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A FREQUENCY DOMAIN ADAPTIVE PID CONTROLLER BASED ON NON-PARAMETRIC PLANT MODEL REPRESENTATION

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Abstract In this paper, we propose a design method of PID adaptive controller based on frequency domain analysis. The method is based on the estimation of a nonparametric process model in the frequency domain and the determination of the PID controller parameters by achieving partial model matching so as to minimize a performance function concerning to relative model error between the loop transfer function of the control system and the desired system. In the design method the process is represented only by a discrete set of points on the Nyquist curve of the process. Therefore it is not necessary to estimate a full order parameterized process model.

Keywords Adaptive control, Frequency domain, PID control, Partial model matching, Least squares estimation

1. INTRODUCTION

Many of the general purpose adaptive controllers have been designed based on process models in terms of transfer functions or parametric representations. This is why the controllers have some severe disadvantages. That is, the estimation is sensitive to unmodelled dynamics, it is necessary to have prior information about plant order, and so on. A method to avoid the problems is to represent the process by a simple nonparametric representation defined on some frequency domain points. Then the estimation of process model results in tracking the transfer function at a few frequencies in some range.

By the way, automatic tuning of PID controllers is well established as industrial controllers and experience has shown that PID controllers can handle many of the industrial problems [2]. Especially, many design procedures use given or estimated information about the points on Nyquist curve (for example, gain margin or phase margin) [1],[3],[5]. However, those PID controllers have the disadvantages of the need of large setpoint change to trigger tuning and the use of a large number of tuning rules. It is also difficult to keep good control performance when dynamic characteristics of the processes change without tuning again.

This paper gives a design method of PID adaptive controller based on a frequency domain identification. The estimation of a process is carried out for a nonparametric representation expressed by some frequency domain points. The tuning rule of the parameters of PID controller is carried out by optimally fitting the Nyquist curve of the loop transfer function to that of pre-chosen optimal loop transfer function on some frequency domain points. The effectiveness of the proposed method is evaluated by a numerical simulation.

2. PROBLEM FORMULATION

Consider the following closed-loop system:

$$G_{cl}(s) = \frac{G(s)C(s)}{1+G(s)C(s)} \tag{1}$$

where $G(s)$ is the stable plant and $C(s)$ is the PID controller which is given by

$$C(s) = a + \frac{b}{s} + cs \tag{2}$$

where a, b and c are the proportional, integral and derivative gains, respectively.

The desired closed-loop transfer function is given by

$$\widetilde{G}_m(s) = \frac{1}{1+\tau s+\alpha_2(\tau s)^2+\cdots+\alpha_n(\tau s)^n}\ . \tag{3}$$

If the coefficients τ and $\alpha_k (k = 2, 3, \ldots, n)$ are chosen appropriately, $\widetilde{G}_m(s)$ gives an optimal form which corresponds to binomial model, Butterworth model, and so on. Assume that the desired feedback closed-loop system has unity feedback, then we can obtain the desired open-loop transfer function as follows,

$$G_m(s) = \frac{1}{\tau s + \alpha_2(\tau s)^2 + \cdots + \alpha_n(\tau s)^n} \,. \tag{4}$$

The objective is to design the PID controller such that the actual open-loop frequency response $G(j\omega)C(j\omega)$ matches to the pre-chosen desired open-loop frequency response $G_m(j\omega)$ on some frequency domain points.

3. TUNING OF PID CONTROLLER

The objective here is to choose the parameters a, b and c such that the loss function [4]

$$J(\boldsymbol{\theta}) = \sum_{k=1}^{M} |\varepsilon(j\omega_k)|^2 + \sum_{k=1}^{M} |\varepsilon(-j\omega_k)|^2 \tag{5}$$

is minimized with respect to $\boldsymbol{\theta}$, where $\varepsilon(j\omega_k)$ is the relative open-loop model error defined [6] by

$$\varepsilon(j\omega_k) = \frac{G_m(j\omega_k) - G(j\omega_k)C(j\omega_k)}{G_m(j\omega_k)} \tag{6}$$

and $\boldsymbol{\theta}$ represents a parametrization of the PID controller parameters. The frequency ω_k is chosen by the user. It can be chosen quite freely, but it should be chosen in the cut-off frequency region of reference model and the frequency region which actually should be attached importance. Because the cut-off frequency characteristics indicate the stability of control system and its robustness.

Rewriting the expression (6) for $\varepsilon(j\omega_k)$ gives

$$\begin{aligned}\varepsilon(j\omega_k) &= 1 - \frac{G(j\omega_k)}{G_m(j\omega_k)}C(j\omega_k) \\ &= 1 - \frac{G(j\omega_k)}{G_m(j\omega_k)}\left(a + \frac{b}{j\omega_k} + j\omega_k c\right) \\ &= 1 - \boldsymbol{\phi}(\omega_k)^T\boldsymbol{\theta} \end{aligned}\tag{7}$$

where

$$\begin{aligned}\boldsymbol{\phi}(\omega_k) &= \left[A(j\omega_k),\ \frac{1}{j\omega_k}A(j\omega_k),\ j\omega_k A(j\omega_k)\right]^T \\ A(j\omega_k) &= \frac{G(j\omega_k)}{G_m(j\omega_k)} \\ \boldsymbol{\theta} &= [a,\ b,\ c]^T \,.\end{aligned}$$

By using the error regression model (7), the loss function (5) can be rewritten as

$$J(\boldsymbol{\theta}) = (\boldsymbol{\psi}_c - \Phi_c\boldsymbol{\theta})^*(\boldsymbol{\psi}_c - \Phi_c\boldsymbol{\theta}) \tag{8}$$

where

$$\begin{aligned}\Phi_c &= [\boldsymbol{\phi}(\omega_1), \cdots, \boldsymbol{\phi}(\omega_M), \boldsymbol{\phi}(-\omega_1), \cdots, \boldsymbol{\phi}(-\omega_M)]^T \\ \boldsymbol{\psi}_c &= [1, \cdots, 1]^T\end{aligned}$$

and Φ^* denotes the complex conjugate transpose of Φ. The explicit solution is given by

$$\widehat{\boldsymbol{\theta}} = (\Phi_c^*\Phi_c)^{-1}\Phi_c^*\boldsymbol{\psi}_c \,. \tag{9}$$

Using the complex conjugate matrix Φ_c gives a real valued solution of $\widehat{\boldsymbol{\theta}}$ in (9) [4].

Remark:
Since the element $\frac{A(j\omega_k)}{j\omega_k}$ in $\boldsymbol{\phi}(\omega_k)$ has $1/\omega_k$, we cannot apply the least squares at $\omega_k = 0$. However, we can solve the problem if $A(j\omega_k)$ are transformed as follows

$$\begin{aligned}A(j\omega_k) &= \frac{G(j\omega_k)}{G_m(j\omega_k)} \\ &= \frac{G(j\omega_k)}{\frac{1}{j\omega_k}G'_m(j\omega_k)} \\ &= j\omega_k A'(j\omega_k)\end{aligned}\tag{10}$$

where

$$\begin{aligned}G'_m(j\omega_k) &= j\omega G_m(j\omega_k) \\ A'(j\omega_k) &= \frac{G(j\omega_k)}{G'_m(j\omega_k)} \,.\end{aligned}$$

4. PARAMETER ESTIMATION ON FREQUENCY DOMAIN

In the previous section, a tuning method of PID controller is described using the information on some frequency domain points of the process. In this section, it will be given an on-line estimation method using frequency domain approach [2] to keep good performance under the change of dynamic characteristics of the process.

The block diagram describing the identification procedure is shown in Fig.1, where u_m is the reference input, and, u and y are the control input and process output, respectively. $\bar{u}$ and $\bar{y}$ are the signals of which u and y are filtered through narrow band-pass filters at the frequency ω_k. The signals $\bar{u}$ and $\bar{y}$ are used by a recursive least-squares estimator which gives an estimate of $G(j\omega_k)$.

4.1 Band-pass filters

The band-pass filter is given by

$$G_{bp}(s) = \frac{s}{s^2 + 2\zeta\omega_k s + \omega_k^2} \ . \quad (11)$$

This filter will give a relatively high gain at the frequency ω_k, and suppress the signals at other frequencies.

4.2 Parameter estimation

The band-pass filters centered at the frequency ω_k produce two signals which can be approximated with two sinusoidal waves having different amplitudes and phases. Then we can estimate $G(j\omega_k)$ by using the method proposed by Hägglund and Åström [2]. Let

$$G(j\omega_k) = \alpha e^{-j\varphi} \quad (12)$$

Then, we can use the recursive least squares algorithm in order to estimate the parameters α and φ. This is done by estimating parameters of the second order model

$$\bar{y}(t) = b_1\bar{u}(t - h_k) + b_2\bar{u}(t - 2h_k) \quad (13)$$

where h_k is the sampling period which is given by

$$h_k = \frac{2\pi}{8\omega_k} \ .$$

The equation (13) can be rewritten as

$$\begin{aligned} \bar{y}(t) &= \boldsymbol{z}^T(t)\boldsymbol{b} \quad (14)\\ \boldsymbol{b} &= [b_1, \ b_2]^T \\ \boldsymbol{z}(t) &= [\bar{u}(t - h_k), \ \bar{u}(t - 2h_k)]^T \end{aligned}$$

By using $\widehat{\boldsymbol{b}}(t)$, the estimate for $\boldsymbol{b}$, the estimated model is defined as

$$\widehat{y}(t) = \boldsymbol{z}^T(t)\widehat{\boldsymbol{b}}(t) \ . \quad (15)$$

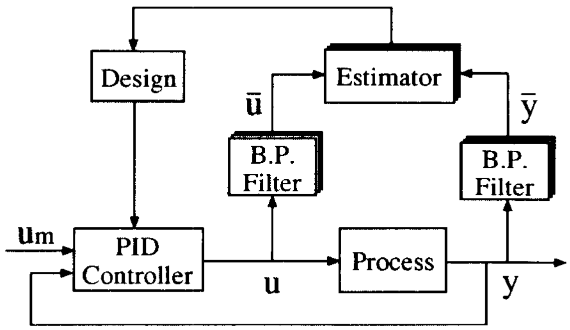

Fig.1: Block diagram describing the identification procedure

The equations (14) and (15) give the following error equation.

$$\begin{aligned} \epsilon(t) &= \widehat{y}(t) - \bar{y}(t) = \boldsymbol{z}^T(t)\boldsymbol{\vartheta}(t) \quad (16)\\ \boldsymbol{\vartheta} &= \widehat{\boldsymbol{b}}(t) - \boldsymbol{b} \end{aligned}$$

Then $\widehat{\boldsymbol{b}}(t)$ can be found by using the recursive least-squares algorithm. Using $\widehat{\boldsymbol{b}}(t) = [\widehat{b}_1, \ \widehat{b}_2]^T$, the amplitude $\widehat{\alpha}$ and phase shift $\widehat{\varphi}$ of the transfer function $G(j\omega_k)$ are given by

$$\widehat{\varphi} = \tan^{-1}\left(\frac{\widehat{b}_1 \sin(\omega_k h_k)}{\widehat{b}_1 \cos(\omega_k h_k) + \widehat{b}_2}\right) - 2\omega_k h_k \quad (17)$$

$$\widehat{\alpha} = \frac{\widehat{b}_1 \sin(\omega_k h_k)}{\sin(2\omega_k h_k + \widehat{\varphi})} \ . \quad (18)$$

It is noted that we need parallel filters and parallel estimators to estimate the frequency response $G(j\omega_k)$ at several frequencies.

5. SIMULATION RESULTS

In this section, the effectiveness of the proposed method is confirmed through a numerical simulation.

Consider the following process and reference model.

Controlled process:

$$G(s) = \frac{1}{(s + a_1)(s + a_2)(s^2 + a_3 s + a_4)}$$

$$0 \le t \le 150 : a_1 = 2, \ a_2 = 3, \ a_3 = 2, \ a_4 = 2$$

$$t > 150 : a_1 = 2 \times 0.9, \ a_2 = 3 \times 0.9, \ a_3 = 2 \times 0.9^2, \ a_4 = 2 \times 0.9^2$$

Reference model:

$$\widetilde{G}_m(s) = \frac{1}{(s+1)^3}$$

$u_m(t)$: rectangular wave with amplitude 1

Here fitting frequencies were chosen as follows

$$\omega_1 = 0.1, \ \omega_2 = 0.3, \ \omega_3 = 0.74$$

In order to attain good estimation performance at $\omega_1 = 0.1$, the sampling period was modified as

$$h_1 = \frac{1}{4} \cdot \frac{2\pi}{8\omega_1} \ .$$

The process and reference output responses are shown in Fig.2. Though the process dynamics changes at $t = 150$, good performance is obtained. Fig.3 shows the time evolution of the frequency point estimates. The dashed curves are the true Nyquist curves of process. It is shown that the estimates follow the change in process dynamics. The curve fitting to the desired Nyquist plots at $t = 400$ shown in Fig.4 seems to be good enough.

6. CONCLUSIONS

This paper has presented a design method of PID adaptive controller which is based on frequency domain identification. In this method we can easily confirm the stability of control system and its robustness since the proposed algorithm is based on the open-loop Nyquist plots. A numerical simulation has been given to illustrate the effectiveness of the proposed method.

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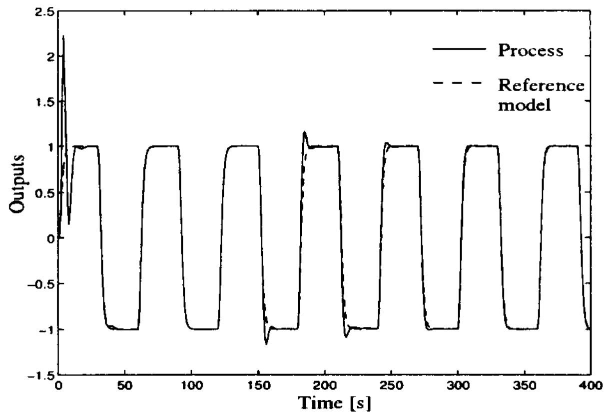

Fig.2: Process and reference outputs

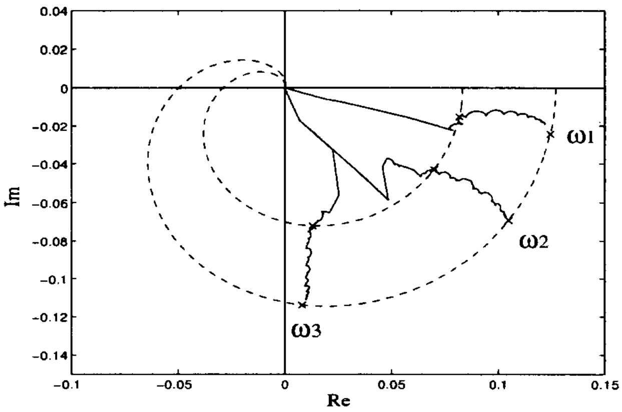

Fig.3: Time evolution of the frequency point estimates

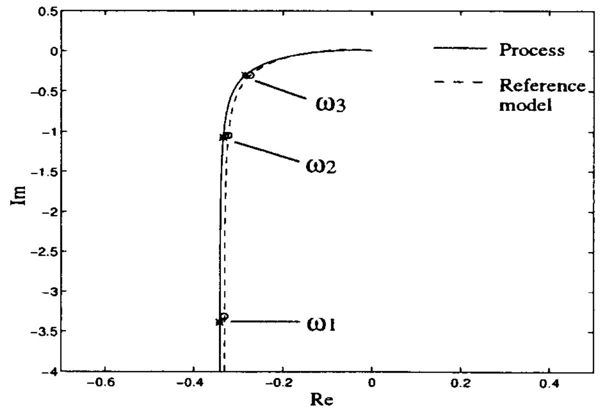

Fig.4: Nyquist curves of the desired and obtained open-loop system at $t = 400$