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IMPROVED EXTENDED KALMAN FILTER DESIGN FOR RADAR TRACKING

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Abstracts A new filtering algorithm for radar tracking is developed based on the fact that correct evaluation of the measurement error covariance can be made possible by doing it with respect to the Cartesian state vector. The new filter may be viewed as a modification of the extended Kalman filter where the variance of the range measurement errors is evaluated in an adaptive manner. The structure of the proposed filter allows sequential measurement processing scheme to be incorporated into the scheme, and this makes the resulting algorithm favorable in both estimation accuracy and computational efficiency.

Keywords State estimation, Radar tracking, Nonlinear estimation, Sequential processing, Estimation accuracy

1. INTRODUCTION

The problem of tracking a moving target is a very interesting application area for state estimation and has received a great deal of treatment in the literature for nearly three decades. Usually the target dynamics can be well described in a Cartesian coordinate frame. Since the measurements made in radar-centered polar coordinates are expressed as nonlinear equations in Cartesian coordinates, the tracking problem is connected with nonlinear estimation. There have been two common approaches to this problem. One method is to apply a linear Kalman filter after converting polar measurements to a Cartesian frame. This method will be referred to as converted measurement Kalman filter (CMKF). The other approach is to use the extended Kalman filter (EKF). Although both of the methods perform well in many cases, it has turned out that they may yield unacceptably large biases or inconsistent estimation results in certain applications [3-5].

Lerro and Bar-Shalom [4] presented a debiasing procedure which accounts for approximation errors of the conventional CMKF. The idea was to modify the expressions for the first two moments of the converted measurements so that the modified expressions may match their true statistics. It was shown in [4] that their modified CMKF (denoted MCMKF) yields consistent estimates at different situations unlike the conventional CMKF and EKF .

In this paper, we propose a new tracking algorithm which is different from the MCMKF. The newly developed filter may be viewed as a modification of the EKF. The derivation of the filter is based in part on a new observation on the relationship between CMKF and EKF. Incorporating the sequential measurement processing scheme introduced in [5] into the new filter reduces the computational requirement of the resulting filter, which makes our developed filter more efficient than the MCMKF. Comparison results obtained from computer simulations using the proposed and other existing methods are presented to show the effectiveness of our proposed methods.

2. PROBLEM STATEMENT

Using the two-dimensional Cartesian coordinate system the state vector is defined by

$$\mathbf{x}_k = [x_k \; y_k \; \dot{x}_k \; \dot{y}_k]'$$

where $[x_k \; y_k]'$ and $[\dot{x}_k \; \dot{y}_k]'$ denote the position and velocity of the target, respectively. The target position is tracked by a radar that provides measurements of range and line-of-sight angle of the target. The measurement equation is described by the following nonlinear discrete equation

$$\mathbf{z}_k = \begin{bmatrix} r_k^m \\ \theta_k^m \end{bmatrix} = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \tan^{-1}(y_k/x_k) \end{bmatrix} + \begin{bmatrix} v_k^r \\ v_k^\theta \end{bmatrix}. \qquad (1)$$

where the superscript m refers to the measured value. Measurement noises v_k^r and v_k^θ are assumed to be mutually uncorrelated, white, and zero mean with variances σ_r^2 and σ_θ^2, respectively. Thus the measurement noise covariance can be written as

$$R_k = \mathrm{diag}\{\sigma_r^2, \sigma_\theta^2\}.$$

The problem of radar tracking is to estimate as accurately as possible the true state of the target from the noisy radar measurements.

3. TWO BASIC APPROACHES: EKF AND CMKF

3.1 Brief Description

The extended Kalman filter (EKF) is a natural extension of the linear Kalman filter for systems with nonlinear dynamics and/or measurement equations. In the EKF, the nonlinear function $\mathbf{h}(\cdot)$ of (1) is approximated as follows.

$$\mathbf{h}(\mathbf{x}_k) \approx \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}) + \mathcal{H}_k(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})$$

where $\mathcal{H}_k$ represents the Jacobian of $\mathbf{h}(\cdot)$ evaluated at the *a priori* state estimate $\hat{\mathbf{x}}_{k|k-1}$, that is,

$$\mathcal{H}_k = \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k|k-1}} = \begin{bmatrix} \cos\bar{\theta}_k & \sin\bar{\theta}_k & 0 & 0 \\ -\frac{\sin\bar{\theta}_k}{\bar{r}_k} & \frac{\cos\bar{\theta}_k}{\bar{r}_k} & 0 & 0 \end{bmatrix}$$

where $\bar{r}_k$ and $\bar{\theta}_k$, *a priori* estimates of the range and azimuth, are defined by $\bar{r}_k = (\hat{x}_{k|k-1}^2 + \hat{y}_{k|k-1}^2)^{1/2}$ and $\bar{\theta}_k = \tan^{-1}(\hat{y}_{k|k-1}/\hat{x}_{k|k-1})$. We can then obtain the EKF equations easily by applying the standard Kalman filter to the resulting linear model.

There exists an alternative approach to the EKF. The method is to convert the polar measurements to a Cartesian coordinate frame so that the resulting measurements may be modeled as linear in the transformed state. This method will be called the converted measurement Kalman filter (CMKF). Transforming (1) to a Cartesian coordinate frame gives the following pseudo-linear measurement model

$$\mathbf{z}_k^c = \begin{bmatrix} x_k^c \\ y_k^c \end{bmatrix} = \begin{bmatrix} r_k^m \cos\theta_k^m \\ r_k^m \sin\theta_k^m \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} + \begin{bmatrix} v_k^x \\ v_k^y \end{bmatrix}. \quad (2)$$

The observation matrix in the measurement equation (2) becomes then

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Let us denote by R_k^C the covariance of the Cartesian coordinate measurement error $[v_k^x \; v_k^y]'$. In the CMKF, R_k^C is approximately evaluated using the latest estimates of the state as follows:

$$R_k^C \approx J_k R_k J_k' \quad (3)$$

with

$$J_k = \begin{bmatrix} \cos\bar{\theta}_k & -\bar{r}_k \sin\bar{\theta}_k \\ \sin\bar{\theta}_k & \bar{r}_k \cos\bar{\theta}_k \end{bmatrix}. \quad (4)$$

3.2 Comparison

The EKF requires the evaluation of the Jacobians to obtain the observation matrix, while the CMKF needs it to compute the measurement error covariance. Since both the filters employ first-order approximations, it would be interesting to examine how EKF and CMKF are related. Since the target dynamics are usually modeled as linear, the EKF and CMKF equations are identical in time update. Accordingly, only the measurement update portion will be considered here.

Let us assume that before the update by the current measurement $\mathbf{z}_k$, EKF and CMKF have identical *a priori* estimates of the state and the corresponding error covariance matrix, denoted $\hat{\mathbf{x}}_{k|k-1}$ and $P_{k|k-1}$. The *a posteriori* estimates of the EKF, denoted $\hat{\mathbf{x}}_{k|k}^E$ and $P_{k|k}^E$, are then obtained as

$$\hat{\mathbf{x}}_{k|k}^E = \hat{\mathbf{x}}_{k|k-1} + K_k^E[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})] \quad (5)$$

$$P_{k|k}^E = [I - \mathcal{K}_k \mathcal{H}_k] P_{k|k-1} \quad (6)$$

with

$$K_k^E = P_{k|k-1}\mathcal{H}_k'[\mathcal{H}_k P_{k|k-1}\mathcal{H}_k' + R_k]^{-1}. \quad (7)$$

On the other hand, the *a posteriori* estimates of the CMKF, denoted $\hat{\mathbf{x}}_{k|k}^C$ and $P_{k|k}^C$, are given by

$$\hat{\mathbf{x}}_{k|k}^C = \hat{\mathbf{x}}_{k|k-1} + K_k^C[\mathbf{z}_k^c - H_k\hat{\mathbf{x}}_{k|k-1}] \quad (8)$$

$$P_{k|k}^C = [I - K_k H_k] P_{k|k-1} \quad (9)$$

with

$$K_k^C = P_{k|k-1}H_k'[H_k P_{k|k-1} H_k' + R_k^C]^{-1}. \quad (10)$$

Noting that

$$H_k = J_k \mathcal{H}_k \quad (11)$$

and using (3), (7), and (10), we have

$$\begin{aligned} K_k^C &= P_{k|k-1}H_k'[H_k P_{k|k-1}H_k' + J_k R_k J_k']^{-1} \\ &= P_{k|k-1}\mathcal{H}_k'[\mathcal{H}_k P_{k|k-1}\mathcal{H}_k' + R_k]^{-1} J_k^{-1} \\ &= K_k^E J_k^{-1}. \end{aligned} \quad (12)$$

Then it follows from (6), (9) and (12) that

$$P_{k|k}^C = P_{k|k}^E.$$

This shows that EKF and CMKF provide the same *a posteriori* estimates for the error covariance matrix from the same *a priori* information. Also, by comparing (5) and (8), it is easy to see that the state estimate of the CMKF will become equal to that of the EKF only if the Cartesian coordinate residual

$$\mathbf{z}_k^c - H_k\hat{\mathbf{x}}_{k|k-1}$$

in the CMKF equation is replaced by

$$J_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})].$$

We remark that it is difficult to decide which of EKF and CMKF is better in estimation accuracy. In fact, the two filters perform comparably in most cases.

4. NEW TRACKING ALGORITHM

For their simple structures, the EKF and CMKF schemes have been widely used for radar tracking problems. However, in case cross-range measurement errors of the target position are large, it has been found that they give considerably degraded estimation results [3, 4]. In the case of the CMKF, this undesirable behavior of the filter can be cured by debiasing the pseudo-linear measurements and re-evaluating the corresponding mean and covariance, which was proposed by Lerro and Bar-Shalom [4]. This modified version of the CMKF (MCMKF) achieves its improved performance at the expense of increased filter complexity since their newly derived expressions for the measurement error statistics are rather complicated to compute. In this section, we will present a new radar tracking filter which is derived by exploiting the idea of the MCMKF method.

We first note that the conventional pseudo-linear measurements of (2) may be rewritten as

$$\begin{bmatrix} x_k^c \\ y_k^c \end{bmatrix} = T(r_k, \theta_k) \begin{bmatrix} (r_k + v_k^r)\cos v_k^\theta \\ (1 + \frac{v_k^r}{r_k})\sin(\theta_k + v_k^\theta) \end{bmatrix} \quad (13)$$

where $T(\cdot,\cdot)$ is defined by

$$T(\alpha, \beta) = \begin{bmatrix} \cos\beta & -\alpha\sin\beta \\ \sin\beta & \alpha\cos\beta \end{bmatrix}.$$

We can see that $T(\bar{r}_k, \bar{\theta}_k) = J_k$. The true Cartesian position of the target can be expressed as

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = T(r_k, \theta_k) \begin{bmatrix} r_k \\ 0 \end{bmatrix}. \quad (14)$$

From (13) and (14), the errors in each coordinate are obtained as

$$\begin{bmatrix} \delta x_k \\ \delta y_k \end{bmatrix} = \begin{bmatrix} x_k^c - x_k \\ y_k^c - y_k \end{bmatrix} = T(r_k, \theta_k) \begin{bmatrix} (r_k + v_k^r)\cos v_k^\theta - r_k \\ (1 + v_k^r/r_k)\sin v_k^\theta \end{bmatrix}.$$

Then the mean and covariance of the Cartesian coordinate measurement errors may be obtained in terms of the true range and azimuth as

$$\mu_k = \begin{bmatrix} E[\delta x_k | r_k, \theta_k] \\ E[\delta y_k | r_k, \theta_k] \end{bmatrix} = T(r_k, \theta_k) \begin{bmatrix} r_k(e^{-\sigma_\theta^2/2} - 1) \\ 0 \end{bmatrix}$$

and

$$\begin{aligned} R_k^\mu &= \begin{bmatrix} \mathrm{var}[\delta x_k | r_k, \theta_k] & \mathrm{cov}[\delta x_k, \delta y_k | r_k, \theta_k] \\ \mathrm{cov}[\delta x_k, \delta y_k | r_k, \theta_k] & \mathrm{var}[\delta y_k | r_k, \theta_k] \end{bmatrix} \\ &= T(r_k, \theta_k)\, \mathrm{diag}\{(r_k^2 + \sigma_r^2)(1 + e^{-2\sigma_\theta^2})/2 - r_k^2 e^{-\sigma_\theta^2}, \\ &\qquad (1 + \sigma_r^2/r_k^2)(1 - e^{-2\sigma_\theta^2})/2\}\, T'(r_k, \theta_k). \end{aligned}$$

Here the expectations were evaluated using the following equalities

$$E[\sin v_k^\theta]=0\,,\qquad E[\cos v_k^\theta]=e^{-\sigma_\theta^2/2}\,,$$

$$E[\sin^2 v_k^\theta]=(1-e^{-2\sigma_\theta^2})/2\,,\quad E[\cos^2 v_k^\theta]=(1+e^{-2\sigma_\theta^2})/2\,.$$

We remark that for all practical radars it is reasonable to assume

$$\sigma_r << r_k \text{ and } \sigma_\theta << 1 \tag{15}$$

where r_k denotes the true range. This assumption allows the expressions for μ_k and R_k^μ to be simplified as follows.

$$\mu_k \approx T(r_k,\theta_k)\begin{bmatrix}-r_k\,\sigma_\theta^2/2\\ 0\end{bmatrix}$$

$$R_k^\mu \approx T(r_k,\theta_k)\,R_k T'(r_k,\theta_k)\,.$$

The "debiased" converted measurement can then be obtained by subtracting the bias μ_k from $\mathbf{z}_k^c$. The associated error covariance is R_k^μ. However, because the true range and azimuth are not available in practice, it is impossible to evaluate μ_k and R_k^μ. Accordingly, it is required to find their best possible approximations. As a reasonable approximation, we propose to take the conditional mean given all the previous data, that is, $E[\mu_k|Z_{k-1}]$ and $E[R_k^\mu|Z_{k-1}]$ where Z_{k-1} represents the set of measurements up through time $k-1$. Let us define

$$\bar{\mu}_k = E[\mu_k|Z_{k-1}] = -\frac{\sigma_\theta^2}{2}\begin{bmatrix}E[r_k\cos\theta_k|Z_{k-1}]\\ E[r_k\sin\theta_k|Z_{k-1}]\end{bmatrix} \tag{16}$$

$$\bar{R}_k^\mu = E[R_k^\mu|Z_{k-1}] = E[T(r_k,\theta_k)R_kT'(r_k,\theta_k)|Z_{k-1}]\,. \tag{17}$$

Then, the measurement update equations for the debiased measurement conversion are obtained as

$$K_k = P_{k|k-1}H_k'[H_kP_{k|k-1}H_k' + \bar{R}_k^\mu]^{-1} \tag{18}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k[\mathbf{z}_k^c - \bar{\mu}_k - H_k\hat{\mathbf{x}}_{k|k-1}] \tag{19}$$

$$P_{k|k} = [I-K_kH_k]P_{k|k-1}\,. \tag{20}$$

Notice that our suggested debiasing scheme is rather different from that of [4] and that because $\bar{\mu}_k$ and $\bar{R}_k^\mu$ cannot be computed easily, the above algorithm is not practical to use.

At this stage, our strategy for obtaining an algorithm which is easy to implement is to modify the measurement residual term by utilizing the relation between EKF and CMKF. That is, we propose to replace the term ($\mathbf{z}_k^c - H_k\hat{\mathbf{x}}_{k|k-1}$) in (19) by its first-order approximation, $J_k[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})]$. Then, from Eqs. (18)~(20), a different update procedure can be obtained as follows:

$$\mathcal{K}_k = P_{k|k-1}\mathcal{H}_k'[\mathcal{H}_kP_{k|k-1}\mathcal{H}_k' + \bar{R}_k^p]^{-1} \tag{21}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathcal{K}_k[\mathbf{z}_k^c - \bar{\mu}_k^p - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})] \tag{22}$$

$$P_{k|k} = [I-\mathcal{K}_k\mathcal{H}_k]P_{k|k-1} \tag{23}$$

where

$$\bar{\mu}_k^p = J_k^{-1}\bar{\mu}_k \tag{24}$$

and

$$\bar{R}_k^p = J_k^{-1}\bar{R}_k^\mu(J_k^{-1})'\,. \tag{25}$$

It is interesting to observe that the measurement update equations derived above are the same as those of the EKF except for the bias term in the measurement residual and the modified expression for the measurement error covariance.

Now, what is left is to evaluate $\bar{\mu}_k^p$ and $\bar{R}_k^p$ defined above. Substitution of Eqs. (16) and (17) into Eqs. (24) and (25) does not immediately lead to explicit expressions for $\bar{\mu}_k^p$ and $\bar{R}_k^p$. They depend on the probability density function for $\mathbf{x}_k$, which is not available in general, and their computation requires an involved numerical integration. However, it will be shown below that we can obtain the approximate expressions for $\bar{\mu}_k^p$ and $\bar{R}_k^p$. For this we proceed as follows.

First let us define

$$\tilde{r}_k = r_k - \bar{r}_k$$
$$\tilde{\theta}_k = \theta_k - \bar{\theta}_k$$
$$\tilde{x}_k = x_k - \hat{x}_{k|k-1}$$
$$\tilde{y}_k = y_k - \hat{y}_{k|k-1}\,.$$

Also, let $\sigma_{\tilde{r}}^2$ and $\sigma_{\tilde{\theta}}^2$ represent filter estimated *a priori* error variances of range and azimuth, respectively. Noting the following first-order Taylor series approximations

$$\tilde{r}_k \approx \frac{\hat{x}_{k|k-1}\tilde{x}_k + \hat{y}_{k|k-1}\tilde{y}_k}{\bar{r}_k} \tag{26}$$

$$\tilde{\theta}_k \approx \frac{-\hat{y}_{k|k-1}\tilde{x}_k + \hat{x}_{k|k-1}\tilde{y}_k}{\bar{r}_k^2} \tag{27}$$

we can approximate $\sigma_{\tilde{r}}^2$ and $\sigma_{\tilde{\theta}}^2$ as

$$\sigma_{\tilde{r}}^2 = \frac{\hat{x}_{k|k-1}^2p_{11} + \hat{y}_{k|k-1}^2p_{22} + 2\hat{x}_{k|k-1}\hat{y}_{k|k-1}p_{12}}{\bar{r}_k^2}$$

$$\sigma_{\tilde{\theta}}^2 = \frac{\hat{y}_{k|k-1}^2p_{11} + \hat{x}_{k|k-1}^2p_{22} - 2\hat{x}_{k|k-1}\hat{y}_{k|k-1}p_{12}}{\bar{r}_k^4}\,.$$

where p_{ij} denotes the (i,j) th element of $P_{k|k-1}$.

It seems reasonable to suppose that $\sigma_{\tilde{r}}^2$ and $\sigma_{\tilde{\theta}}^2$ are smaller than the corresponding measurement noise variances, σ_r^2 and σ_θ^2, while the filter is operating normally, so that from (15) it follows that

$$\sigma_{\tilde{r}} << r_k\,(\approx \bar{r}_k) \text{ and } \sigma_{\tilde{\theta}} << 1 \tag{28}$$

Using (4), (16), (26), (27), and (28) in Eq. (24), we can then obtain

$$\bar{\mu}_k^p = -\frac{\sigma_\theta^2}{2}\begin{bmatrix}E[r_k\cos\tilde{\theta}_k|Z_{k-1}]\\ E[r_k\sin\tilde{\theta}_k/\bar{r}_k|Z_{k-1}]\end{bmatrix}$$
$$\approx -\frac{\sigma_\theta^2}{2}\begin{bmatrix}\bar{r}_k\,E[\cos\tilde{\theta}_k|Z_{k-1}]\\ E[\sin\tilde{\theta}_k|Z_{k-1}]\end{bmatrix} = -\frac{\sigma_\theta^2}{2}\begin{bmatrix}\bar{r}_k\,e^{-\sigma_{\tilde{\theta}}^2/2}\\ 0\end{bmatrix}$$
$$\approx \begin{bmatrix}-\bar{r}_k\sigma_\theta^2/2\\ 0\end{bmatrix}.$$

In a similar manner, we have

$$\begin{aligned}\bar{R}_k^\mu &= E[J_k^{-1}T(r_k,\theta_k)R_kT'(r_k,\theta_k)(J_k^{-1})'|Z_{k-1}]\\ &\approx \mathrm{diag}\{\sigma_r^2 + \bar{r}_k^2\sigma_\theta^2\sigma_{\tilde{\theta}}^2,\ \sigma_\theta^2\}.\end{aligned} \tag{29}$$

This completes the derivation of approximate explicit expressions for $\bar{\mu}_k^p$ and $\bar{R}_k^p$.

By comparing Eq. (29) with R_k, some useful observations may be made. In particular, it is not difficult to expect that nonlinear effects will not be significant so long as the target range $r_k\,(\approx \bar{r}_k)$ remains relatively small compared with the ratio $\sigma_r/\sigma_\theta\sigma_{\tilde{\theta}}$.

Let us now present a filter initialization procedure. Using the first single measurement, the initial estimate for the state may be obtained as

$$\hat{\mathbf{x}}_{0|0} = \begin{bmatrix}r_0^m\cos\theta_0^m\\ r_0^m\sin\theta_0^m\end{bmatrix}$$

where r_0^m and θ_0^m denote the range and azimuth measurements at time $k=0$, respectively. The corresponding error covariance can be obtained from Eqs. (21) and (23) by applying the matrix inversion lemma and letting $P_{0|-1}^{-1}=0$ (which means the absence of *a priori* information before the arrival of $\mathbf{z}_0$):

$$P_{0|0}=\mathcal{H}_0^{-1}\bar{R}_0^{\mu}(\mathcal{H}_0^{-1})' .$$

Here, the matrix $\bar{R}_0^{\mu}$ is given by

$$\bar{R}_0^{\mu}=\mathrm{diag}\{\sigma_r^2+(r_0^m)^2\sigma_\theta^4,\ \sigma_\theta^2\} . \qquad (30)$$

Note here that because $\sigma_{\tilde{\theta}}$ cannot be computed at the initialization stage, σ_θ is used instead.

Finally, the fact that R_k^p is a diagonal matrix can be further utilized for the development of a more efficient algorithm. It is known that when the measurement error covariance is diagonal, it is possible to sequentially process scalar components of the measurement vector $\mathbf{z}_k$ instead of processing it as a single data. The scheme of sequential measurement processing leads to considerable computational savings [1].

5. SIMULATION RESULTS

To verify the performance of the proposed method via a computer simulation, a numerical example is provided. It is assumed that the target is initially located at a range of 200 km, and that it moves straight with a nearly constant speed of 125 m/s. The target trajectory is modeled by the two-dimensional piecewise constant white acceleration model [2]. The standard deviation of the piecewise constant acceleration errors is set as $0.5\ \mathrm{m/sec^2}$ for each coordinate. The target is tracked by a radar that provides measurements of range and azimuth at the sampling interval of $T=10$ s. The measurement noise processes are regarded as zero-mean white Gaussian noise sequences. The noise process standard deviations of range and azimuth measurements are 50 m and 2 deg, respectively.

The performance of the new algorithm is compared with that of the standard extended Kalman filter (EKF) and an iterated extended Kalman filter (ItEKF). For the ItEKF, we assumed a maximum of 10 iterations per step since little further improvement of estimation accuracy was achieved by additional iterations for the given problem.

For a practical implementation of filter initialization, an initial state estimate and the corresponding error covariance are obtained using the first two measurements in a similar manner of the two-point differencing method of [2]. Recall that in the case of the proposed filter, the modified measurement error covariance $\bar{R}_0^{\mu}$ given by (30) is used, in place of R_0, for the initialization.

A Monte Carlo simulation of 100 runs was carried out to obtain the root mean square (RMS) errors in position and velocity for each filter. The results are depicted in Figs. 1 and 2. As seen, the EKF exhibits large biases and slow error decrease early in track. In fact, the estimates that the EKF produces are found to be inconsistent. Although the performance of the ItEKF is better than that of the EKF, one can observe that by using the proposed filter there results a significant performance improvement as compared with the other filters, especially for the transient response.

On the other hand, we have compared the performance of the proposed method with that of the MCMKF for this example, and we have found that the proposed method is slightly better than the MCMKF in estimation accuracy. It should be also stressed that the computational load of the proposed method is smaller than that of the MCMKF.

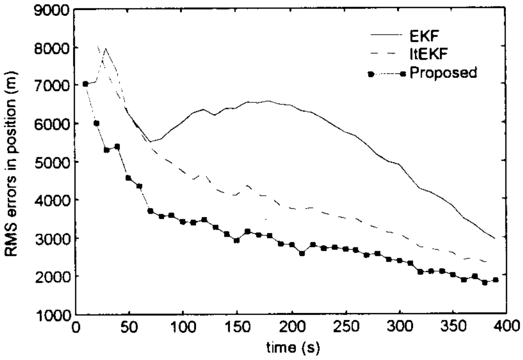

Fig. 1. RMS position errors

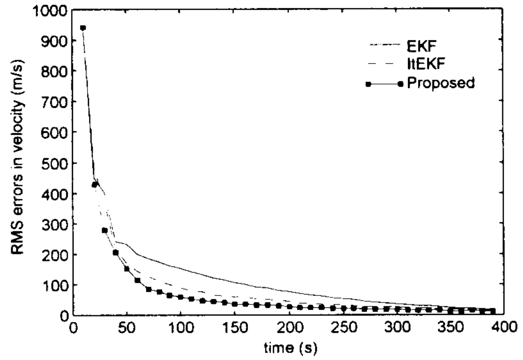

Fig. 2. RMS velocity errors

6. CONCLUSIONS

In this paper, we presented a new radar tracking filter based on the correct evaluation of the measurement error covariance with respect to the Cartesian state vector. The newly proposed filter tunes the measurement error variance in an adaptive manner to effectively account for the measurement nonlinearities, and employs the sequential measurement processing scheme. The new algorithm is simple in form and gives an easy indication of how sensor accuracy and target geometry are related to measurement accuracy viewed by (linearization-based) Kalman filters. A numerical example was given which shows that the proposed method reduces effectively the nonlinear effect of the polar measurements while requiring a relatively small computational burden.

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