# Nonlinear Control System using Universal Learning Network with Random Search Method of Variable Search Length

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Abstract: In this paper, a new optimization method which is a kind of random searching is presented. The proposed method is called RasVal which is an abbreviation of Random Search Method with Variable Search Length and it can search for a global minimum based on the probability density functions of searching, which can be modified using informations on success or failure of the past searching in order to execute intensified and diversified searching. By applying the proposed method to a nonlinear crane control system which can be controlled by the Universal Learning Network with radial basis function(R.B.F.), it has been proved that RasVal is superior in performance to the commonly used back propagation learning algorithm.

Keywords: Universal Learning Network, Random Search Method, Neural Network, Nonlinear Control.

#### 1. INTRODUCTION

Universal Learing Network (U.L.N.) is a new-type of network which can be used to model and control large-scale complicated systems such as economic, social and living phenomena as well as industrial plants. Universal Learning Network consists of nonlinearly operated nodes and multi-branches that may have different time delays between the nodes. A new control method has been already presented for nonlinear systems using Universal Learning Network with radial basis function(R.B.F.) and it has been compared to the commonly used control method using neural networks. In the above system, as learning algorithm of parameter variables in the controller was based on the gradient method, the problem of falling into a local minimum that leads to low efficiency of learning could not be solved. In this paper, a new learning algorithm that can find a global minimum is presented and it is applied to build the optimal controller of a nonlinear control system. The proposed learning algorithm is called RasVal which is an abbreviation of Random Search with Variable Search Length and it can search for a global minimum systematically and effectively in a single framework which is not a combination of different methods. RasVal is a kind of random search based on the probability density function of searching, which can be modified using informations on the results of the past searching in order to execute intensified and diversified searching. The features of RasVal are such that it does not require differential calculation as gradient method, therefore, it takes a shorter calculation time than gradient method, and ramdom search with intensification and diversification is carried out in order to solve the local minimum problem. By applying the proposed method to a nonlinear crane control system, it has been proved that RasVal is superior in performance to the back propagation learning algorithm.

### 2. UNIVERSAL LEARNING NETWORK

# 2.1 Structure of Universal Learning Network

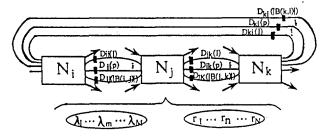


Fig.1 Structure of U.L.N. with Multiplex Branch

Universal Learning Network (U.L.N.)<sup>[1]</sup> is a new-type of network which can be used to model and control large-scale complicated systems such as economic, social and living phenomena as well as industrial plants. It consists of nonlinearly operated nodes and branches that may have arbitrary time delays including zero or minus ones. Structure of U.L.N. is shown in Fig.1.

Basic equation of U.L.N. with Multiplex Branch is represented by Eq.(1):

$$h_{j}(t) = O_{j}(\{h_{i}(t - D_{ij}(p)) | i \in JF(j), p \in B(i, j)\}, \{r_{n}(t) | n \in N(j)\}, \{\lambda_{m}(t) | m \in M(j)\})$$
(1)  
$$i \in J. \quad t \in T$$

where.

 $h_j(t)$ : output value of j node at time t;

 $\lambda_m(t)$ : value of mth parameter variable at time t;

 $r_n(t)$ : value of nth external input variable at time t;

 $O_j$ : nonlinear function of j node;

 $D_{ij}(p)$ : time delay of pth branch from i node to j node;

JF(j): set of node numbers whose outputs are connected to j node;

JB(j): set of node numbers whose inputs are connected from j node;

B(i,j): set of branches from i node to j node;

to j node;

N: set of external input variables;

M(j): set of parameter variable numbers that are included in j node;

M: set of parameter variable numbers;

J: set of node numbers;

T: set of sampling times;

Let a criterion function be written in Eq.(2):

$$E = E(\{h_r(s)\}, \{\lambda_m(s)\})$$

$$r \in R_0, m \in M_0, s \in S_0$$
(2)

where

 $R_0$ : set of node numbers related with evaluation;

 $M_0$ : set of parameter variable numbers related with evaluation:

 $S_0$ : set of sampling times related with evaluation.

The important features of U.L.N. are that function of the nodes can take any nonlinear function and the nodes can be connected arbitrarily. So the structure of U.L.N. is a general one in the sense that U.L.N. with sigmoid functions and one sampling time delays corresponds to the recurrent neural network.

# 2.2 Learning of Universal Learning Network with Radial Basis Function

U.L.N. with R.B.F. can be expressed as follows.

$$h_j(t) = \sum_{m \in L(j)} f_{jm}(x_{jm}) + b_j$$
 (3)

$$f_{jm}(x_{jm}) = k_{jm} exp(x_{jm}) \tag{4}$$

$$x_{jm} = -\frac{1}{2} \sum_{i \in JF(j)} \sum_{p \in B(i,j)} \left( \frac{h_i(t - D_{ij}(p)) - h_{jm}^i(p)}{\sigma_{jm}^i(p)} \right)^2$$
(5)

L(j): number of nonlinear functions of j node;  $k_{jm}, h_{jm}^{i}(p), \sigma_{jm}^{i}(p), b_{j}$ : parameters for j node.

Learning algorithms of U.L.N. with R.B.F. based on gradient method is shown below.

$$k_{jm} \leftarrow k_{jm} - \gamma \left\{ \sum_{tl \in T} \left[ \frac{\partial h_j(tl)}{\partial k_{jm}} \delta(j, tl) \right] + \frac{\partial E}{\partial k_{jm}} \right\}$$
 (6)

$$h_{jm}^{i}(p) \leftarrow h_{jm}^{i}(p) - \gamma \left\{ \sum_{t \in T} \left[ \frac{\partial h_{j}(tt)}{\partial h_{jm}^{i}(p)} \delta(j, tt) \right] + \frac{\partial E}{\partial h_{jm}^{i}(p)} \right\}$$
(7)

B(i, j): set of branches from 1 node to 1 node;  
N(j): set of external input variables that are connected 
$$\sigma_{jm}^{i}(p) \leftarrow \sigma_{jm}^{i}(p) - \gamma \left\{ \sum_{t t \in T} \left[ \frac{\partial h_{j}(tt)}{\partial \sigma_{jm}^{i}(p)} \delta(j, tt) \right] + \frac{\partial E}{\partial \sigma_{jm}^{i}(p)} \right\}$$
to j node;

$$b_j \leftarrow b_j - \gamma \left\{ \sum_{t \in T} \left[ \frac{\partial h_j(tt)}{\partial b_j} \delta(j, tt) \right] + \frac{\partial E}{\partial b_j} \right\}$$
 (9)

$$\delta(j,t) = \sum_{k \in JB(j)} \sum_{p \in B(j,k)} \frac{\partial h_k(t + D_{jk}(p))}{\partial h_j(t)} \delta(k,t + D_{jk}(p)) + \frac{\partial E}{\partial h_j(t)}$$
(10)

where, E: criterion function;

# RANDOM SEARCH METHOD WITH VARIABLE SEARCH LENGTH (RasVal)

RasVal is a kind of random search method based on the probability density functions of searching, which can be modified using informations on success or failure of the past searching.

The features of RasVal are such that it does not require differential calculation as gradient method, therefore, it takes a shorter calculation time than gradient method, and random search with intensification and diversification can lead to solve the local minimum problem.

Calculation procedure of RasVal is as follows.

$$if \quad E(\lambda + x) < E(\lambda) \Longrightarrow \lambda \longleftarrow \lambda + x; \qquad (11)$$

(searching is success)

if 
$$E(\lambda + x) \ge E(\lambda) \Longrightarrow \lambda \longleftarrow \lambda$$
. (12)

(searching is failure)

where, E: criterion function;

 $\lambda = [\lambda_1 ... \lambda_m ... \lambda_M]^T$ : parameter variable vector;  $x = [x_1 ... x_m ... x_M]^T$ : parameter variable search vector. The probability density function  $f(x_m)$  of searching  $x_m$  (see Fig.2) is represented by Eq.(13),(14) and (15):

$$f(x_m) = p_m \beta e^{\beta x_m}, x_m \le 0; \tag{13}$$

$$f(x_m) = q_m \beta e^{-\beta x_m}, x_m > 0; \tag{14}$$

$$p_m + q_m = 1.0 (15)$$

Therefore  $x_m$  can be calculated as follows:

$$if \quad 0 \le z \le p_m \quad \Longrightarrow x_m = \frac{1}{\beta} \ln(\frac{z}{p_m})$$
 (16)

$$if \quad p_m < z \le 1.0 \Longrightarrow x_m = -\frac{1}{\beta} \ln(\frac{1-z}{q_m})$$
 (17)

where, z: random numbers in [0,1].

Parameter variables  $\beta$ ,  $p_m$ ,  $q_m$  of  $f(x_m)$  which are related to searching range and direction are modified based on the informations of success or failure of the past searching as follows.

In case of negative direction searching:

$$p_m \longleftarrow \alpha p_m + (1 - \alpha) \cdot SF$$
 (18)

In case of positive direction searching:

$$q_m \longleftarrow \alpha q_m + (1 - \alpha) \cdot SF$$
 (19)

$$\beta = \bar{\beta}e^{-\eta n} + \underline{\beta} \tag{20}$$

In case of failure,  $n \leftarrow n+1$ ;

In case of success,  $n \leftarrow n$ , n = 0;  $n \leftarrow n - 1$ ,  $0 < n \le n_0$ ;

 $n \longleftarrow n_0, \qquad n > n_0$ 

where, SF = 1.0, in case of success;

SF = 0.0, in case of failure;

 $\alpha$ : exponential filter coefficient;

 $\bar{\beta} + \beta$ : upper limit of  $\beta$ ;

 $\underline{\beta}$ : lower limit of  $\beta$ ;

 $\eta$ : coefficient.

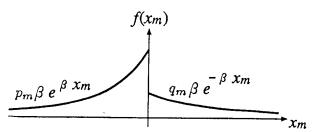


Fig.2 Probability Density Function of Searching  $x_m$ 

From Eq.(11)  $\sim$  (20), intensification and diversification

of the search can be realized such that when there is quite a possibility of finding good solutions around the current one, intensified search for the vicinity of the current solution is carried out; on the other hand, when there is no possibility of finding good solutions, diversified search is executed in order to find good solutions in the region far from the current solution.

#### 4. SIMULATION

### 4.1 Nonlinear Crane Control System

A nonlinear crane system(Fig.3) was studied in order to compare the performance with RasVal and gradient method. The aim of control is to bring the trolley to the target position, and to winch the load to the target height at the same time to minimize the criterion function.

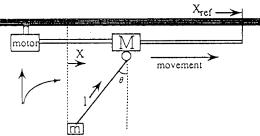


Fig.3 Nonlinear Crane System

The equation of the crane system is represented in the followings:

$$\ddot{x} = \frac{mg}{M}\theta - \frac{D+G}{M}\dot{x} + \frac{G}{M}u_d \tag{21}$$

$$\ddot{\theta} = \frac{M+m}{lM}g\theta - \frac{D+G}{lM}\dot{x} + \frac{G}{lM}u_d \qquad (22)$$

$$\ddot{l} = \frac{c + G_m}{m}\dot{l} + \frac{G_m}{m}u_m \tag{23}$$

where, M: mass of the trolley; m: mass of the load; l: height of the load from intial position;  $\theta$ : angle of the load; x: location of the trolley; C, D: coefficients of the friction.  $u_d, u_m$  are input voltage control vaules from the controller to the crane system.

Assuming as follows,

$$h_1(t) = x(t)$$
  $h_2(t) = \dot{x}(t)$   
 $h_3(t) = \theta(t)$   $h_4(t) = \dot{\theta}(t)$ 

 $h_5(t) = l(t)$   $h_6(t) = \dot{l}(t)$ 

then, equations can be expressed in the discrete form.

$$h_1(t) = a_{11}h_1(t-1) + a_{21}h_2(t-1)$$
 (24)

$$h_2(t) = a_{22}h_2(t-1) + a_{32}h_3(t-1) + b_1u_d(t)$$
(25)

$$h_3(t) = a_{33}h_3(t-1) + a_{43}h_4(t-1)$$

$$h_4(t) = a_{24}\frac{h_2(t-1)}{h_5(t-1)} + a_{34}\frac{h_3(t-1)}{h_5(t-1)}$$
(26)

$$+ \quad a_{44}h_4(t-1) + \frac{b_1}{h_5(t-1)}u_d(t) \qquad (27)$$

$$h_5(t) = a_{55}h_5(t-1) + a_{65}h_6(t-1)$$
 (28)

$$h_6(t) = a_{66}h_6(t-1) + b_2u_m(t) (29)$$

The structure of the nonlinear crane control system is shown in Fig.4. The controller is constructed by the radial basis function network. The arbitrary time delay is assumed to be 1.0 sampling time.

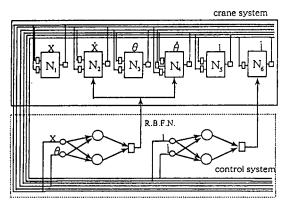


Fig.4 Structure of Nonlinear Crane Control System

# 4.2 Simulation Results

Fig.5, Fig.6, Fig.7 and Fig.8 show the simulation results. The real line was obtained using RasVal; the dotted line was obtained using gradient method.

In simulations, control time is 40 seconds, and the goal of the task is to bring x from 0.0m to 0.5m then to 1.0m and l from 2.0m to 1.50m then to 2.0m, where as  $\theta$  should be as small as possible. Therefore the criterion function can be expressed as follows.

$$E = \frac{1}{2} \sum_{t=0}^{T} [Q_1(l_{ref} - l(t))^2 + Q_2(x_{ref} - x(t))^2 + Q_3\theta^2(t) + Q_4\dot{\theta}^2(t) + Q_5u_m^2(t) + Q_6u_d^2(t)] + \frac{1}{2}(Q_7\dot{x}^2(t_f) + Q_8\dot{t}^2(t_f))$$
(30)

where,  $l_{ref}$ ,  $x_{ref}$ : target value of l, x;  $t_f$ : final sampling time;  $Q_i$ : coefficient of criterion function.

From simulation results, it is shown that the learning speed and performance of RasVal are better than that of gradient method, and from Fig.7, Fig.8, it is also shown that intensification and diversification of searching can be realized by RasVal.

#### 5. CONCLUSION

In this paper, a new optimization method called RasVal is presented which can obtain the optimal value by intensified and diversified searching. And simulations were carried out in order to compare the learning speed and performance of RasVal with that of gradient method. It has been proved that a nonlinear crane control system using RasVal has better performance than that of the system using gradient method.

#### REFERENCES

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