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Adaptive Fuzzy Learning Control for a Class of Second Order Nonlinear Dynamic Systems

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Abstract

This paper presents an iterative fuzzy learning control scheme which is applicable to a broad class of nonlinear systems. The control scheme achieves system stability and boundedness by using the linear feedback plus adaptive fuzzy controller and achieves precise tracking by using the iterative learning rules. The switching mode control unit is added to the adaptive fuzzy controller in order to compensate for the error that has been inevitably introduced from the fuzzy approximation of the nonlinear part. It also obviates any supervisory control action in the adaptive fuzzy controller which normally requires high gain signal. The learning control algorithm obviates any output derivative terms which are vulnerable to noise.

1 Introduction

There have been substantial amount of research works in the development of various iterative learning control methods. The concept of iterative learning control has been introduced in order to improve tracking performance in an iterative manner by attempting to execute a desired motion repeatedly. The control effort in each attempt is improved using the tracking error signals obtained from previous trial. Arimoto *et al.* [2] proposed a general iterative learning control method for a class of nonlinear systems whose input and output gain matrices are of linear time-invariant form. In their scheme, the time derivative of the current output error is used to update the learning control input for the next iteration. Ahn *et al.* [1] and Jang *et al.* [4] proposed iterative learning control laws based on the relative degree of nonlinear system introduced by Isidori [3]. Their controllers still uses derivatives of output errors and its convergence condition is implicitly coupled with the decoupling matrix of the nonlinear system. Kuc and Lee [5] presented a simple and efficient adaptive learning control law for robot systems based on the linear parametrization technique. In their control scheme, they updated not only the learning control input but also the parameter estimates. This control scheme, however, is not extendable to general nonlinear systems for which a convenient linear parametrization form is not available in general. In order to overcome the problem, a fuzzy logic representation technique is introduced in this paper where the nonlinear system is approximated by a fuzzy logic system which can be formulated in the linear parametrization form using the fuzzy basis function approach.

The research on the fuzzy logic control [8] has been motivated by papers on the linguistic approach and the system analysis using the theory of fuzzy sets [12]. Fuzzy logic, which forms the basis of fuzzy control methods, is similar to human thinking and natural language representation, and provides effective means of capturing approximated and inexact nature of the real word. From a mathematical point of view, a fuzzy logic system is in fact a mapping from the input space to the output space that approximates the nonlinear function within a given accuracy [7]. It describes the nonlinear function by using the linguistic descriptions obtained either from the experts or from the experimental results, and uses the fuzzy inference technique to obtain the results. Wang [11] introduced the universal approximation theorem for a class of fuzzy logic systems: any continuous or L_2 nonlinear function can be approximated by a fuzzy logic system with arbitrary accuracy. This fact was extensively used in his other paper to develop adaptive control methods for a class of nonlinear systems [10]. Since the universal approximation theory holds only in the compact set, he added yet another supervisory control term to keep the state of the system within the compact set. Su and Stepanenko [9] presented an adaptive fuzzy controller with switching control technique and proved that the system errors converge to zero.

Applying the fuzzy logic technique to the iterative learning control method, we propose an adaptive fuzzy learning control scheme in this paper which is applicable to a broad class of nonlinear dynamic systems. The adaptive fuzzy control block compensates for the nonlinear part of the system, thereby reducing the load from the feedback control block and keeping the feedback gain reasonably small. It also helps eliminate the conditions that are normally required to prove the error convergence of the system [5]. In order to compensate for the error that has been inevitably introduced from the fuzzy approximation of the feedforward nonlinear term, the switching mode control algorithm is added to the fuzzy controller. Since the gain of the switching mode control algorithm decreases to zero as the error converges to zero, its effect reduces to zero. The learning controller updates the input signals by using the feedback signals obtained from linear feedback control block. In contrast with many other learning control schemes [1, 4], the presented scheme updates iteratively not only the learning control signal but also the parameters in the fuzzy logic system by using a regressor vector obtained from the fuzzy basis function.

2 Problem Formulation

The nonlinear dynamic systems that are considered in this paper are of the form:

$$g(x(t))\ddot{x}(t) + f(x(t), \dot{x}(t)) = u(t), \qquad (1)$$

where $x(t) : R \rightarrow R$, $g(x(t)) : R \rightarrow R$ and $f(x(t), \dot{x}(t)) : R^2 \rightarrow R$. In order to simplify the presentation, the system is assumed to be the single-input single-output(SISO) systems, but can be easily extended to multi-input multi-output(MIMO) systems. The results in the subsequent sections are developed based on the following assumptions that hold in many practical systems.

Assumption 1 For all $(x, \dot{x}) \in U$ where U is a compact set in R^2,

$$l_1 \leq g(x) \leq l_2 \quad \text{and} \quad \left|\frac{dg(x)}{dx}\right| \leq r_g,$$

for some l_1, l_2 and $r_g > 0$.

Assumption 2 $f(x, \dot{x})$ satisfy the following Lipschitz continuity conditions:

$$|f(x_1, \dot{x}_1) - f(x_2, \dot{x}_2)| \leq r_f(|x_1 - x_2| + |\dot{x}_1 - \dot{x}_2|)$$

for all $(x_1, \dot{x}_1), (x_2, \dot{x}_2) \in U$, where U is a compact set in R^2.

Note that the system (1) describes a class of nonlinear systems which include holonomic mechanical systems as its subset.
The desired control input u_d that generates the desired trajectory x_d in (1) satisfies the following assumption.

Assumption 3 The bound u_d^b of $|u_d|$ is known.

Under these assumptions, the fundamental learning control problem is the following.
Problem Statement. Suppose that $x_d(t) \in C^2[0, t_f]$, the trajectory of system (1), is in the interior of a domain U, which is a compact and simply connected set of R^2. Then find a sequence of piecewise continuous control input $u^j(t) \in R$ for uncertain system (5) with which the system trajectory $x^j(t)$ converges to $x_d(t)$. In other words, for a given $\epsilon > 0$ and $t \in [0, t_f]$, find a sequence of $u^j(t)$ such that there exists $N > 0$ such that

$$|x_d(t) - x^j(t)| \leq \epsilon, \quad \text{for all } j > N.$$

Our approach to the above problem is the approximation of nonlinear functions $g(x)$ and $f(x, \dot{x})$ with fuzzy logic systems and the use of learning control strategy to achieve precise tracking.

3 Fuzzy Logic Description of Nonlinear Systems

As stated in the universal approximation theorem [10, 11], any continuous nonlinear system can be approximated by a fuzzy logic system with arbitrary accuracy in the compact domain. Thus, the nonlinear functions $f(x, \dot{x})$ and $g(x)$ in (1) can be approximated by the fuzzy logic systems as follows:

$$\begin{aligned} g(x) &= \hat{g}(x|\theta_g^*) + w_g(x) = \zeta_g^T(x)\theta_g^* + w_g(x) \quad \text{and} \\ f(x, \dot{x}) &= \hat{f}(x, \dot{x}|\theta_f^*) + w_f(x, \dot{x}) \\ &= \zeta_f^T(x, \dot{x})\theta_f^* + w_f(x, \dot{x}), \end{aligned} \tag{2}$$

where $\zeta_g(x)$ and $\zeta_f(x, \dot{x})$ are respectively the fuzzy basis function vectors of $\hat{g}(x|\theta_g^*)$ and $\hat{f}(x, \dot{x}|\theta_f^*)$; $w_g(x)$ and $w_f(x, \dot{x})$ are the corresponding fuzzy approximation errors. Here the optimum parameter vectors θ_g^* and θ_f^* are defined by

$$\begin{aligned} \theta_g^* &= argmin_{\theta_g \in \Omega_g}\left[sup_{(x,\dot{x}) \in U}|g(x) - \hat{g}(x|\theta_g)|\right] \quad \text{and} \\ \theta_f^* &= argmin_{\theta_f \in \Omega_f}\left[sup_{(x,\dot{x}) \in U}|f(x, \dot{x}) - \hat{f}(x, \dot{x}|\theta_f)|\right], \end{aligned}$$

where Ω_g and Ω_f are the bounded feasible sets of θ_g and θ_f respectively.
For notational brevity, we will use in the sequel g for $g(x)$; f for $f(x, \dot{x})$; g^j for $g(x^j)$; f^j for $f(x^j, \dot{x}^j)$; f_d for $f(x_d, \dot{x}_d)$; g_d for $g(x_d)$; $\hat{g}^*$ for $\hat{g}(x|\theta^*)$; $\hat{f}^*$ for $\hat{f}(x, \dot{x}|\theta^*)$; $\hat{g}^{*^j}$ for $\hat{g}(x^j|\theta^*)$; $\hat{f}^{*^j}$ for $\hat{f}(x^j, \dot{x}^j|\theta^*)$; $\hat{g}_d^*$ for $\hat{g}(x_d|\theta^*)$; $\hat{f}_d^*$ for $\hat{f}(x_d, \dot{x}_d|\theta^*)$; w_g for $w_g(x)$; w_f for $w_f(x, \dot{x})$; w_g^j for $w_g(x^j)$; w_f^j for $w_f(x^j, \dot{x}^j)$; w_{g_d} for $w_g(x_d)$; w_{f_d} for $w_f(x_d, \dot{x}_d)$.
Note here that w_f and w_g are Lipschitz continuous since $g(x), f(x, \dot{x}), \zeta_g(x)$ and $\zeta_f(x, \dot{x})$ are Lipschitz continuous from Assumption 1 and 2, and from the universal approximation theorem [10, 11]. Thus, it follows that

$$\begin{aligned} |w_g(x) - w_g(x_d)| &\leq l_g|x - x_d| \quad \text{and} \\ |w_f(x, \dot{x}) - w_f(x_d, \dot{x}_d)| &\leq l_f(|x - x_d| + |\dot{x} - \dot{x}_d|), \end{aligned}$$

for some positive constants l_g and l_f.
Now, let x^j and x_d be respectively the states of (1) due to inputs u^j and u_d. Then, it follows from (1) that

$$\begin{aligned} & g^j(\ddot{x}^j - \ddot{x}_d) + (g^j - g_d)\ddot{x}_d + f^j - f_d \\ = \; & g^j\dot{z}^j + (g^j - g_d)\ddot{x}_d - a\dot{e}^j g^j + f^j - f_d \\ = \; & u^j - u_d, \end{aligned} \tag{3}$$

where $e^j = x^j - x_d$, $z^j = \dot{e}^j + ae^j$ and a is a positive constant.
Substituting (2) into (3), we have

$$\begin{aligned} & g^j\dot{z}^j + (\hat{g}^{*^j} - \hat{g}_d^*)\ddot{x}_d - a\dot{e}^j\hat{g}^{*^j} - a\dot{e}w_g^j + (w_g^j - w_{g_d})\ddot{x}_d \\ & + \hat{f}^{*^j} - \hat{f}_d^* + (w_f^j - w_{f_d}) = u^j - u_d, \end{aligned} \tag{4}$$

which can be rearranged further to

$$\begin{aligned} & g^j\dot{z}^j - a\dot{e}^j\hat{g}^{*^j} + g_e^{*^j}\ddot{x}_d + f_e^{*^j} \\ = \; & u^j - u_d - (w_g^j - w_{g_d})\ddot{x}_d - w_f^j + w_{f_d} + a\dot{e}^j w_g^j, \end{aligned} \tag{5}$$

where $g_e^{*^j} = \hat{g}^{*^j} - \hat{g}_d^*$ and $f_e^{*^j} = \hat{f}^{*^j} - \hat{f}_d^*$. By using the fuzzy basis function representation in (2), equation (5) can be represented as

$$g^j\dot{z}^j + Y_s^j\theta^* = u^j - u_d - w_e^j, \tag{6}$$

where $Y_s^j = (\zeta_g^T(x^j)(\ddot{x}_d - a\dot{e}^j) - \zeta_g^T(x_d)\ddot{x}_d, \zeta_f^T(x^j, \dot{x}^j) - \zeta_f^T(x_d, \dot{x}_d))$, $\theta^{*^T} = (\theta_g^{*^T}, \theta_f^{*^T})$ and $w_e^j = (w_g^j - w_{g_d})\ddot{x}_d + w_f^j - w_{f_d} - a\dot{e}^j w_g^j$. Since w_f^j and w_g^j are Lipschitz continuous, the combined fuzzy approximation error term w_e^j satisfies for all $j \geq 1$ that

$$\begin{aligned} |w_e^j| &\leq |(w_g^j - w_{g_d})\ddot{x}_d| + |w_f^j - w_{f_d}| + a|e^j w_g^j| \\ &\leq l_g|\ddot{x}_d||e^j| + l_f(|e^j| + |\dot{e}^j|) + l_2|ae^j| \leq l_e(|e^j| + |\dot{e}^j|), \end{aligned}$$

where $l_e = l_g|\ddot{x}_d| + l_f + al_2$.
Note here that (6) is in the linear form with respect to the parameter θ^* with additional fuzzy approximational error w_e^j. It will be used extensively in the following sections to prove the stability and the convergence of the closed loop system.

4 Fuzzy Learning Control

Now, based on the previous fuzzy system representation (6) of nonlinear dynamical systems (1), we construct a fuzzy learning control strategy. We assume in this section that the parameter θ^* which realizes the optimum fuzzy representation is fully known. The next section deals with the case when the parameters are not completely known. Now, we propose the fuzzy learning control law for (1) as follows:

$$u^j = c^j + u_f^j + u_s^j + h^j, \tag{7}$$

where

$$\begin{aligned} c^j &= Y_s^j\theta^*; \quad u_f^j = -(\beta k + k_g|\dot{x}^j|)z^j; \\ u_s^j &= l_e(|e^j| + |\dot{e}^j|)sgn(z^j); \\ h^j &= proj(\bar{h}^j); \quad \bar{h}^j = h^j - \beta k z^{j-1}; \end{aligned} \tag{8}$$

where $k_g \geq r_g$ and $sgn(z^j)$ is 1 when $z^j \geq 0$ and -1 when $z^j < 0$; $proj(\bar{h}^j)$ is the projection operator defined by

$$proj(\bar{h}^j) = \begin{cases} u_d^b & \text{if } \bar{h}^j \geq u_d^b \\ -u_d^b & \text{if } \bar{h}^j \leq -u_d^b \\ \bar{h}^j & \text{otherwise.} \end{cases}$$

u_f^j is a linear feedback term that stabilizes the overall closed loop system. c^j is a fuzzy logic representation of the nonlinear term $(g^j - g_d)\ddot{x}_d - a\dot{e}^j g^j + f^j - f_d$. Its main effect is to reduce the control load from the feedback term u_f^j and maintains small feedback gain k. Since k is also used in the learning controller (8), small value of k can help achieve smoother performance of the learning rule (8). u_s^j is a switching type control term that compensates for the fuzzy approximation error. Note that the gains of u_s^j converges to zero as error converges to zero. h^j is a learning rule that estimates and compensates for the desired control input u_d. The projection operator guarantees that h^j is bounded. Note that the proposed learning control law does not use any derivative of output feedback [1, 2, 4].
Substituting (7) into (6), we have

$$g^j\dot{z}^j + \beta k z^j + k_g|\dot{x}^j|z^j = \tilde{u}^j - w_e^j + u_s^j, \tag{9}$$

where $\tilde{u}^j = h^j - u_d$. Now, we prove that the proposed controller (7) keeps the tracking errors z^j, e^j and $\dot{e}^j$ bounded and drives them to zero.

Theorem 1 The fuzzy learning control system (9) is bounded;

$$|z^j(t)| \leq (\frac{1}{\beta l_1}v^1(t))^{\frac{1}{2}},$$

and converges as follows:

$$\begin{aligned} &i) \quad \lim_{j\to\infty} v^j(t) = v(t) \\ &ii) \quad \lim_{j\to\infty} z^j(t) = 0 \quad \text{for all } t \in [0, t_f], \end{aligned}$$

where $v^j(t) = \int_0^t \frac{1}{k}\tilde{u}^{j^2}(\tau)d\tau$ for all $t \in [0, t_f]$ and for all $j \geq 1$.

Proof. First, we show that $|z^j|$ is bounded. Let $\bar{\tilde{u}}^j$ be $\bar{h}^j - u_d$. Then, $|\bar{\tilde{u}}^j| \geq |\tilde{u}^j|$, and $v^{j+1}(t) - v^j(t) \leq \bar{v}^{j+1}(t) - v^j(t)$, where $\bar{v}^j(t) = \int_0^t \frac{1}{k}\bar{\tilde{u}}^{j^2} d\tau$. Let $\Delta\tilde{u}^j$ be $\bar{\tilde{u}}^{j+1} - \tilde{u}^j$. Then, $\Delta\tilde{u}^j = \bar{\tilde{u}}^{j+1} - \tilde{u}^j = \bar{h}^{j+1} - h^j = -\beta k z^j$, and

$$\begin{aligned} & v^{j+1}(t) - v^j(t) \leq \bar{v}^{j+1}(t) - v^j(t) \\ = & \int_0^t (\frac{1}{k}\bar{\tilde{u}}^{j+1^2} - \frac{1}{k}\tilde{u}^{j^2})d\tau \\ = & \int_0^t (\frac{1}{k}\Delta\tilde{u}^{j^2} + \frac{2}{k}\Delta\tilde{u}^j\tilde{u}^j)d\tau \\ = & \int_0^t (\beta^2 k z^{j^2} - 2\beta z^j(g\dot{z}^j + \beta k z^j + k_g|\dot{x}^j|z^j - w_e^j + u_s^j) \\ \leq & \int_0^t (-\beta^2 k z^{j^2} - 2\beta z^j g^j \dot{z}^j - k_g|\dot{x}^j|z^{j^2})d\tau \\ \leq & -\beta z^{j^2} g^j. \end{aligned}$$

Thus, $v^1 \geq v^j - v^{j+1} \geq \beta g^j z^{j^2}$ for all $j \geq 1$, which proves that $|z^j|$ is bounded. Next, since v^j is positive definite and monotonically decreasing, it converges to some function v, which is (i). From the convergence of v^j, it follows that there exist a positive integer N for all $\sigma > 0$ such that $|v^{j+1}(t) - v^j(t)| \leq \sigma$ for all $j \geq N$. If we choose $\sigma = \beta l_1\epsilon^2$ given any $\epsilon > 0$, then we have

$$\beta l_1|z^j(t)|^2 \leq v^j(t) - v^{j+1}(t) \leq \beta l_1\epsilon^2 \qquad \text{for all } j \geq N. \tag{10}$$

Hence, it follows that $|z^j(t)| \leq \epsilon$ for any $\epsilon > 0$, from which (ii) follows. Q.E.D.

Since z^j converges to zero and $e^j(0) = \dot{e}^j(0) = 0$, e^j and $\dot{e}^j$ also converge to zero. Note that v^1 decreases as feedback gain k increases. Thus, we can keep the state $(x^j, \dot{x}^j)$ within a compact set U by increasing the feedback gain k. In establishing that the states are bounded, our proof is much simpler than that of Kuc *et al.* [6].

5 Adaptive Fuzzy Learning Control

Since the optimum parameter vector θ^* is not known in general, we need an additional learning algorithm that search for the optimum θ^* and then the nonlinear term $(g^j - g_d)\ddot{x}_d - a\dot{e}^j g^j + f^j - f_d$ as closely as possible. We, thus, introduce a parameter learning algorithm in this section based on the following assumption.

Assumption 4 The bound θ^b of $|(\theta_g^{*T}, \theta_f^{*T})|$ is known.

In order to simplify the convergence proof, let us first in this section modify the feedback control law u_f^j as,

$$u_f^j = -(\beta Y_s^j Y_s^{j^T} + \beta k + k_g|\dot{x}^j|)z^j. \tag{11}$$

Since the parameter vector θ^* is not known, the estimated parameter $\hat{\theta}^j$ is used instead to compute the adaptive fuzzy control input c^j:

$$c^j = Y_s^j \hat{\theta}^j. \tag{12}$$

Substituting (11) and (12) into (7), and then (7) into (6), we have

$$\begin{aligned} & g^j z^j + \beta k z^j + \beta Y_s^j Y_s^{j^T} z^j + k_g|\dot{x}^j|z^j \\ = \;& Y_s^j \tilde{\theta}^j + \tilde{u}^j - w_e^j + u_s^j, \end{aligned} \tag{13}$$

where $\tilde{\theta}^j = \hat{\theta}^j - \theta^*$. At this point, we propose a parameter learning rule as follows:

$$\hat{\theta}^{j+1^T} = proj\{\hat{\bar{\theta}}^{j+1}\} = \{proj(\hat{\bar{\theta}}_1^{j+1}), \cdots, proj(\hat{\bar{\theta}}_N^{j+1})\} \text{ and}$$
$$\hat{\bar{\theta}}^{j+1} = \hat{\theta}^j + \beta Y_s^{j^T} z^j, \tag{14}$$

where

$$proj(\hat{\bar{\theta}}_i^{j+1}) = \begin{cases} \theta^b & \text{if } \hat{\bar{\theta}}^{j+1} \geq \theta_i^b \\ -\theta^b & \text{if } \hat{\bar{\theta}}^{j+1} \leq -\theta_i^b \\ \hat{\bar{\theta}}_i^{j+1} & \text{otherwise.} \end{cases}$$

When the proposed parameter learning rule is used in conjunction with the learning rule (8) in the controller, we can prove the stability and the convergence of the closed-loop system as given below.

Theorem 2 The adaptive fuzzy learning controller with the learning control rule (8) and the parameter learning rule (14) for the uncertain dynamic system (13) is bounded as follows:

$$|z^j(t)| \leq (\frac{1}{\beta l_1} v_a^1(t))^{\frac{1}{2}},$$

and converges as follows:

$$i) \quad \lim_{j\to\infty} v_a^j(t) = v_a(t)$$

$$ii) \quad \lim_{j\to\infty} z^j(t) = 0 \quad \text{for all } t \in [0, t_f],$$

where $v_a^j(t) = \int_0^t (\frac{1}{k}\tilde{u}^{j^2}(\tau) + \tilde{\theta}^{j^T}(\tau)\tilde{\theta}^j(\tau))d\tau$ for all $t \in [0, t_f]$ and $j \geq 1$.

Proof: Similar to that of Theorem 1

Note here that, v^1 decreases as feedback gain k increases or $\hat{\theta}^j$ comes closer to θ^*. Thus, we can keep the state $(x^j, \dot{x}^j)$ within the preset compact set U by increasing the feedback gain k or by choosing $\hat{\theta}^j$ appropriately.

6 Conclusions

An adaptive fuzzy learning control scheme is presented for a class of nonlinear dynamic systems where an exact linear parametrization of the dynamics is not possible. The controller compensates for the nonlinear term via adaptive fuzzy system. The fuzzy compensation term reduces the control load from the feedback controller term and keep the feedback gain values reasonably small. The learning controller achieves precise tracking without using any output derivative terms which are vulnerable to noise. In contrast with other learning controllers, our parameter learning rule achieves the error convergence under virtually no conditions.

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