_r(z)[H_2(z)(zI - F_2)^{-1}G_2 + E_2] \quad (21)$$
$$Q^{-1}(z)D_d(z)R(z)N_f(z) = N_r(z)[H_3(z)(zI - F_3)^{-1}G_3 + E_3] \quad (22)$$
$$Q^{-1}(z)T(z)N_m(z) = N_r(z)[H_4(z)(zI - F_4)^{-1}G_4 + E_4] \quad (23)$$

and the followings must be satisfied.

$$\xi_1(k+1) = F_1\xi_1(k) + G_1\mathbf{u}(k) \quad (24)$$
$$\xi_2(k+1) = F_2\xi_2(k) + G_2\mathbf{y}(k) \quad (25)$$
$$\xi_3(k+1) = F_3\xi_3(k) + G_3\mathbf{f}(\mathbf{v}(k)) \quad (26)$$
$$\xi_4(k+1) = F_4\xi_4(k) + G_4\mathbf{r}_m(k) \quad (27)$$
$$|zI - F_i| = |Q(z)|,\ i=1,2,3,4. \quad (28)$$

$\mathbf{u}(k)$ of (18) is obtained from $\mathbf{e}(k)$=0. Model following control system can be realized if system internal states are bounded.

3. PROOF OF BOUNDNESS OF INTERNAL STATES

System inputs are reference input signal $\mathbf{r}_m(k)$ and disturbances $\mathbf{d}(k)$, $\mathbf{d}_0(k)$ which are all assumed to be bounded. The boundness can be easily proved if there is no nonlinear part $\mathbf{f}(\mathbf{v}(k))$. But if $\mathbf{f}(\mathbf{v}(k))$ exists, the boundness has relation with it.

First, the overall system can represented by state space in (29),(30),(31),(32).

$$\mathbf{z}(k+1) = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix}\mathbf{z}(k)$$
$$+ \begin{bmatrix} BH_4 \\ G_1H_4 \\ 0 \\ 0 \end{bmatrix}\xi_4(k) + \begin{bmatrix} B_f - BE_3 \\ -G_1E_3 \\ 0 \\ G_3 \end{bmatrix}\mathbf{f}(\mathbf{v}(k)) + \begin{bmatrix} BE_4 \\ G_1E_4 \\ 0 \\ 0 \end{bmatrix}\mathbf{r}_m(k)$$
$$+ \begin{bmatrix} \mathbf{d}(k) - BE_2\mathbf{d}_0(k) \\ -G_1E_2\mathbf{d}_0(k) \\ G_2\mathbf{d}_0(k) \\ 0 \end{bmatrix} \quad (29)$$

$$\xi_4(k+1) = F_4\xi_4(K) + G_4\mathbf{r}_m(k) \quad (30)$$
$$\mathbf{v}(k) = C_f\mathbf{x}(k) \quad (31)$$
$$\mathbf{y}(k) = C\mathbf{x}(k) + \mathbf{d}_0(k) \quad (32)$$

Where $z^T(k) = (\mathbf{x}^T(k), \xi_1^T(k), \xi_2^T(k), \xi_3^T(k))$. Obviously, $\xi_4(k)$ is bounded. Here, necessary part about boundness is considered.(29)-(31) can be simplified as

$$\mathbf{z}(k+1) = A_s\mathbf{z}(k) + B_s\mathbf{f}(\mathbf{v}(k)) + \mathbf{d}_s(k) \quad (33)$$
$$\mathbf{v}(k) = C_s\mathbf{z}(k) \quad (34)$$

(29)-(31) and (33)-(34) is identical. The contents of $A_s, B_s, C_s, \mathbf{d}_s(k)$ are shown clearly in (29)-(31).In order to obtain desired conclusion, it is sufficient to prove that $\mathbf{z}(k)$ is bounded.

Then, we prove that A_s is stable . A_s and its characteristic polynomial is calculated as the followings[1].

$$A_s = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix} \quad (35)$$

$$|zI - A_s| = |Q(z)|^2 V_s(z)T^l(z)D_m^l(z) \quad (36)$$

Where, $V_s(z)$ is the zero point polynomial of $C(zI - A)^{-1}B = W(z)^{-1}U(z)$ (left coprime decomposition),that is, $V_s(z) = |U(z)|/|N_r|$. As |Q(z)|, $V_s(z)$,T(z), $D_m(z)$ are all stable polynomial. Therefore, A_s is a stable system matrix.

3.1 *In the case of* $0 \le \gamma < 1$

We construct the Lyapunov function $V(k)$ consisting of quadratic $\mathbf{z}(k)$,

$$V(k) = \mathbf{z}^T(k)P\mathbf{z}(k) \quad (37)$$
$$\Delta V(k) = \mathbf{z}^T(k+1)P\mathbf{z}(k+1) - V(k)$$
$$= [A_s\mathbf{z}(k) + B_s\mathbf{f}(\mathbf{v}(k)) + \mathbf{d}_s(k)]^T P$$
$$[A_s\mathbf{z}(k) + B_s\mathbf{f}(\mathbf{v}(k)) + \mathbf{d}_s(k)] - V(k)$$
$$A_s^T P A_s - P = -Q \quad (38)$$

Where P,Q are symmetric positive definite matrices and (38) is satisfied. If A_s is stable matrix, we can get an unique P from (38) if only Q is given. As $\mathbf{d}_s(k)$ is bounded and $0 \le \gamma < 1$, $\Delta V(k)$ satisfies the next inequality.

$$\Delta V(k) \le -\mathbf{z}^T(k)Q\mathbf{z}(k) + \mu_2\|\mathbf{f}(\mathbf{v}(k))\|^{1+\gamma} + M_1$$
$$\le -\mu_1\|\mathbf{z}(k)\|^2 + \mu_2\|\mathbf{f}(\mathbf{v}(k))\|^{1+\gamma} + M_2$$
$$\le -\mu_m V(k) + M \quad (39)$$

Where, $0 < \mu_1 = \lambda_{\min}(Q)$, $0 < \mu_m < 1$. μ_2, M_1, M_2, M are positive constants. As the result of (39), it is known that $V(k)$ is bounded.

$$V(k) \le V(0) + M/\mu_m \quad (40)$$

Hence, $\mathbf{z}(k)$ is also bounded. The above result is summarized as

Theorem 1: In the nonlinear system,

$$\mathbf{z}(k+1) = A\mathbf{z}(k) + B\mathbf{f}(\mathbf{v}(k)) + \mathbf{d}(k) \quad (41)$$
$$\mathbf{v}(k) = C\mathbf{z}(k) + \mathbf{d}_0(k) \quad (42)$$

Where $\mathbf{z}(k) \in R^n$, $\mathbf{v}(k) \in R^{l_f}$, $\mathbf{f}(\mathbf{v}(k)) \in R^{l_f}$. A is a stable system matrix, and disturbances of $\mathbf{d}(k)$, $\mathbf{d}_0(k)$ are bounded.

$\mathbf{z}(k)$, $\mathbf{v}(k)$ is bounded if nonlinear function $\mathbf{f}(\mathbf{v}(k))$ satisfies below. $\|\mathbf{f}(\mathbf{v}(k)\| \le \alpha + \beta\|\mathbf{v}(k)\|^{\gamma}$ where, $\alpha \ge 0, \beta \ge 0, 0 \le \gamma < 1$.

3.2 *In the case of* $\gamma = 1$ *and* $l_f = 1$

In this case, $\mathbf{v}(k)$ and $\mathbf{f}(\mathbf{v}(k))$ are scalars. To use regular transformation $\mathbf{z}(k) = T\overline{\mathbf{z}}(k)$, (33) and (34) can be transformed to Kalman canonical form.

$$\overline{\mathbf{z}}(k+1) = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} & \overline{A}_{13} & \overline{A}_{14} \\ 0 & \overline{A}_{22} & 0 & \overline{A}_{24} \\ 0 & 0 & \overline{A}_{33} & \overline{A}_{34} \\ 0 & 0 & 0 & \overline{A}_{44} \end{bmatrix} \overline{\mathbf{z}}(k) + \begin{bmatrix} \overline{B}_1 \\ \overline{B}_2 \\ 0 \\ 0 \end{bmatrix} f(v(k)) + \begin{bmatrix} \overline{d}_{s1}(k) \\ \overline{d}_{s2}(k) \\ \overline{d}_{s3}(k) \\ \overline{d}_{s4}(k) \end{bmatrix} \quad (43)$$

$$v(k) = \begin{bmatrix} 0 & \overline{C}_2 & 0 & \overline{C}_4 \end{bmatrix} \overline{\mathbf{z}}(k) \quad (44)$$

Where, $\overline{z}^T(k) = \left[\overline{z}_1^{\,T}(k), \overline{z}_2^{\,T}(k), \overline{z}_3^{\,T}(k), \overline{z}_4^{\,T}(k)\right]$. Since A_s is stable matrix, then $\overline{A}_{ii}$ (i =1,2,3,4) also are stable. Obviously, $\overline{z}_3(k)$, $\overline{z}_4(k)$ are bounded. $\overline{z}_2(k)$ can be rewritten as followings.

$$\overline{\mathbf{z}}_2(k+1) = \overline{A}_{22}\overline{\mathbf{z}}_2(k) + \overline{B}_2 f(v(k)) + \overline{\mathbf{d}}_2(k) \quad (45)$$

$$v(k) = \overline{C}_2\overline{\mathbf{z}}_2(k) + \overline{d}_v(k) \quad (46)$$

Where, $\overline{\mathbf{d}}_2(k), \overline{d}_v(k)$ are bounded. Subsequently, the transfer function from $f(v(k))$ to $v(k)$ can be calculated as

$$H(z) = \overline{C}_2(zI - \overline{A}_{22})^{-1}B_2 \quad (47)$$

It can be also calculated in terms of original system parameter[1].

$$H(z) = C_f(zI - A)^{-1}B_f - C_f(zI - A)^{-1}B$$
$$[C(zI - A)^{-1}B]C(zI - A)^{-1}B_f \quad (48)$$

Thus, $v(k)$ can be rewritten as following.

$$v(k) = H(z)f(v(k)) + d_v(k) \quad (49)$$

Where $d_v(k)$ is bounded. Let $\hat{v}(k) = v(k) - d_v(k)$, we can get $\hat{f}(\hat{v}(k)) = f(v(k)) = f(\hat{v}(k) + d_v(k))$.

$$H(z) = \frac{\hat{v}(k)}{\hat{f}(\hat{v}(k))} = \frac{g_{n'}z^{n'-1} + g_{n'-1}z^{n'-2} + \cdots + g_2 z + g_1}{z^{n'} + h_{n'}z^{n'-1} + \cdots + h_2 z + h_1} \quad (50)$$

The state space observable canonical form of (50) can be written as following[3].

$$\hat{\mathbf{z}}(k+1) = \begin{bmatrix} 0 & \cdots & 0 & -h_1 \\ 1 & \ddots & 0 & -h_2 \\ & \ddots & 0 & \vdots \\ 0 & & 1 & -h_{n'} \end{bmatrix} \hat{\mathbf{z}}(k) + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n'} \end{bmatrix} \hat{f}(\hat{v}(k))$$
$$= \hat{A}\hat{\mathbf{z}}(k) + \mathbf{g}\hat{f}(\hat{v}(k)) \quad (51)$$

$$\hat{v}(k) = [0, \cdots, 0, 1]\hat{\mathbf{z}}(k) = \hat{z}_{n'}(k) \quad (52)$$

Lemma : Consider system (51),(52) ,Assume that $\hat{f}(\hat{v}(k))$ satisfies the constraint as below.

$$\left|\hat{f}(\hat{v}(k))\right| \le \beta\left|\hat{v}(k)\right| + M, \ \beta \ge 0, \ \mathrm{M} > 0. \quad (53)$$

if the following condition holds, then $\hat{\mathbf{z}}(k)$ is bounded.

Condition:

$$S = \sum_{i=1}^{n'}(\beta|g_i| + |h_i|) \le \mu < 1 \quad (54)$$

Proof: To construct Lyapunov function as following.

$$V(k) = \sum_{i=1}^{n'} r_i|\hat{z}_i(k)| \quad (55)$$

Where, $r_0 > 0$, $\mu^{1/2n'} \le \eta < 1$, $r_i = \eta^i r_0$ ($i = 1, 2, \cdots, n'$). Suppose that $\hat{z}_0(k) = 0$, the next inequality can be derived.

$$\Delta V(k) = \sum_{i=1}^{n'} r_i|\hat{z}_i(k+1)| - V(k)$$
$$\le \sum_{i=1}^{n'-1}\left(\frac{r_{i+1}}{r_i} - 1\right)r_i|\hat{z}_i(k)| + \left[\sum_{i=1}^{n'}\frac{r_i}{r_{n'}}\left(\beta|g_i| + h_i\right) - 1\right]r_{n'}|\hat{z}_{n'}(k)| + M_g$$
$$\le \sum_{i=1}^{n'-1}(\eta - 1)r_i|\hat{z}_i(k)| + \left[\eta^{-n'}\sum_{i=1}^{n'}\left(\beta|g_i| + h_i\right) - 1\right]r_{n'}|\hat{z}_{n'}(k)| + M_g$$
$$\le \sum_{i=1}^{n'-1}(\eta - 1)r_i|\hat{z}_i(k)| + \left[\eta^{-n'}\mu - 1\right]r_{n'}|\hat{z}_{n'}(k)| + M_g$$
$$\le -r_m V(k)|\hat{z}_i(k)| + M_g \quad (56)$$

Where $0 < r_m < 1$, $M_g > 0$. It is similar to (39) to prove that $V(k)$ is bounded. Hence, $\hat{\mathbf{z}}(k)$ is bounded.

Base on above lemma, the theorem 2 can be described as following.

Theorem 2 : Consider the system

$$\mathbf{z}(k+1) = A\mathbf{z}(k) + Bf(v(k)) + \mathbf{d}(k) \quad (57)$$

$$v(k) = C\mathbf{z}(k) + d_0(k) \quad (58)$$

Where, $\mathbf{z}(k) \in R^n$, $v(k) \in R$, $f(v(k)) \in R$, $\mathbf{d}(k) \in R^n$, $d_0(k) \in R$. A is a stable system matrix, and disturbance of $\mathbf{d}(k)$, $d_0(k)$ are bounded. The nonlinear function $f(v(k))$ satisfies the following. $|f(v(k))| \le \alpha + \beta|v(k)|$ where, $\alpha \ge 0, \beta \ge 0$.

If the following condition is satisfied, then $\mathbf{z}(k)$, $v(k)$ is bounded.

Condition:

$$S = \sum_{i=1}^{n'}(\beta|g_i| + |h_i|) \le \mu < 1 \quad (59)$$

Where, g_i, h_i is coefficient of $H(z)$

$$H(z) = C(zI - A)^{-1}B$$
$$= \frac{g_{n'}z^{n'-1} + g_{n'-1}z^{n'-2} + \cdots + g_2 z + g_1}{z^{n'} + h_{n'}z^{n'-1} + \cdots + h_2 z + h_1} \quad (60)$$

Proof: $v(k)$ can be derived from (57) and (58) as following.

$$v(k) = C(zI - A)^{-1}Bf(v(k)) + C(zI - A)^{-1}\mathbf{d}(k) + d_0(k)$$
$$= H(z)f(v(k)) + d_v(k) \quad (61)$$

Since A is stable, thus $d_v(k)$ is bounded. Let $\hat{v}(k) = v(k) - d_v(k)$, hence, $\hat{f}(\hat{v}(k)) = f(v(k)) = f(\hat{v}(k) + d_v(k))$, the inequality

$$\left|\hat{f}(\hat{v}(k))\right| \le \beta\left|\hat{v}(k)\right| + M,\ \beta \ge 0,\ \mathrm{M} > 0$$

can be obtained. The rest of proof is same as lemma .

Corollary: in the theorem 2,the conclusion also holds if $f(v(k))$ satisfies following.

$$\left|f(v(k))\right| \le v(k)g(v(k)) + q(v(k)),$$
$$\left|g(v(k))\right| \le \beta, q(v(k)) < M \tag{62}$$

4.NUMERICAL SIMULATION

A example is given as following. We show a result of simulation in **Fig.1.**

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -0.4 & 1.3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} f(v(k)) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d(k)$$
$$v(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$
$$y(k) = \begin{bmatrix} 0.6 & 1 \end{bmatrix} \mathbf{x}(k) + d_0(k)$$
$$f(v(k)) = 0.3v(k)\sin(v(k)) + 1$$

Reference model is given as

$$\mathbf{x}_m(k+1) = \begin{bmatrix} 0 & 1 \\ -0.12 & 0.7 \end{bmatrix} \mathbf{x}_m(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(k)$$
$$r_m(k) = \sin(k\pi / 16) + 1$$
$$y_m(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}_m(k)$$

In this system, $H(z)=1/(z+0.6)$, and S= $\beta + 0.6 < 1$, here $\beta = 0.3$, the condition is satisfied. Disturbance d(k), $d_0(k)$ are step disturbances. The responses of system are shown in Fig.1. It can be concluded that output signal follows the reference even though disturbance existed in system.

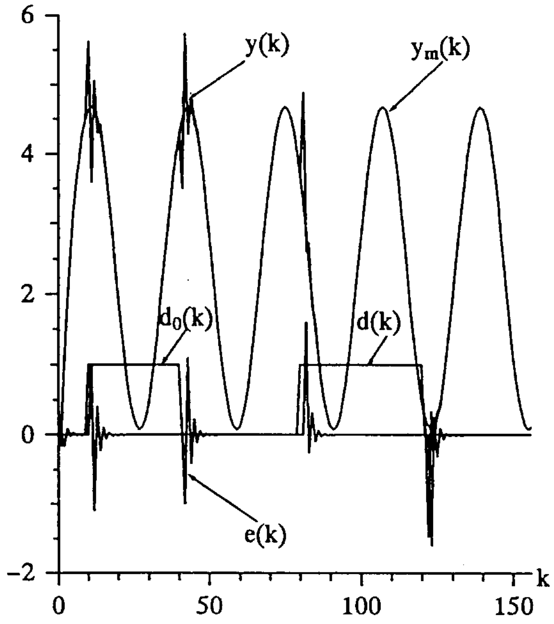

Fig.1. Simulation results (f(v)=v**sin**(v)+1)

REFERENCES

[1] S.Okubo, "A Design of Nonlinear Model Following Control System with Disturbance," Trans. of the Society of Instrument and Control Engineers,vol.21, No.8, pp.792-799,1985(in Japanese).

[2] S.Okubo, "A Nonlinear Model Following Control System with Containing Input in Nonlinear Parts," Trans. of the Society of Instrument and Control Engineers, vol.22,No.6, pp.714-716, 1986(in Japanese).

[3] T.Mita, "Digital Control Theory," Syokodo,1991(in Japanese).