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OUTPUT FEEDBACK-BASED MODEL REFERENCE ADAPTIVE CONTROL FOR MIMO PLANTS

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Abstract This paper deals with the design problem of model reference adaptive controllers for MIMO plants with unknown orders. A design scheme for an adaptive control system based on CGT theorem, which has hierarchical structures derived from backstepping strategies, is proposed for MIMO plants with unknown orders but with known relative MacMillan degrees(relative degrees for SISO plants). It is also shown that all the signals in the resulting control system are bounded, and that the asymptotic tracking is achieved in the case where reference inputs are step.
keywords model reference adaptive control, almost strictly positive realness, MIMO plant, backstepping, command generator tracker, parallel inverted pendulum

1. INTRODUCTION

Recently, several types of output feedback-based model reference adaptive control methods, which do not depend on the order of the plant, have been proposed under the condition that the controlled plant is ASPR(almost strictly positive real)[1],[4],[5]. The plant is said to be ASPR if there exists a static output feedback such that the resulting closed-loop transfer function is SPR(strictly positive real)[1]. Because the sufficient condition for m-input/m-output nth order plant to be ASPR is that (a)the plant is minimum phase, (b)the relative MacMillan degree is $(n-m)/n$ and (c)for the plant(A, B, C), CB is positive definite, the ASPR condition places a severe restriction on practical plants. As a countermeasure to this problem, it has been shown that the problem for relative degrees on ASPR conditions can be alleviated by using "backstepping" strategies for SISO systems[3],[4].

In this paper, a new design scheme for output feedback-based model reference adaptive control for MIMO plants is proposed. The proposed control algorithm is based on the CGT(command generator tracker) theory[2] and has hierarchical structures derived from backstepping strategy. The effectiveness of the proposal is confirmed through experiments of positioning control by a parallel inverted pendulum.

2. PROBLEM SETUP

Consider the following controllable and observable m-input/m-output LTI plant:

$$\dot{x}(t)=Ax(t)+Bu(t)\ ,\ y(t)=Cx(t). \tag{1}$$

The transfer function matrix of the plant(1) is given by

$$G(s)=C(sI-A)^{-1}B. \tag{2}$$

We make the following assumptions on the plant.

Assumption 1:

(1) $G(s)$ is minimum-phase.

(2) The relative degree γ_i of each diagonal (i,i)element of $G(s)$ is known.

(3) There exists a matrix M_0 defined by

$$M_0=\lim_{s\to\infty} G(s)diag[s^{\gamma_1}, s^{\gamma_2},\dots,s^{\gamma_m}] \tag{3}$$

and $M_0>0$(symmetric positive definite).

The objective of this paper is to design an adaptive controller such that the plant outputs $y(t)$ track the outputs $y_m(t)$ of a reference model:

$$\dot{x}_m(t)=A_m x_m(t)+B_m u_m(t)\ ,\ y_m(t)=C_m x_m(t). \tag{4}$$

Now, represent the plant(1) with transfer function matrix $G(s)$ as follows:

$$\begin{aligned}
y(t) &= G(s)F(s)F(s)^{-1}[u(t)]=G_v(s)[u_{f1}(t)] && (5)\\
G_v(s) &= G(s)F(s)=C_v(sI-A_v)^{-1}B_v && (6)\\
u_{f1}(t) &= F(s)^{-1}[u(t)] && (7)\\
F(s) &= \mathrm{diag}[f_1(s), f_2(s),\dots,f_m(s)] && (8)\\
f_j(s) &= \prod_{i=1}^{\gamma_j-1}(s+\lambda_{ij}),\ \lambda_{ij}>0 && (9)
\end{aligned}$$

where the notation $G(s)[u(t)]$ denotes the output of a system at time t with transfer function matrix $G(s)$ and input $u(t)$. Since the plant(1) satisfies the assumption 1, the virtual plant(5) with transfer function $G_v(s)$ and input $u_{f1}(t)$ is ASPR.

For this ASPR plant and the reference model(4), make the following assumptions:

Assumption 2:

(1)There exist solutions Ω_{ij} of the matrix equation.

$$\begin{bmatrix} A_v & B_v \\ C_v & 0 \end{bmatrix}\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}=I \tag{10}$$

(2) No eigenvalue of Ω_{11} is equal to the inverse of an eigenvalue of A_m.

Under Assumption 2, if we could design a control signal $u_{f1}(t)$ for $G_v(s)$ as:

$$\begin{aligned}
u_{f1}(t) &= K(t)z(t) && (11)\\
z(t) &= [e_y(t)^T, x_m(t)^T, u_m(t)^T]^T && (12)\\
e_y(t) &= y_m(t)-y(t) && (13)\\
\dot{K}(t) &= [\dot{k}_1(t),\dots,\dot{k}_m(t)]^T=z(t)e_y(t)^T\Gamma-\sigma K(t),\\
&\quad \Gamma=\Gamma^T>0\ ,\sigma>0 && (14)
\end{aligned}$$

then, we can achieve the control objective according to the conventional Simple Adaptive Control(SAC) strategy[1].

In the following section, we will propose a controller design scheme based on backstepping strategy[3],[4] which virtually realizes the controller given in (11) for MIMO plants.

3. THE DESIGN PROCEDURE

3.1 Filtered Signal and Virtual ASPR Plant

Define a new signal $u_{fi}(t), i=1,\dots,\gamma^*$ as follows:

$$\Delta_{i+1}\dot{u}_{fi}(t) = -\Lambda_i u_{fi}(t) + u_{fi+1}(t) \quad (15)$$

where

$$\Delta_i = \mathrm{diag}[\delta_{i1},\dots,\delta_{im}],\ \delta_{ij} = \begin{cases} 1\ ,\ j\in\{j|\gamma_j \geq i\} \\ 0\ ,\ j\in\{j|\gamma_j \leq i-1\} \end{cases} \quad (16)$$

and

$$\Lambda_i = \mathrm{diag}[\bar{\lambda}_{i1},\bar{\lambda}_{i2},\dots,\bar{\lambda}_{im}],\ \bar{\lambda}_{ij} = \begin{cases} \lambda_{ij}\ ,\ j\in\{j|\gamma_j \geq i+1\} \\ 0\ ,\ j\in\{j|\gamma_j \leq i\} \end{cases}. \quad (17)$$

The signal $u_{f1}(t)$ given in (7) can be successively generated from (15). Further, since the virtual plant $G_v(s)$ is ASPR, we have the following input-output representation for the virtual system(5) with input $u_{f1}(t)$.

$$\dot{y}(t) = \Theta y(t) + B_0 u_{f1}(t) + \eta(t) \quad (18a)$$
$$\dot{x}_2(t) = A_2 y(t) + B_2 y(t) \quad (18b)$$
$$\eta(t) = C_2 x_2(t) \quad (18c)$$

where, Θ, B_0, B_2 and C_2 are appropriate dimensional matrices, and A_2 is a stable matrix corresponding to zeros of $G_v(s)$.

3.2 Controller Design

Step 1:

First of all, we introduce a virtual input $\alpha_1(t) = [\alpha_{11}(t),\dots,\alpha_{1m}(t)]^T$ corresponding to $u_{f1}(t)$.

$$\alpha_1(t) = K(t)z(t) \quad (19)$$

If $\gamma_j = 1$ for any j, then, for $j\in\{j|\gamma_j=1\}$, the j element $u_j(t)$ of the control input vector $u(t)$ is given in this step by

$$u_j(t) = u_{f1j}(t) = \alpha_{1j}(t)\ ,\ \ j\in\{j|\gamma_j=1\}. \quad (20)$$

The design procedure is completed if $\gamma_j = 1$ for all j. Otherwise, in case where there exist elements such that $\gamma_j \geq 2$, introduce a new variable $\varepsilon_2(t) = [\varepsilon_{21}(t),\dots,\varepsilon_{2m}(t)]^T$:

$$\varepsilon_2(t) = u_{f1}(t) - \alpha_1(t). \quad (21)$$

Step 2:

Introduce a virtual input $\alpha_2(t) = [\alpha_{21}(t),\dots,\alpha_{2m}(t)]^T$ corresponding to $u_{f2}(t)$ as follows:

$$\begin{aligned}\alpha_{2j}(t) &= \beta_{2j}(t) + r_{1j}(t)^T\Big(\widehat{\Theta}(t)y(t) + \widehat{B}_0(t)u_{f1}(t) \\ &\quad + \widehat{S}(t)\bar{z}_m(t)\Big) - \bar{\gamma}_{\varepsilon 2j}\varepsilon_{2j}(t) - \bar{\bar{\gamma}}_{\varepsilon 2j}\bar{r}_{\varepsilon 1j}(t)^T\varepsilon_2(t) \\ &\quad + \lambda_{1j}u_{f1j}(t)\ ,\ j\in\{j|\gamma_j\geq 2\} \quad (22a)\end{aligned}$$

$$\beta_{2j}(t) = \frac{\partial\alpha_{1j}}{\partial k_j}\dot{k}_j(t) + \left[\frac{\partial\alpha_{1j}}{\partial y_m}, \frac{\partial\alpha_{1j}}{\partial x_m}, \frac{\partial\alpha_{1j}}{\partial u_m}\right]\dot{z}_m(t) \quad (22b)$$

$$z_m(t) = \left[y_m(t)^T, x_m(t)^T, u_m(t)^T\right]^T \quad (22c)$$

$$\bar{z}_m(t) = [x_m(t)^T, u_m(t)^T] \quad (22d)$$

$$r_{1j}(t)^T = \frac{\partial\alpha_{1j}}{\partial y} \quad (22e)$$

$$\alpha_{2j}(t) = 0\ ,\ \ j\in\{j|\gamma_j=1\} \quad (22f)$$

$$\begin{cases} \bar{\gamma}_{\varepsilon 2j} > 1,\ \bar{\bar{\gamma}}_{\varepsilon 2j} > 0\ \ ,\ \ j\in\{j|\gamma_j\geq 2\} \\ \bar{\gamma}_{\varepsilon 2j} = \bar{\bar{\gamma}}_{\varepsilon 2j} = 0\ \ ,\ \ j\in\{j|\gamma_j=1\} \end{cases}$$

where, $\widehat{\Theta}(t)$ and $\widehat{B}_0(t)$ are the estimated values of Θ and B_0. $\widehat{S}(t)\bar{z}_m(t)$ is the estimated signal of $\eta(t)$. The adjusting laws of $\widehat{\Theta}(t)$, $\widehat{B}_0(t)$ and $\widehat{S}(t)$ are to be determined later. The vector $\bar{r}_{\varepsilon 1j}(t)^T$ is a j row of $\bar{R}_1(t)\bar{R}_1(t)^T$ where

$$\bar{R}_1(t) = \Delta_2 R_1(t)\ ,\ R_1(t) = [r_{11}(t),\dots,r_{1m}(t)]^T. \quad (23)$$

If $\gamma_j = 2$ for any j, then, for $j\in\{j|\gamma_j=2\}$, the j element $u_j(t)$ of the control input vector $u(t)$ is given in this step by

$$u_j(t) = u_{f2j}(t) = \alpha_{2j}(t)\ ,\ \ j\in\{j|\gamma_j=2\}. \quad (24)$$

The design procedure is completed if $\gamma_j = 2$ for all j. Otherwise, in the case where there exist elements such that $\gamma_j \geq 3$, introduce a new variable $\varepsilon_2(t) = [\varepsilon_{21}(t),\dots,\varepsilon_{2m}(t)]^T$:

$$\varepsilon_3(t) = u_{f2}(t) - \alpha_2(t). \quad (25)$$

Step i $(3 \leq i \leq \gamma^*-1, \gamma^* = \max_j(\gamma_j))$**:**

By using the same procedure as in Step 2, set a virtual input $\alpha_i(t) = [\alpha_{i1}(t),\dots,\alpha_{im}(t)]^T$ corresponding to $u_{fi}(t)$ as follows:

$$\begin{aligned}\alpha_{ij}(t) &= \beta_{ij}(t) + r_{i-1j}(t)^T\Big(\widehat{\Theta}(t)y(t) + \widehat{B}_0(t)u_{f1}(t) \\ &\quad + \widehat{S}(t)\bar{z}_m(t)\Big) + \sum_{k=1}^{m}\phi_{\theta i-1jk}(t)^T\Gamma_\theta\tau_{\theta ik}(t) \\ &\quad + \sum_{k=1}^{m}\phi_{bi-1jk}(t)^T\Gamma_b\tau_{bik}(t) + \sum_{k=1}^{m}\phi_{si-1jk}(t)^T\Gamma_s\tau_{sik}(t) \\ &\quad - \bar{\gamma}_{\varepsilon ij}\varepsilon_{ij}(t) - \bar{\bar{\gamma}}_{\varepsilon ij}\bar{r}_{\varepsilon i-1j}(t)^T\varepsilon_i(t) \\ &\quad + \lambda_{i-1j}u_{fi-1j}(t) + \tilde{\alpha}_{ij}(t), j\in\{j|\gamma_j\geq i\} \quad (26a)\end{aligned}$$

$$\alpha_{ij}(t) = 0\ ,\ \ j\in\{j|\gamma_j\leq i-1\} \quad (26b)$$

where

$$\begin{cases} \bar{\gamma}_{\varepsilon ij} > 1\ ,\ \ \bar{\bar{\gamma}}_{\varepsilon ij} > 0,\ j\in\{j|\gamma_j\geq i\} \\ \bar{\gamma}_{\varepsilon ij} = \bar{\bar{\gamma}}_{\varepsilon ij} = 0,\ j\in\{j|\gamma_j\leq i-1\} \end{cases} \quad (27)$$

$$\Gamma_\theta = \Gamma_\theta^T > 0\ ,\ \Gamma_b = \Gamma_b^T > 0\ ,\ \Gamma_s = \Gamma_s^T > 0 \quad (28)$$

and

$$\begin{aligned}\beta_{ij}(t) &= \sum_{k=1}^{m}\frac{\partial\alpha_{i-1j}}{\partial k_k}\dot{k}_k(t) \\ &\quad + \sum_{h=1}^{i-1}\left[\frac{\partial\alpha_{i-1j}}{\partial y_m^{(h)}}, \frac{\partial\alpha_{i-1j}}{\partial x_m^{(h)}}, \frac{\partial\alpha_{i-1j}}{\partial u_m^{(h)}}\right]\dot{z}_m^{(h)}(t) \\ &\quad + \sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1j}}{\partial\varepsilon_l}\big(-\Lambda_{l-1}u_{fl-1}(t) + u_{fl}(t) - \Delta_l\beta_l(t)\big) \\ &\quad + \sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1j}}{\partial u_{fl-1}}\big(-\Lambda_{l-1}u_{fl-1}(t) + u_{fl}(t) \\ &\quad + (\Delta_{l-1}-\Delta_l)\beta_l(t)\big), j\in\{j|\gamma_j\geq i-1\} \quad (29a)\end{aligned}$$

$$\beta_{ij}(t) = 0\ ,\ \ j\in\{j|\gamma_j\leq i-2\} \quad (29b)$$

$$\beta_l(t) = [\beta_{l1}(t),\beta_{l2}(t),\dots,\beta_{lm}(t)]^T. \quad (30)$$

Further,

$$\begin{aligned}r_{ij}(t)^T &= \frac{\partial\alpha_{ij}}{\partial y} - \sum_{l=2}^{i}\frac{\partial\alpha_{ij}}{\partial\varepsilon_l}\bar{R}_{l-1}(t) \\ &\quad + \sum_{l=2}^{i}\frac{\partial\alpha_{ij}}{\partial u_{fl-1}}(\Delta_{l-1}-\Delta_l)R_{l-1}(t), \\ &\quad j\in\{j|\gamma_j\geq i\} \quad (31a)\end{aligned}$$

$$r_{ij}(t)^T = 0\ ,\ \ j\in\{j|\gamma_j\leq i-1\} \quad (31b)$$

$$\bar{R}_l(t) = \Delta_{l+1} R_l(t)\ ,\ R_i(t) = [r_{l1}(t), \ldots, r_{lm}(t)]^T \tag{32}$$

$$\bar{r}_{\varepsilon ij}(t)^T : j \text{ row of } \bar{R}_i(t)\bar{R}_i(t)^T \tag{33}$$

and $\phi_{\theta ijk}(t), \phi_{bijk}(t)$ and $\phi_{sijk}(t)$ are given by the following equation for $w = \theta,\ b$ and s respectively.

$$\begin{aligned} \phi_{wijk}(t)^T &= \left\{ \frac{\partial \alpha_{ij}}{\partial \widehat{w}_k} - \sum_{l=3}^{i} \left(\frac{\partial \alpha_{ij}}{\partial \varepsilon_l} \bar{\Phi}_{wl-1k}(t) \right.\right. \\ &\left.\left. + \frac{\partial \alpha_{ij}}{\partial u_{fl-1}} (\Delta_{l-1} - \Delta_l) \Phi_{wl-1k}(t) \right) \right\} , \\ & j \in \{j|\gamma_j \geq i\} \end{aligned} \tag{34a}$$

$$\phi_{wijk}(t)^T = 0\ ,\ j \in \{j|\gamma_j \leq i-1\} \tag{34b}$$

$$\bar{\Phi}_{wlk}(t) = \Delta_{l+1}\Phi_{wlk}(t)\ ,\ \Phi_{wlk}(t) = [\phi_{wl1k}(t), \ldots, \phi_{wlmk}(t)]^T \tag{35}$$

where $\widehat{w}_k(t)$ for $w = \theta, b$ and s are k rows of $\widehat{\Theta}(t)$, $\widehat{B}_0(t)$ and $\widehat{S}(t)$ respectively. Further $\tau_{\theta ij}(t)^T$, $\tau_{bij}(t)^T$ and $\tau_{sij}(t)^T$ are j rows vectors of $\mathcal{T}_{\theta i}(t)$, $\mathcal{T}_{bi}(t)$, $\mathcal{T}_{si}(t)$, respectively, which are given as follows:

$$\begin{cases} \mathcal{T}_{\theta i}(t) = -\bar{R}_{i-1}(t)^T \varepsilon_i(t) y(t)^T + \mathcal{T}_{\theta i-1}(t) \\ \mathcal{T}_{\theta 2}(t) = -\bar{R}_1(t)^T \varepsilon_2(t) y(t)^T \end{cases} \tag{36}$$

$$\begin{cases} \mathcal{T}_{bi}(t) = -\bar{R}_{i-1}(t)^T \varepsilon_i(t) u_{f1}(t)^T + \mathcal{T}_{bi-1}(t) \\ \mathcal{T}_{b2}(t) = -\bar{R}_1(t)^T \varepsilon_2(t) u_{f1}(t)^T \end{cases} \tag{37}$$

$$\begin{cases} \mathcal{T}_{si}(t) = -\bar{R}_{i-1}(t)^T \varepsilon_i(t) \bar{z}_m(t)^T + \mathcal{T}_{si-1}(t) \\ \mathcal{T}_{s2}(t) = -\bar{R}_1(t)^T \varepsilon_2(t) \bar{z}_m(t)^T \end{cases} \tag{38}$$

and $\tilde{\alpha}_{ij}(t)$ ($\tilde{\alpha}_{ij}(t) = 0, j \in \{j|\gamma_j \leq i-1\}$) is an auxiliary signal which is given by, for $i = 3$

$$\begin{aligned} \tilde{\alpha}_{ij}(t) &= -\sum_{k=1}^{m} \phi_{\theta 2jk}(t)\sigma_\theta \hat{\theta}_k(t) - \sum_{k=1}^{m} \phi_{b2jk}(t)\sigma_b \hat{b}_{0k}(t) \\ &-\sum_{k=1}^{m} \phi_{s2jk}(t)\sigma_s \hat{s}_k(t)\ ,\ j \in \{j|\gamma_j \geq 3\} \end{aligned} \tag{39a}$$

$$\tilde{\alpha}_{ij}(t) = 0\ ,\ j \in \{j|\gamma_j \leq 2\} \tag{39b}$$

for $4 \leq i \leq \gamma^* - 1$

$$\begin{aligned} \tilde{\alpha}_{ij}(t) &= -\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{\theta l-1k}(t) \right) \Gamma_\theta y(t) \bar{r}_{i-1kj}(t) \\ &-\sum_{k=1}^{m} \phi_{\theta i-1jk}(t)\sigma_\theta \hat{\theta}_k(t) \\ &-\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{bl-1k}(t) \right) \Gamma_b u_{f1}(t) \bar{r}_{i-1kj}(t) \\ &-\sum_{k=1}^{m} \phi_{bi-1jk}(t)\sigma_b \hat{b}_{0k}(t) \\ &-\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{sl-1k}(t) \right) \Gamma_s \bar{z}_m(t) \bar{r}_{i-1kj}(t) \\ &-\sum_{k=1}^{m} \phi_{si-1jk}(t)\sigma_s \hat{s}_k(t)\ ,\ j \in \{j|\gamma_j \geq i\} \end{aligned} \tag{40a}$$

$$\tilde{\alpha}_{ij}(t) = 0\ ,\ j \in \{j|\gamma_j \leq i-1\} \tag{40b}$$

where $\bar{r}_{ikj}(t)$ is the j element of k row vector of $\bar{R}_{l-1}(t)^T$.

If $\gamma_j = i$ for any j, then, for $j \in \{j|\gamma_j = i\}$, the j element $u_j(t)$ of the control input vector $u(t)$ is given in this step by

$$u_j(t) = u_{fij}(t) = \alpha_{ij}(t)\ ,\ j \in \{j|\gamma_j = i\} \tag{41}$$

and for $j \in \{j|\gamma_j \leq i\}$ the j elements of $u(t)$ are determined by this step i. In the case where there there exist elements such that $\gamma_j \geq i+1$, introduce a new variable $\varepsilon_{i+1}(t) = [\varepsilon_{i+11}(t), \ldots, \varepsilon_{i+1m}(t)]^T$:

$$\varepsilon_{i+1}(t) = u_{fi}(t) - \alpha_i(t). \tag{42}$$

Step γ^* ($\gamma^* = \max_j(\gamma_j)$):

This step is the final step. We determine the remaining control input $u_j(t), j \in \{j|\gamma_j = \gamma^*\}$, as follows:

$$\begin{aligned} u_j(t) &= \alpha_{\gamma^* j}(t) \\ &= \beta_{\gamma^* j}(t) + r_{\gamma^*-1j}(t)^T \Big(\widehat{\Theta}(t) y(t) + \widehat{B}_0(t) u_{f1}(t) \\ &+ \widehat{S}(t)\bar{z}_m(t) \Big) + \sum_{k=1}^{m} \phi_{\theta\gamma^*-1jk}(t)^T \dot{\hat{\theta}}_k(t) \\ &+ \sum_{k=1}^{m} \phi_{b\gamma^*-1jk}(t)^T \dot{\hat{b}}_{0k}(t) + \sum_{k=1}^{m} \phi_{s\gamma^*-1jk}(t)^T \dot{\hat{s}}_k(t) \\ &-\bar{\gamma}_{\varepsilon\gamma^* j}\varepsilon_{\gamma^* j}(t) - \bar{\bar{\gamma}}_{\varepsilon\gamma^* j} \bar{r}_{\varepsilon\gamma^*-1j}(t)^T \varepsilon_{\gamma^*}(t) \\ &+\lambda_{\gamma^*-1j} u_{f\gamma^*-1j}(t) + \tilde{\alpha}_{\gamma^* j}(t)\ ,\ j \in \{j|\gamma_j = \gamma^*\} \\ &(\alpha_{\gamma^* j} = \tilde{\alpha}_{\gamma^* j}(t) = 0,\ \ j \in \{j|\gamma_j \leq \gamma^* - 1\}) \end{aligned} \tag{43}$$

$$\begin{cases} \bar{\gamma}_{\varepsilon\gamma^* j} > 1\ ,\ \bar{\bar{\gamma}}_{\varepsilon\gamma^* j} > 0\ ,\ j \in \{j|\gamma_j = \gamma^*\} \\ \bar{\gamma}_{\varepsilon\gamma^* j} = \bar{\bar{\gamma}}_{\varepsilon\gamma^* j} = 0\ ,\ j \in \{j|\gamma_j \leq \gamma^* - 1\} \end{cases}$$

where $\beta_{\gamma^* j}(t)$ is given in (29) with $i = \gamma^*$, and $r_{\gamma^*-1}(t)$, $\phi_{\theta\gamma^*-1jk}(t)$, $\phi_{b\gamma^*-1jk}(t)$, $\phi_{s\gamma^*-1jk}(t)$ and $\bar{r}_{\varepsilon\gamma^*-1j}(t)$ have been defined in Step$i = \gamma^* - 1$. The auxiliary input is given by

$$\begin{aligned} \tilde{\alpha}_{\gamma^* j}(t) &= -\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{\theta l-1k}(t) \right) \Gamma_\theta y(t) \bar{r}_{i-1kj}(t) \\ &-\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{bl-1k}(t) \right) \Gamma_b u_{f1}(t) \bar{r}_{i-1kj}(t) \\ &-\sum_{k=1}^{m} \left(\sum_{l=3}^{i-1} \varepsilon_l(t)^T \bar{\Phi}_{sl-1k}(t) \right) \Gamma_s \bar{z}_m(t) \bar{r}_{i-1kj}(t), \\ & j \in \{j|\gamma_j = \gamma^*\} \end{aligned} \tag{44a}$$

$$\tilde{\alpha}_{\gamma^* j}(t) = 0\ ,\ j \in \{j|\gamma_j \leq \gamma^* - 1\} \tag{44b}$$

where $\bar{r}_{\gamma^* kj}(t)$ is given in (32) with $i = \gamma^*$. Further, in final step, $\widehat{\Theta}(t), \widehat{B}_0(t)$ and $\widehat{S}(t)$ are adjusted by

$$\begin{cases} \dot{\widehat{\Theta}}(t) = \mathcal{T}_{\theta\gamma^*}(t)\Gamma_\theta - \sigma_\theta \widehat{\Theta}(t) \\ \dot{\widehat{B}}_0(t) = \mathcal{T}_{b\gamma^*}(t)\Gamma_b - \sigma_b \widehat{B}_0(t) \\ \dot{\widehat{S}}(t) = \mathcal{T}_{s\gamma^*}(t)\Gamma_s - \sigma_s \widehat{S}(t) \end{cases} \tag{45}$$

where σ_θ , σ_b and σ_s are 'small' positive constants and $\mathcal{T}_{\theta\gamma^*}(t)$, $\mathcal{T}_{b\gamma^*}(t)$, $\mathcal{T}_{s\gamma^*}(t)$ are given in (36)~(38) with $i = \gamma^*$ respectively.

Finally, the control input is determined as follows:

$$\begin{aligned} u(t) &= (\Delta_1 - \Delta_2)\alpha_1(t) + (\Delta_2 - \Delta_3)\alpha_2(t) \\ &+ \cdots + (\Delta_{\gamma^*-1} - \Delta_{\gamma^*})\alpha_{\gamma^*-1}(t) \\ &+ \Delta_{\gamma^*}\alpha_{\gamma^*}(t). \end{aligned} \tag{46}$$

We have the following theorems concerning the tracking performance and the stability of the control system with control input(46).

Theorem 1: *Under the assumptions 1,2, all the signals in the system composed of control laws in Step1∼Stepγ^* are bounded*

Theorem 2: *In case where the reference input $u_m(t)$ is step, if we chose $\sigma = \sigma_\theta = \sigma_b = 0$, the asymptotic tracking*

$$\lim_{t\to\infty} e_y(t) = 0 \tag{47}$$

is achieved.

4. EXPERIMENTAL EXAMPLE

We will confirm the effectiveness of the proposed method through a experiment by a parallel inverted pendulums device. Fig.1 illustrates the experimental equipment. These inverted pendulums are coupled with springs(Fig.1). The mathematical model of the parallel inverted pendulum from input torque to the angles is given as follows:

$$G(s) = \frac{1}{d(s)}N(s) \tag{48}$$

$$d(s)=s^6 + 3(\zeta^2 - \psi^2)s^2 - \zeta^3 + 3\zeta\psi^2 - \psi^3$$

$$N(s) = \frac{1}{ml^2}\begin{bmatrix} n_1(s) & n_2(s) & n_2(s) \\ n_2(s) & n_1(s) & n_2(s) \\ n_2(s) & n_2(s) & n_1(s) \end{bmatrix}$$

$$n_1(s) = s^4 - 2\zeta s^2 + \zeta^2 - \psi^2,\ n_2(s)=\psi s^2 + \psi^2 - \zeta\psi$$

$$\zeta = \frac{g}{l} - \frac{2ka}{ml^2},\ \psi = \frac{ka}{ml^2}$$

g : gravity acceleration

$$k = 0.042[\text{kg/cm}],\ m = 0.05[\text{kg}]$$

$$,l = 12.0[\text{cm}],\ a=4.0[\text{cm}]$$

Since $G(s)$ is minimum-phase and all relative degree of diagonal elements are two, the number of steps in our design procedure is two. Design parameters in control laws are chosen as follows:

$$\Gamma = \text{diag}\,[20000, 30000, 20000],\ \sigma = 0.01$$

$$\Gamma_\theta = \text{diag}\,[100, 100, 100],\ \sigma_\theta = 0.001$$

$$\Gamma_b = \text{diag}\,[100, 100, 100],\ \sigma_b = 0.001$$

$$\bar{\gamma}_{\epsilon 11} = \bar{\gamma}_{\epsilon 12} = \bar{\gamma}_{\epsilon 12} = 5,\ \bar{\bar{\gamma}}_{\epsilon 11}=\bar{\bar{\gamma}}_{\epsilon 12} = \bar{\bar{\gamma}}_{\epsilon 12} = 2$$

$$\lambda_{11} = \lambda_{12} = \lambda_{13} = 10$$

and the reference model is given by

$$G_m(s) = diag\,[g_{m1}(s), g_{m2}(s), g_{m3}(s)] \tag{49}$$

$$g_{mi}(s) = \frac{1}{s+1},\ i = 1,2,3\ . \tag{50}$$

further the reference inputs are set by

$$u_{mi}(t) = \begin{cases} s_i\ , & (0 < t < 6.5) \\ -s_i\ , & (6.5 \le t \le 10) \end{cases} \quad i = 1,2,3 \tag{51}$$

$$s_1 = 0.2\ ,\ s_2 = 0.15\ ,\ s_3 = -0.2\ .$$

The plant and model output are shown in Fig.2. It is shown that plant outputs track the model outputs very well.

5. CONCLUSIONS

In this paper, we proposed the design method of a model reference adaptive control using backstepping strategy for MIMO plants. The effectiveness of the proposed method confirmed through the experimental example for the parallel inverted pendulums.

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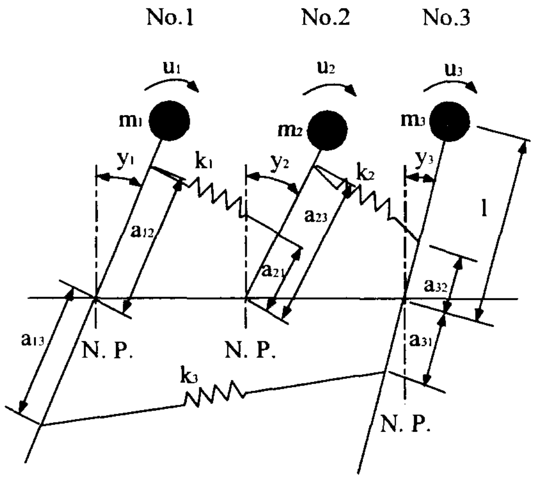

Fig.1 A parallel inverted pendulums

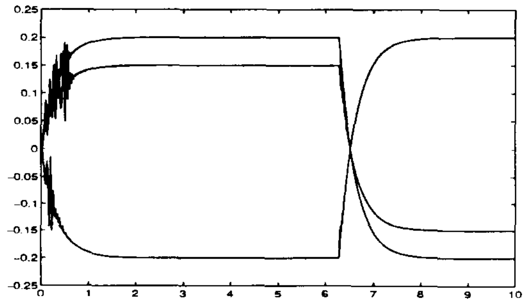

Fig.2 The plant and model outputs