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A Robust Noise Rejector in a Small Cavity

Suk-Bong Seo, Woo-Hyun Ahn, Tae-Jin Chung, Chan-Soo Chung

Dept. of Electrical Engineering
Soongsil University
1-1, Sangdo 5 Dongjak-gu, Seoul, 156-743
South Korea
tarzan@control.soongsil.ac.kr

Abstract

This paper studies on an active noise control to reduce noise sound level in a small cavity. Ideally, high gain control solves this problem, but, in practice, there exist nonlinear characteristics and modelling errors of the small cavity, which make the control more complicated. H_∞ control can be used in an uncertain system after determining uncertain boundary and solved algebraically or numerically. In this paper, the numerical one, LMI(Linear Matrix Inequality), is used to get controller. Finally, experiment result shows the performance of the controller.

1. Introduction

Acoustic noise is one of the main sources of environmental pollution and known to give one physical and/or psychological damages. The most significant health problem caused by noise is hearing loss. People, under noisy condition, tend to be mentally unstable.

Many researchers have been tried to solve noise reduction/cancellation problem in many areas, like an air duct system, automobile, airplane, etc. The idea of noise control in a small cavity had suggested by Olson(1956) and studied by Wheeler(1986) and Carme(1987). They measured the electroacoustic frequency response of the small cavity and designed a controller based on the frequency response. Ideally, their method was pretty simple and efficient but they didn't consider nonlinear characteristics and modelling errors of the cavity which we cannot ignore to prove a controller's performance.

One of emerging control theory to handle modelling error is H_∞ control theory suggested by Zame[4] and the noise control problem can be, easily, interpreted to H_∞ regulating problem. The paper suggested by Chung in 1995 used H_∞ control synthesis by solving two linear algebraic Riccati equations.

In this paper, we solve the H_∞ regulating problem by LMI with time constraints.

2. A Small Cavity and Noise Characteristic

The structure of the main plant used in this study is a small cavity shown in Figure 2.1. The plant consists of a plastic cavity, a condenser microphone and a loud speaker. The block diagram of the small cavity system is shown in Figure 2.2.

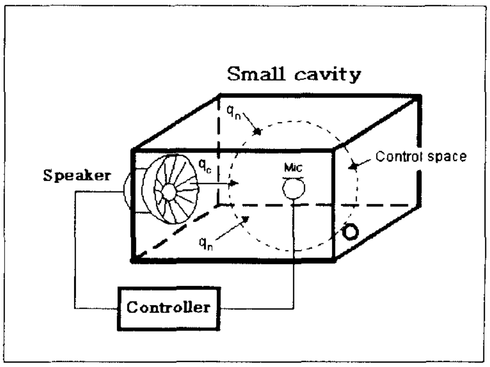

Fig 2.1 The structure of the small cavity system
(q_n : noise signal , q_c : control signal)

In the small cavity system, the input is a noise signal caused by external noise sources and the output is an error signal which is measured by the error microphone.

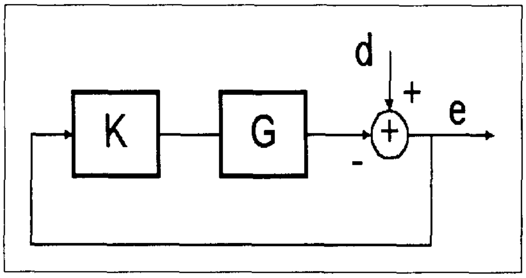

Fig 2.2 Block diagram of the small cavity system (K : controller , G : plant , d : noise signal , e : error signal)

In the figure 2.2, G is the transfer function of an acoustic path which included a loud speaker, a cavity space, and a microphone. The controller K makes a control signal. The transfer function of the block diagram in Figure 2.2 is

$$\frac{E(s)}{D(s)} = \frac{1}{1+K(s)G(s)} = S(s) \qquad (2.1)$$

In equation (2.1), if the controller's gain is increased to infinity, the magnitude of the transfer function $\|D/E(s)\|_\infty$ converges to zero, which means the noise in the small cavity is cancelled to zero.

But the actual system is so complicate that just increasing the magnitude of the controller leads the whole system unstable easily. So, to adapt H_∞ control, it is needed to re-interpreted the control problem. One can see that the transfer function in Figure 2.2 is equal to a sensitivity function of total transfer function. Thus, the noise control problem in the small cavity can be re-interpreted to H_∞ regulating problem which is to minimize the infinite norm of the sensitivity function.

3. H_∞ control and LMI [7,8]

When there are uncertainties in the system model, one of well known methods to express the uncertain system is multiplicative model, shown equation (3.1).

$$\tilde{P} = (1+\triangle W_2)\cdot P \qquad (3.1)$$

where P : nominal plant transfer function
$\tilde{P}$: perturbed plant transfer function
$\triangle$: scaling factor
W_2 : weighting function

Before solving H_∞ control problem, we set an augmented system with weight functions, W1 and W2, to express the characteristic of uncertainty and out design performance.

Given the augmented system with

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{aligned} \qquad (3.2)$$

an LMI stability region D, and some H_∞ performance $\gamma > 0$, find and a control u=K(s)y s.t.

- The closed-loop poles lie in D.
- $\|T_{wz}\|_\infty < \gamma$ where $T_{wz}(s)$ denote the closed loop transfer function from w to z.

If the controller is represented by

$$\begin{aligned} \dot{x}_K(t) &= A_K x_K(t) + B_K y(t) \\ u(t) &= C_K x_K(t) + D_K y(t) \end{aligned} \qquad (3.3)$$

Then $T_{wz}(s) = D_{cl} + C_{cl}(sI - A_{cl})^{-1} B_{cl}$ with the notation

$$\begin{aligned} A_{cl} &= \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix} \\ B_{cl} &= \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix} \\ C_{cl} &= [\, C_1 + D_{12} D_K C_2, \; D_{12} C_K] \\ D_{cl} &= D_{11} + D_{12} D_K D_{21} \; . \end{aligned} \qquad (3.4)$$

Inequality (3.5) must be satisfied for any $X_D > 0$ for pole placement constraint.

$$[\, \alpha_{kl} X_D + \beta_{kl} A_{cl} X_D + \beta_{lk} X_D A_{cl}^T]_{1 \le k,l \le m} < 0, \qquad (3.5)$$

And inequality (3.6) must be satisfied for any $X_\infty > 0$ for H_∞ control constraints.

$$\begin{bmatrix} A_{cl} X_\infty + X_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl}^T \\ B_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} X_\infty & D_{cl} & -\gamma I \end{bmatrix} < 0 \qquad (3.6)$$

So, we will find any $X > 0$ that satisfy inequalities (3.5) and (3.6) with $X = X_D = X_\infty$, then the controller satisfy our H_∞ control constraint and pole placement constraint.

We solve these inequalities using Matlab's *LMI Control Toolbox* [11].

4. Controller design in a Small Cavity

To derive analytically the transfer function G is very complicate. So, this paper gets the transfer characteristic experimentally and derives its parameters using system identifications.

The plant transfer function G(s) is expressed like this.

$$\hat{G}(s) = \frac{NumG(s)}{DenG(s)} \text{ (1)} \qquad (4.1)$$

To design the controller effective as a practical active noise controller, the design specifications for noise control in the small cavity is defined. The design specification is below.

1. The complementary sensitivity function T has -20dB gain over about 1.5kHz.
2. The sensitivity function S has magnitude as small as possible below about 600Hz.

To solve the control problem, LMI control toolbox is used. Figure 4.1 shows that the resulting controller satisfied our design specifications

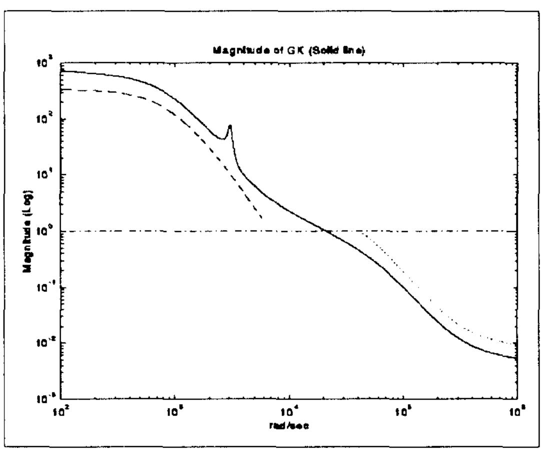

Fig 4.1 open loop characteristics of the system

—— : GK

······ : Robust stability

---- : Performance

Figure 4.2 shows the sensitivity function and its complementary sensitivity function. In figure 3.5, the magnitude of sensitivity below 600Hz is 10^{-2}. This means that the noise can be reduced by about 100 times. And the small size of the complementary sensitivity function of high frequency region means that the closed loop system takes little effect by uncertainties in high frequency range.

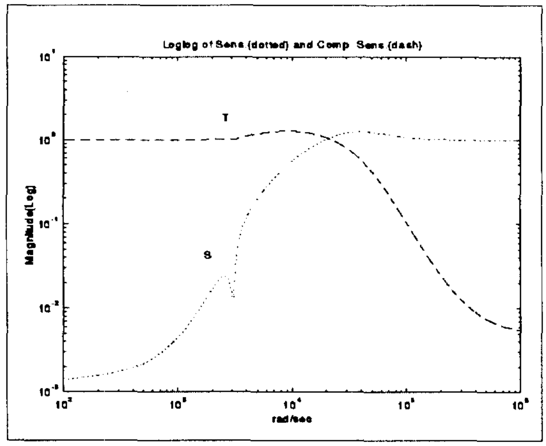

Fig 4.2 Bode plot of sensitivity funciton and its complementary sensitivity function

5. Experimental Results

The designed controller is realized by analog devices, like OP-Amp's.

For experiment of the controller, the external noise is given by using another loud speaker. The total system for experiment is shown in figure 5.1.

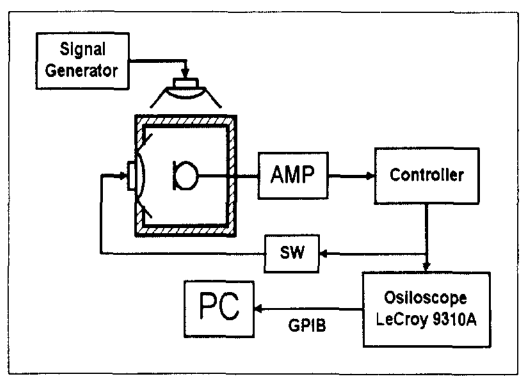

Fig 5.1 Overall system for experiment

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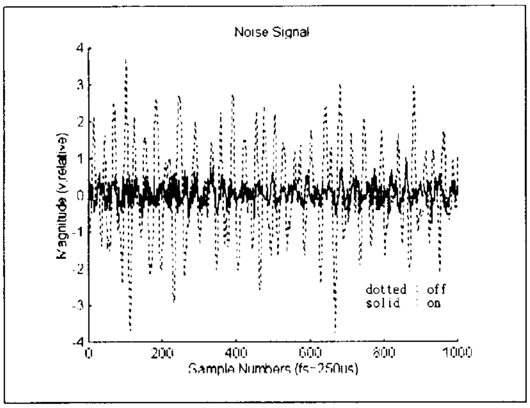
(1)

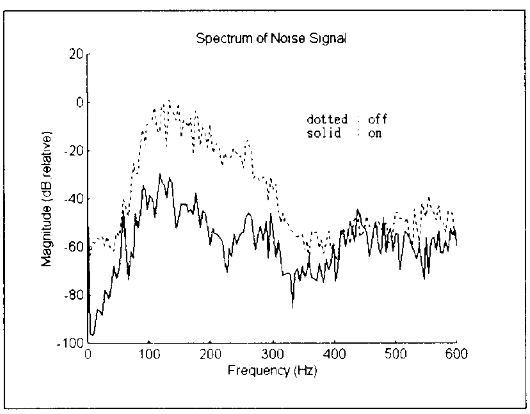
$$NumG(s) = 0.2s^{12} + 1.212e4s^{11} + 3.319e8s^{10} + 5.536e12s^{9} + 6.418e16s^{8} + 5.552e20s^{7} + 3.668e24s^{6} + 1.825e28s^{5} + 6.661e31s^{4} + 1.767e35s^{3} + 3.35e38s^{2} + 3.693e41s + 3.549e43$$

$$DenG(s) = s^{12} + 4.061e4s^{11} + 1.174e9s^{10} + 1.605e13s^{9} + 2.086e17s^{8} + 1.477e21s^{7} + 1.17e25s^{6} + 4.507e28s^{5} + 2.184e32s^{4} + 4.208e35s^{3} + 1.206e39s^{2} + 8.589e41s + 3.683e44$$

To measure the performance of the controller, we compared the magnitudes of thime and frequency characteristics of noise when the controller is on and off. Figure 5.2 shows the noise signal when the external noise is a running automobile's noise and its spectrum characteristic. When the controller is on, the magnitude of noise signal is reduced about -20dB in the range of 10Hz to 400Hz.



(a) Time domain (average of 5 cases)



(b) Frequency domain (average of 5 cases)

Fig 5.2 The signals of the ANC system when external noise is a running automobile's noise

6. Conclusion

This paper studied on a robust active noise controller design for a small cavity to control the noise induced in th cavity. There exist nonlinear characteristics and modeling error of the small cavity, which make the control more complicated.

To solve these problems, this paper designed the robust controller to minimize H_∞ norm of the mixed sensitivity function by using H_∞ control theory. H_∞ control can be used in an uncertain boundary and solved numerically using LMI(Linear Matrix Inequality). The results shown that the designed controller has better stability and performance about -20dB noise reduction.

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