Proceedings of the 11th KACC, October 1996

On the Robust Adaptive Linearizing Control for Unknown and Analytic Relay Nonlinearity

Jae-kwan Lee and Ken-ichi Abe

Department of Electrical Engineering, Faculty of Engineering, Tohoku University, Sendai 980-77, JAPAN.
Tel:+81-22-217-7075; Fax:+81-22-263-9290; E-mail:lee@abe.ecei.tohoku.ac.jp

Abstract : *The purpose of this paper is to design a robust adaptive control algorithm for a class of systems having continuous relay nonlinearity. This continuous relay nonlinearity can be defined as an analytic nonlinear function having unknown parameters and bounded unmodeling part. By this mathematical modeling, the whole system can be considered as a nonlinear system having unknown parameters and bounded perturbation. The control algorithm of this paper,* **RALC**, *can be constructed by robust adaptive law, feedback linearization, and indirect robust adaptive control. By this* **RALC**, *we can obtain that the output of given system can follow that of a stable reference linear model made by designer and the boundedness of all signals in closed-loop system can be maintained. Therefore, we can confirm a robust adaptive control for a class of systems having continuous relay nonlinearity.*

Keywords : Continuous relay nonlinearity, Robust adaptive law, Feedback linearization, Indirect adaptive controller, Computer simulation

1. Introduction

In many applications of control theories, we are confronted with handling for continuous relay nonlinearity. This property is usually caused by sensors and actuators, which are essential components on control system performances. Up to the present, this nonlinearity has been studied by linear method[1,2,3]. This linear method is the method that assumes the nonlinearity to be described by a linear system with unmodeling part and then applies a proper linear control theory for the linearized system. In this method, however, if the linearized system has large unmodeling part, the closed-loop system having linear controller will easily lead to instability. Therefore, in the presence of this nonlinearity, this defect is difficult to be derived from the linear method. Furthermore, when parameter parts in modeling are not completely known, any regulation design for whole system may be complex increasingly.

Hence, such complex situations demand a new controller which has a combination construction of adaptive control theory[2,3], robust control theory[1,2] and nonlinear system theory[4,5].

The main purpose of this paper is to design such a controller for a class of systems having continuous relay nonlinearity. This continuous relay nonlinearity can be defined as an analytic nonlinear function having unknown parameters and unmodeling part. By this mathematical modeling, the whole system can be considered as a nonlinear system having unknown parameters and bounded perturbation. This controller is studied under the following assumptions: 1) the output and state vector of given system are observable, 2) the reference input satisfies persistent excitation during parameter estimation, 3) the relay nonlinearity exists in analytic vector field, and 4) the slope range of relay nonlinearity is known in advance.

The designed controller is called Robust Adaptive Linearizing Controller, **RALC**, which is constructed by robust adaptive law[1,2], feedback linearization[4,5,7], and indirect adaptive controller[3].

This paper is organized as follows. In Section 2, we develop theories for design and analysis of RALC. In Section 3, we present simulation results for the proposed control scheme. In Section 4, we summarize our conclusions and outline future researches.

2. RALC

The continuous relay nonlinearity $f_{cr}(\cdot)$ can be considered as characteristic like the following figure, approximately.

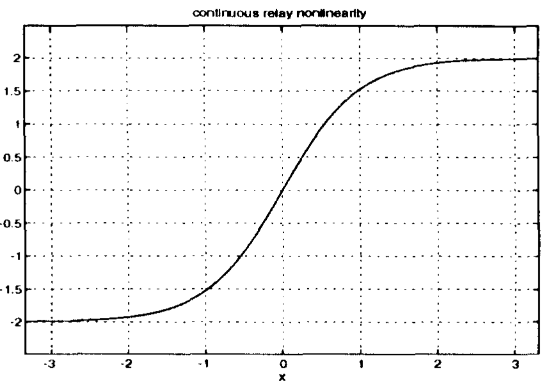

Figure 1: Continuous relay nonlinearity

As seen from Fig. 1, the nonlinearity can be classified as C^∞ vector field and can be represented as an analytic nonlinear function $f_{Rnf}(\cdot)$ having unknown parameter matrix a and bounded unmodeling part $\Delta f_{cr}(\cdot)$:

$$f_{cr}(x_p) \equiv a\, f_{Rnf}(x_p) + \Delta f_{cr}(x_p) \tag{1}$$

Let us now describe the whole system including the nonlinearity by the following nonlinear differential equation:

$$\begin{aligned} \dot{x}_p &= f_n(x_p) + a\, f_{Rnf}(x_p) + \Delta f_{cr}(x_p) + g_n(x_p)\, v \\ y_p &= h_n(x_p) \end{aligned} \tag{2}$$

where $x_p \in \Re^n$ is the observable state vector, $y_n \in \Re$ is the measured output, f_n, g_n, h_n are observable C^∞ vector fields, and $v : \Re^+ \rightarrow \Re$ is a bounded piecewise continuous input.

The main object of our study in the rest of this paper is that the output y_p is required to track a desired output y_{lm} of the following stable reference linear model:

$$\begin{aligned} \dot{x}_{lm} &= a_{lm}\ x_{lm} + b_{lm}\ r \\ y_{lm} &= c_{lm}^T\ x_{lm} \end{aligned}$$

or

$$y_{lm}(s) = G_{lm}(s)\ r(s) = k_{lm}\ \frac{Z_{lm}(s)}{P_{lm}(s)}\ r(s) \tag{3}$$

where a_{lm} is a n×n asymptotically stable matrix.

In order to achieve such a purpose, we can construct the following block diagram.

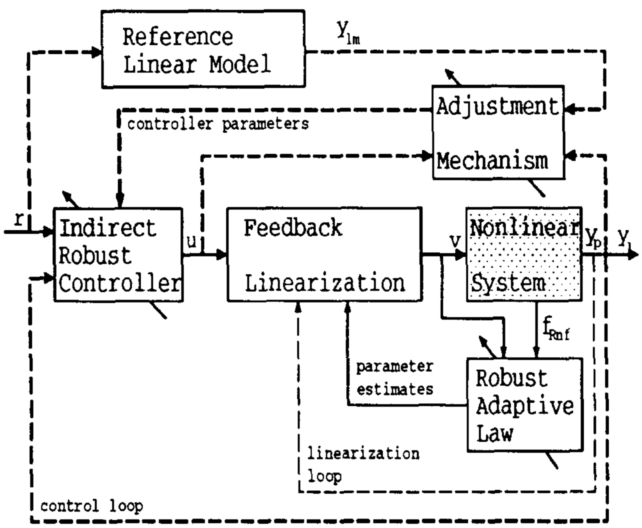

Figure 2: Block diagram of **RALC**

In Fig. 2, the parameter estimator having robust adaptive law[1,2] works as the part to estimate unknown parameter a, the feedback linearization[4,5,7] part replaces previous nonlinear differential equation with an equivalent linear differential equation, and the last part is the indirect robust controller[3] which calculates control parameters to satisfy control goal.

The control algorithm developed in this paper, **RALC**, can be explained as the following order.

2.1 Parameter Estimator

Using the Laplace transform, we can represent the system (2) as the following $s-domain$ form:

$$(s+1)\ \bar{x}_{ni} = \bar{f}_n + a\ \bar{f}_{Rnf} + \Delta\bar{f}_{cr} + \bar{g}_n\ v + \bar{x}_p \tag{4}$$

where $\bar{x}$, $\bar{f}_n$, $\bar{f}_{Ref}$, $\Delta\bar{f}_{cr}$ and $\bar{g}_n$ describe $s-domain$ representations for x, f_n, f_{Rnf}, Δf_{cr} and g_n, respectively.

This equation can be arranged as

$$\begin{aligned} \bar{x}_p &= \frac{1}{s+1}\ [([I\ a\ I]\ [\bar{f}_n\ \bar{f}_{Rnf}\ \bar{x}_p]^T + \bar{g}_n\ v) \\ &\quad + \Delta\bar{f}_{cr}] \end{aligned} \tag{5}$$

Then we can define the following linear expression having strictly proper SPR transfer function[3] $W(s) \equiv \frac{1}{s+1}$, time invariant parameter matrix $\theta^{*T} \equiv [I\ a\ I]$, state variable matrix $\omega \equiv [\bar{f}_n\ \bar{f}_{Rnf}\ \bar{x}_p]^T$, and piecewise continuous signal input $z_0 \equiv \bar{g}_n\ v$:

$$\bar{x}_p = W(s)\ [\theta^{*T}\ \omega + z_0 + \Delta\bar{f}_{cr}] \tag{6}$$

According to the linear expression, the estimate $\hat{x}_p$ of $\bar{x}_p$ can be constructed as

$$\hat{x}_p = W(s)\ [\theta^T\ \omega + z_0] \tag{7}$$

where θ is the estimate result of θ^* at time t.

By the results, the estimation error ϵ_1 and the normalized estimation error ϵ can be defined as the following form:

$$\epsilon_1 \equiv \hat{x}_p - \bar{x}_p \tag{8}$$

$$\begin{aligned} \epsilon &\equiv \epsilon_1 - W(s)\ [\epsilon\ n_s^2] \\ &= \hat{x}_p - \bar{x}_p - W(s)\ [\epsilon\ n_s^2] \\ &= W(s)\ [\phi^T\ \omega - \epsilon\ n_s^2 - \Delta\bar{f}_{cr}] \end{aligned} \tag{9}$$

where $\phi \equiv \theta - \theta^*$ and n_s is called the parameter error and the normalizing signal, respectively.

In the next place, we can transfer this transfer function form into a state space form:

$$\begin{aligned} \dot{e} &= a_c\ e + b_c\ (\phi^T\ \omega - \epsilon\ n_s^2 - \Delta\bar{f}_{cr}) \\ \epsilon &= h_c\ e \end{aligned} \tag{10}$$

where $(h_c,\ a_c,\ b_c)$ is a minimal state representation of $W(s) = h_c\ (sI - a_c)^{-1}\ b_c$.

The adaptive law for generating parameter estimate $\theta(t)$ is developed by considering the next Lyapunov function:

$$V(\phi, e) = \frac{e^T\ P_c\ e}{2} + \frac{\phi^T\ \Gamma^{-1}\ \phi}{2} \tag{11}$$

where $\Gamma = \Gamma^T > 0$ and $P_c = P_c^T > 0$. The matrix P_c is constructed so as to satisfy the following relation:

$$\begin{aligned} P_c\ a_c + a_c^T\ P_c &= -q\ q^T - \nu\ L \\ P_c\ b_c &= h_c \end{aligned} \tag{12}$$

for some vector $q, L = L^T > 0$ and a small constant $\nu > 0$.

Then, by choosing

$$\dot{\phi} = \dot{\theta} = -\ \Gamma\ \epsilon\ \omega - \Gamma\ w(t)\ \theta \tag{13}$$

where $w(t) \geq 0$ is called the leakage and a signal to be designed by

$$w(t) \equiv |\epsilon\ m|\ \nu_o \tag{14}$$

where $\nu_o > 0$ is a design constant, we can obtain $\epsilon,\ \theta \in L_\infty$ and $\dot{V}(\phi,\ e) < 0$.

2.2 Feedback Linearization

Let x_{ne} be a singular point of $\check{f}_n$. We shall assume $\Delta f_{cr}(x_{ne}) = 0$, $h_n(x_{ne}) = 0$, and that the following differential equation:

$$\begin{aligned} \dot{x}_p &= \check{f}_n(x_p) \equiv f_n(x_p) + a\ f_{Rnf}(x_p) \\ y_p &= h_n(x_p) \end{aligned} \tag{15}$$

satisfies the condition

$$rank\{d(L_{\check{f}_n}^j\ h_n(x_{ne}) :\ 0 \leq j \leq n-1\} = n \tag{16}$$

which implies the local observability condition[6,8,9].

Then, we can define a neighborhood vector field κ of x_{ne} satisfying

$$\langle L^j_{f_n} dh_n, \kappa \rangle = \begin{cases} 0 & \text{for } 0 \le j \le n-2 \\ 1 & \text{for } j = n-1 \end{cases} \tag{17}$$

If a global state space diffeomorphism T[6,8,9]

$$x_l = T(x_n),\ T(0) = 0,\ x_l \in \Re^n \tag{18}$$

where

$$T^{-1}(x_l) = \Phi^{ad^{n-1}_{-f_n}\kappa}_{x_{l1}} \circ \Phi^{ad^{n-2}_{-f_n}\kappa}_{x_{l2}} \circ \ldots \circ \Phi^{\kappa}_{x_{ln}}(x_{ne})$$

satisfies the following conditions

- $[ad^i_{f_n}\kappa,\ ad^j_{f_n}\kappa] = 0, \text{for } 0 \le i, j \le n-1$
- $[ad^i_{f_n}\kappa,\ g_n] = 0, \text{for } 0 \le i \le n-2$
- $[ad^i_{f_n}\kappa,\ \Delta f_{cr}] = 0, \text{for } 0 \le i \le n-2,$

the nonlinear differential equation (2) can be transformed into the following equivalent linear differential equation having another bounded perturbation $\Delta_l \in L_\infty$:

$$\begin{aligned} \dot{x}_l &= A_l\, x_l + \Delta_l + b_l\, u \\ y_l &= y_n = c_l^T\, x_l \end{aligned} \tag{19}$$

2.3 Indirect Robust Controller

The objective of indirect robust controller[2,8] is to control the output of perturbation-free part for the equivalent linear system (19) to track that of the stable reference linear model (3) without influence of bounded perturbation.

In order to achieve such a goal, an indirect robust controller can be constructed as follows

$$\begin{aligned} \dot{w}_{c1}(t) &= \Lambda\, w_{c1}(t) + \iota\, u(t) \\ \dot{w}_{c2}(t) &= \Lambda\, w_{c2}(t) + \iota\, y_l(t) \\ u(t) &= \theta_c^T(t)\, w_c(t) \end{aligned} \tag{20}$$

with

$$\begin{aligned} w_c(t) &\equiv [r(t),\ w_{c1}^T(t),\ y_l(t),\ w_{c2}^T]^T \\ \theta_c(t) &\equiv [k_c(t),\ \theta_{c1}^T(t),\ \theta_{c3}(t),\ \theta_{c2}^T(t)]^T \end{aligned}$$

where $k_c : \Re^+ \to \Re,\ \theta_{ci}(t) : \Re^+ \to \Re^{n-1},\ i = 1,\ 2,\ 3$ are the controller parameters which are determined at each time t, and $(\Lambda,\ \iota)$ is controllable.

Then an adaptive control law[2,8] for control parameters can be designed as the next differential equation:

$$\begin{aligned} \dot{k}_c(t) &= -sgn(k_{lp})\, e_y(t)\, r(t) \\ &\quad -\gamma\ |e_y(t)|\ k_c(t) \\ \dot{\theta}_{c3}(t) &= -sgn(k_{lp})\, e_y(t)\, y_l(t) \\ &\quad -\gamma\ |e_y(t)|\ \theta_{c3}(t) \\ \dot{\theta}_{c1}(t) &= -sgn(k_{lp})\, e_y(t)\, w_{c1}(t) \\ &\quad -\gamma\ |e_y(t)|\ \theta_{c1}(t) \\ \dot{\theta}_{c2}(t) &= -sgn(k_{lp})\, e_y(t)\, w_{c2}(t) \\ &\quad -\gamma\ |e_y(t)|\ \theta_{c2}(t) \end{aligned} \tag{21}$$

where

$$e_y(t) \equiv y_l(t) - y_{lm}(t)$$

$$sgn(x) \equiv \begin{cases} +1 & \text{if } x \ge 0 \\ -1 & \text{if } x < 0 \end{cases}$$

3. Simulation and Results

The system to be controlled has linear part with unknown parameters and continuous relay characteristic $f_{cr}(\cdot)$

$$\begin{aligned} \dot{x}_{p1}(t) &= a_{11}\ x_{p1}(t) + a_{12}\ x_{p2}(t) + v(t) \\ \dot{x}_{p2}(t) &= a_{21}\ x_{p2}(t) + a_{22}\ x_{p2}(t) + f_{cr}(x_{p2}) \\ y_p(t) &= 2\ x_{p1}(t) + x_{p2}(t) \end{aligned} \tag{22}$$

where $x_p(t) \equiv [x_{p1}(t)\ x_{p2}(t)]^T$ is the observable state vector, $v(t)$ is the input, $y_p(t)$ is the measured output, and a_{ii}, $i = 1,\ 2$ are unknown parameters given by $a_{11} = -1.5$, $a_{12} = 1$, $a_{21} = 1$ and $a_{22} = -2$.

If we assume the continuous relay nonlinearity as a nonlinear function $f_{Rnf}(\cdot)$ and a bounded unmodeling part $\Delta f_{cr}(\cdot)$

$$f_{cr}(x_{p2}) \equiv a\ f_{Rnf}(x_{p2}) + \Delta f_{cr}(x_{p2}) \tag{23}$$

where

$$\begin{aligned} f_{Rnf}(x_{p2}) &\equiv \left(\frac{e^{b\ x_{p2}} - e^{b\ x_{p2}}}{e^{b\ x_{p2}} + e^{b\ x_{p2}}}\right) \\ &= \left(1 - \frac{2}{e^{2\ b\ x_{p2}} + 1}\right) \\ b &= 1, \end{aligned}$$

then given system can be considered as a nonlinear system having unknown parameters, a and a_{ii}, $i = 1,\ 2$ and bounded perturbation $\Delta f_{cr}(\cdot)$.

Here, the main purpose of **RALC** is to control the output $y_p(t)$ to track the output $y_{lm}(t)$ of a stable reference linear model defined as follows

$$\begin{aligned} \dot{x}_{lm1}(t) &= -2\ x_{lm1}(t) - x_{lm2}(t) \\ \dot{x}_{lm2}(t) &= x_{lm1}(t) - 3\ x_{lm2}(t) + r(t) \\ y_{lm}(t) &= x_{lm1}(t) + 2\ x_{lm2}(t) \end{aligned} \tag{24}$$

where $r(t) = 5\ sin(t) + 7\ cos(3t)$.

We can know that, when **RALC** is not worked, $y_p(t)$ does not track to $y_{lm}(t)$ from Fig. 3.

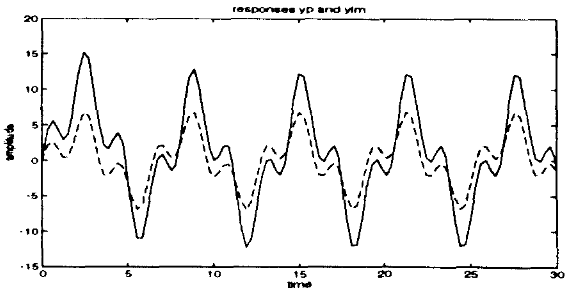

Figure 3: $y_p(t)$ and $y_{lm}(t)$ (RALC-$\times$: solid line $= y_p(t)$ and dashed line $= y_{lm}(t)$)

According to **RALC**, some responses for closed-loop system are shown in Fig. 4 $\sim$ Fig. 7, respectively. It is clear that with **RALC**, the tracking error e_y can be significantly reduced.

4. Conclusions

By the controller of this paper, we can obtain the following results.

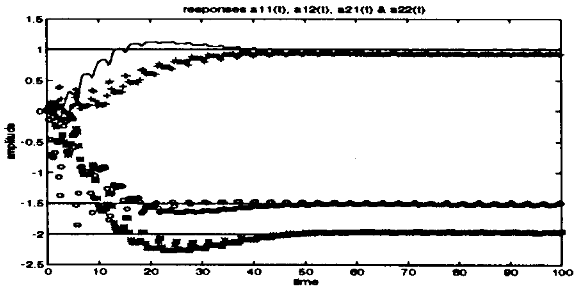

Figure 4: System responses : parameter estimation result (1) - $a_{11}(t)$, $a_{12}(t)$, $a_{21}(t)$ and $a_{22}(t)$ (RALC-$\circ$: $\circ = a_{11}(t)$, $+ = a_{12}(t)$, $-- = a_{21}(t)$, $* = a_{22}(t)$ and solid line = a_{11}, a_{12}, a_{21} & a_{22})

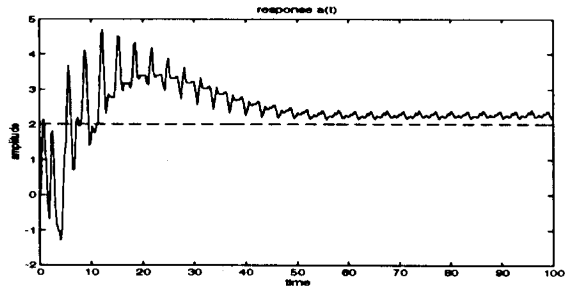

Figure 5: System response : parameter estimation result (2) - $a(t)$ (RALC-$\circ$: solid line = $a(t)$ and dashed line = a)

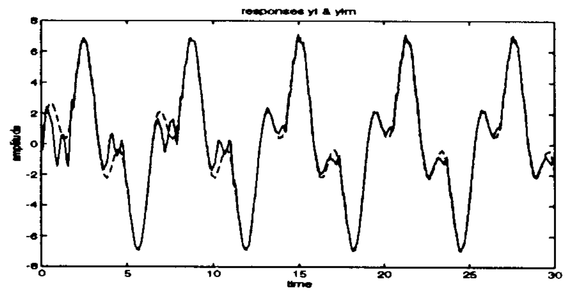

Figure 6: System responses : closed-loop system result (1) - $y_l(t)$ and $y_{lm}(t)$ (RALC-$\circ$: solid line = $y_l(t)$ and dashed line = $y_{lm}(t)$)

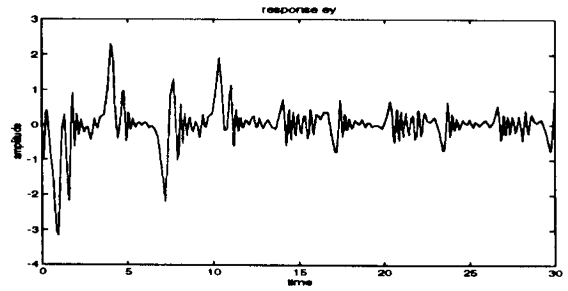

Figure 7: System response : closed-loop system result (2) - $e_y(t)$ (RALC-$\circ$)

First, when estimated parameters are taken into feedback linearization part repeatedly, the whole system can be transferred to an equivalent linear system with another bounded perturbation form. Then, the robust adaptive control for the equivalent linear system having perturbation can be possible, that is, the transfer function of perturbation-free equivalent linear system can follow to that of a stable reference linear model made by designer.

Second, while the output of closed-loop system is tracking that of given stable reference linear model, the boundedness of all signals in estimator and controller can be satisfied. Therefore, we can get a robust adaptive control for a class of systems having continuous relay nonlinearity.

As a future research, there is an application for electronic control unit, **ECU**, of automobile engine.

References

[1] K. S. Narendra and A. M. Annaswamy, *Robust adaptive control in the presence of bounded disturbances*, IEEE Trans. Automat. Contr., vol. 31, pp. 306-315, April 1987.

[2] P. A. Ioannou and A. Datta, *Robust Adaptive Control: A Unified Approach*, IEEE, vol. 79, no. 12, pp. 1736-1768, Dec. 1991.

[3] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-hall Inc., 1989.

[4] A. Isidori, *Nonlinear Control Systems*, 3nd., Springer-Verlag, 1993.

[5] M. Vidyasagar, *Nonlinear Systems Analysis*, Prentice-hall Inc., 1993.

[6] R. Marino and P. Tomei, *Global adaptive output-feedback control of nonlinear systems, Part I: Linear parameterization*, IEEE Trans. Automat. Contr., vol. 38, no. 1, pp. 17-32, 1993.

[7] J. -K. Lee and K. -I. Abe, *Model reference adaptive control of nonlinear system using feedback linearization*, SICE'95, July, 1995.

[8] K. -H. Nam, *An output feedback stabilization scheme using the filtered coordinate transformation*, IEEE Trans. Automat. Contr., vol. 40, no. 11, Nov. 1995.

[9] J. -K. Lee and K. -I. Abe, *Robust adaptive control for nonlinear system with bounded perturbations*, SICE'96, July, 1996.