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Repetitive Learning Method for Trajectory Control of Robot Manipulators Using Disturbance Observer

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Abstracts A novel iterative learning control scheme comprising a unique feedforward learning controller and a disturbance observer is proposed. Disturbance observer compensates disturbance due to parameter variations, mechanical nonlinearities, unmodeled dynamics and external disturbances. The convergence and robustness of the proposed controller is proved by the method based on Lyapunov stability theorem. The results of numerical simulation are shown to verify the effectiveness of the proposed control scheme.

Keywords Learning control, Disturbance observer, Robot manipulators, Lyapunov stability

1. INTRODUCTION

Most robots used in practice are designed to do same work repeatedly. Learning control is a new control scheme arises from the recognition of this feature. Learning controller compensates nonlinearity and improves system performance continuously without accurate modeling about the system using previous information of repetitive operation. Motivated by the idea that robot can learn autonomously from previous measurement data, Arimoto and his research group[1] proposed a learning control scheme based on a simple iteration rule. To optimize control input, optimal learning control scheme was proposed[3, 7]. Bondi *et al.*[2] proposed learning control theory for robot manipulators using high gain feedback. The robustness problems of learning controller have been investigated by a number of researchers[4].

To cancel out the disturbance and to make it easy to design learning controller, we use a disturbance observer. Ohnishi[6] proposed firstly the concept of disturbance observer as compensating unknown disturbance. Umeno *et al.*[8] redesigned disturbance observer using TDOF(Two-degree of freedom) controller.

In this paper, combining disturbance observer and a learning controller, we design a novel repetitive learning control method which has the advantages of each control scheme.

For the design of controllers, we assume the following properties.

ASSUMPTION 1 *Following properties are satisfied in the operations.*

1. *Every operation of the system is periodic and ends in a finite time interval T i.e., $t \in [0, T]$.*
2. *The system dynamics is maintained through operations.*
3. *Each time the output trajectory $\boldsymbol{y}^i(t)$ may contain small high frequency noise $\boldsymbol{\xi}(t)$.*

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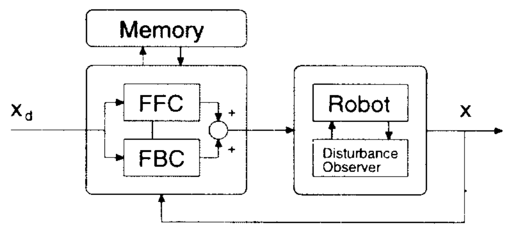


Fig. 1 Block diagram of control system

Fig. 1 shows the overall controller structure which has disturbance observer and learning controller. Each part will be explained in the next sections.

2. DISTURBANCE OBSERVER

Consider the dynamics of rigid robot manipulators described by

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q} \in \Re^n$ is the vector of joint coordinates, $\boldsymbol{M}(\boldsymbol{q}) \in \Re^{n\times n}$ is a positive definite inertia matrix, $\boldsymbol{\tau} \in \Re^n$ is a vector of the generalized joint forces, and $\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^n$ is a vector of nonliner forces written by

$$\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{v}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{B}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{\tau}_{ud}. \tag{2}$$

$\boldsymbol{v}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \Re^n$ is vector of centrifugal, Coriolis forces. $\boldsymbol{B}\dot{\boldsymbol{q}} \in \Re^n$ is a vector of viscous frictional forces, $\boldsymbol{g}(\boldsymbol{q}) \in \Re^n$ is a vector of gravity forces, $\boldsymbol{\tau}_{ud} \in \Re^n$ is a vector due to unmodeled dynamics or unknown disturbance.

From Eq.(1), the dynamic equations of motion of robot manipulators can be expressed as

$$\boldsymbol{M}_n\ddot{\boldsymbol{q}} + \boldsymbol{B}_n\dot{\boldsymbol{q}} = \boldsymbol{\tau} - \boldsymbol{\tau}_d \tag{3}$$

$$\text{with} \quad \boldsymbol{\tau}_d = \triangle\boldsymbol{M}\ddot{\boldsymbol{q}} + \triangle\boldsymbol{B}\dot{\boldsymbol{q}} + \boldsymbol{\tau}_{nd} \tag{4}$$

where $\boldsymbol{M}_n \in \Re^{n\times n}$ is a positive definite inertia matrix of nominal system, $\boldsymbol{B}_n \in \Re^{n\times n}$ is a positive definite viscous term of nominal system, $\boldsymbol{\tau}_d \in \Re^n$ is a vector of total disturbance force, $\triangle\boldsymbol{M} = \boldsymbol{M} - \boldsymbol{M}_n \in \Re^{n\times n}$ is a matrix due to

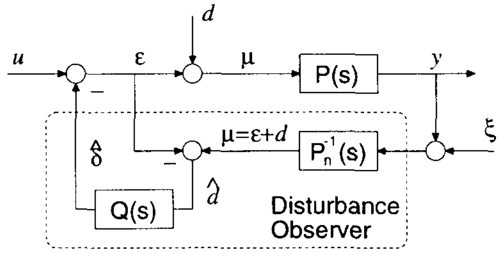


Fig. 2 Realizable form of disturbance observer

uncertainty in inertia, $\triangle \boldsymbol{B} = \boldsymbol{B} - \boldsymbol{B}_n \in \Re^{n\times n}$ is a matrix due to uncertainty in viscous friction, and $\boldsymbol{\tau}_{nd} \in \Re^n$ is a vector of the remained nonlinear forces.

The realizable form of disturbance observer for the SISO system is shown in Fig. 2[5]. In Fig. 2, $P(s)$ is the real system, $P_n(s)$ is the nominal system, and $Q(s)$ is a low-pass filter.

From the block diagram in Fig. 2, following equation can be obtained.

$$y = G_{uy}(s)u + G_{dy}d + G_{\xi y}(s)\xi \tag{5}$$

where

$$G_{uy} = \frac{PP_n}{P_n + (P - P_n)Q} \tag{6}$$

$$G_{dy} = \frac{PP_n(1-Q)}{P_n + (P - P_n)Q} \tag{7}$$

$$G_{\xi y} = -\frac{PQ}{P_n + (P - P_n)Q}. \tag{8}$$

If $Q(s) \approx 1$, then from Eq.(6)-(8)

$$G_{uy} \approx P_n, \;\; G_{dy} \approx 0, \;\; G_{\xi y} \approx -1. \tag{9}$$

This indicates that low frequency disturbance are canceled and mismatch between real system and nominal system is compensated for command signals in the low frequency. And hence, disturbance observer make behavior of the real system as a nominal system. On the other hand, if $Q(s) \approx 0$, these transfer functions are written as

$$G_{uy} \approx P, \;\; G_{dy} \approx P, \;\; G_{\xi y} \approx 0. \tag{10}$$

And we know that sensor noise is blocked, and the open loop dynamics will be observed. Therefore, we design $Q(s)$ to remain close to 1 at low frequencies for disturbance rejection and model uncertainties, and $Q(s)$ to be very small in high frequency region to reject sensor noise.

3. DESIGN OF LEARNING CONTROLLERS

In low frequency, the dynamics of robot manipulator is given by

$$\boldsymbol{M}_n\ddot{\boldsymbol{q}} + \boldsymbol{B}_n\dot{\boldsymbol{q}} + \tilde{\boldsymbol{\tau}}_d = \boldsymbol{\tau} \tag{11}$$

where $\tilde{\boldsymbol{\tau}}_d \in \Re^n$ is a vector due to nonlinearity of the compensated system by disturbance observer, and we can know that the dynamics given by

$$\boldsymbol{\tau}_r = \boldsymbol{M}_n\ddot{\boldsymbol{q}}_d + \boldsymbol{B}_n\dot{\boldsymbol{q}}_d + \tilde{\boldsymbol{\tau}}_d \tag{12}$$

are repeatable if the desired trajectory is periodic. Therefore we assumed previously that the desired trajectory is periodic with period T. This assumption allows us to write

$$\boldsymbol{\tau}_r(t) = \boldsymbol{\tau}_r(t-T) \tag{13}$$

since the dynamics represented by $\boldsymbol{\tau}_r(t)$ depend only on periodic quantities.

Firstly, we design feedback control law as follows

$$\boldsymbol{\tau}_{fb} = \boldsymbol{K}_v\boldsymbol{r} + \boldsymbol{K}_p\boldsymbol{e} + (\boldsymbol{M}_n - \boldsymbol{B}_n)\dot{\boldsymbol{e}} \tag{14}$$

where $\boldsymbol{K}_v \in \Re^{n\times n}, \boldsymbol{K}_p \in \Re^{n\times n}$ are constant diagonal control gains, and the filtered tracking error is defined as

$$\boldsymbol{r} = \boldsymbol{e} + \dot{\boldsymbol{e}}. \tag{15}$$

As using the property of the dynamics given by Eq.(12), we design learning control law as follows

$$\boldsymbol{\tau}_l(t) = \boldsymbol{\tau}_l(t-T) + \boldsymbol{K}_l\boldsymbol{r} \tag{16}$$

where $\boldsymbol{K}_l \in \Re^{n\times n}$ is a positive control gain. This learning law is used to compensate for the repeatable dynamics $\tilde{\boldsymbol{\tau}}_d(t)$. Therefore the control law is formulate as

$$\boldsymbol{\tau}_c = \boldsymbol{\tau}_l + \boldsymbol{K}_v\boldsymbol{r} + \boldsymbol{K}_p\boldsymbol{e} + (\boldsymbol{M}_n - \boldsymbol{B}_n)\dot{\boldsymbol{e}}. \tag{17}$$

We can write the learning law given in Eq.(16) in terms of the learning error, which is defined as

$$\tilde{\boldsymbol{\tau}}_r(t) = \boldsymbol{\tau}_r(t) - \boldsymbol{\tau}_l(t). \tag{18}$$

From Eq.(16), we can formulate as follows

$$\boldsymbol{\tau}_r(t) - \boldsymbol{\tau}_l(t) = \boldsymbol{\tau}_r(t) - \boldsymbol{\tau}_l(t-T) - \boldsymbol{K}_l\boldsymbol{r}. \tag{19}$$

By utilizing the periodic assumption given by Eq.(13), we can write Eq.(19) as

$$\boldsymbol{\tau}_r(t) - \boldsymbol{\tau}_l(t) = \boldsymbol{\tau}_r(t-T) - \boldsymbol{\tau}_l(t-T) - \boldsymbol{K}_l\boldsymbol{r} \tag{20}$$

which gives the learning error update rule

$$\tilde{\boldsymbol{\tau}}_r(t) = \tilde{\boldsymbol{\tau}}_r(t-T) - \boldsymbol{K}_l\boldsymbol{r}. \tag{21}$$

To analyze the stability of the controller given by Eq.(17), we must form the corresponding error system. First, we rewrite Eq.(11) in terms of $\boldsymbol{r}$ defined in Eq.(15). That is, we have

$$\boldsymbol{M}_n\dot{\boldsymbol{r}} = \boldsymbol{\tau}_a - \boldsymbol{\tau} \tag{22}$$

where $\boldsymbol{\tau}_a$ is given as

$$\boldsymbol{\tau}_a = \boldsymbol{M}_n(\ddot{\boldsymbol{q}}_d + \dot{\boldsymbol{e}}) + \boldsymbol{B}_n\dot{\boldsymbol{q}} + \tilde{\boldsymbol{\tau}}_d. \tag{23}$$

Adding and subtracting the term $\boldsymbol{\tau}_a$ on the right-hand side of Eq.(22) yields

$$\boldsymbol{M}_n\dot{\boldsymbol{r}} = \boldsymbol{\tau}_r(t) + \tilde{\mathcal{T}} - \boldsymbol{\tau} \tag{24}$$

where $\tilde{\mathcal{T}}$ is defined as

$$\tilde{\mathcal{T}} = \boldsymbol{\tau}_a(t) - \boldsymbol{\tau}_r(t). \tag{25}$$

This difference between $\boldsymbol{\tau}_a(t)$ and $\boldsymbol{\tau}_r(t)$ can be written as follows

$$\tilde{\mathcal{T}} = (\boldsymbol{M}_n - \boldsymbol{B}_n)\dot{\boldsymbol{e}}. \tag{26}$$

Substituting the control law given by Eq.(17) into Eq.(24) yields the error system

$$\boldsymbol{M}_n\dot{\boldsymbol{r}} = \boldsymbol{\tau}_r(t) + \tilde{\mathcal{T}} - (\boldsymbol{\tau}_l + \boldsymbol{K}_v\boldsymbol{r} + \boldsymbol{K}_p\boldsymbol{e} + (\boldsymbol{M}_n - \boldsymbol{B}_n)\dot{\boldsymbol{e}}) \tag{27}$$

where, from the Eq.(26) we can write this equation as

$$\boldsymbol{M}_n\dot{\boldsymbol{r}} = -\boldsymbol{K}_v\boldsymbol{r} - \boldsymbol{K}_p\boldsymbol{e} + \tilde{\boldsymbol{\tau}}_r(t). \tag{28}$$

1) Analysis of the stability

We now analyze the stability of the error system given by Eq.(28) with the Lyapunov-like function

$$V = \frac{1}{2}\boldsymbol{r}^T\boldsymbol{M}_n\boldsymbol{r} + \frac{1}{2}\boldsymbol{K}_p\boldsymbol{e}^T\boldsymbol{e} + \frac{1}{2\boldsymbol{K}_l}\int_{t-T}^{t} \tilde{\boldsymbol{\tau}}_r^T(\sigma)\tilde{\boldsymbol{\tau}}_r(\sigma)d\sigma. \tag{29}$$

Differentiating Eq.(29) with respect to time yields

$$\begin{aligned}\dot{V} = & \frac{1}{2}\boldsymbol{r}^T\dot{\boldsymbol{M}}_n\boldsymbol{r} + \boldsymbol{r}^T\boldsymbol{M}_n\dot{\boldsymbol{r}} + \boldsymbol{K}_p\boldsymbol{e}^T\dot{\boldsymbol{e}} \\ & + \frac{1}{2\boldsymbol{K}_l}\left(\tilde{\boldsymbol{\tau}}_r^T(t)\tilde{\boldsymbol{\tau}}_r(t) - \tilde{\boldsymbol{\tau}}_r^T(t-T)\tilde{\boldsymbol{\tau}}_r(t-T)\right). \end{aligned} \tag{30}$$

Substituting the error system given by Eq.(28) into Eq.(30) yields

$$\begin{aligned}\dot{V} = & -\boldsymbol{K}_v\boldsymbol{r}^T\boldsymbol{r} - \boldsymbol{K}_p\boldsymbol{r}^T\boldsymbol{e} + \boldsymbol{K}_p\boldsymbol{e}^T\dot{\boldsymbol{e}} + \boldsymbol{r}^T\tilde{\boldsymbol{\tau}}_r(t) \\ & + \frac{1}{2\boldsymbol{K}_l}\left(\tilde{\boldsymbol{\tau}}_r^T(t)\tilde{\boldsymbol{\tau}}_r(t) - \tilde{\boldsymbol{\tau}}_r^T(t-T)\tilde{\boldsymbol{\tau}}_r(t-T)\right). \end{aligned} \tag{31}$$

The second line in Eq.(31) can be written as follows

$$\begin{aligned}\boldsymbol{r}^T\tilde{\boldsymbol{\tau}}_r(t) & + \frac{1}{2\boldsymbol{K}_l}\left(\tilde{\boldsymbol{\tau}}_r^T(t)\tilde{\boldsymbol{\tau}}_r(t) - \tilde{\boldsymbol{\tau}}_r^T(t-T)\tilde{\boldsymbol{\tau}}_r(t-T)\right) \\ & = -\frac{1}{2}\boldsymbol{K}_l\boldsymbol{r}^T\boldsymbol{r}. \end{aligned} \tag{32}$$

Therefore, we can simplify Eq.(31)

$$\dot{V} = -\boldsymbol{K}_p\boldsymbol{e}^T\boldsymbol{e} - (\boldsymbol{K}_v + \frac{1}{2}\boldsymbol{K}_l)\boldsymbol{r}^T\boldsymbol{r}. \tag{33}$$

From Eq.(33), we can place an upper bound on $\dot{V}$ in the following manner

$$\dot{V} \leq -\boldsymbol{K}_p\|e\|^2 - (\boldsymbol{K}_v + \frac{1}{2}\boldsymbol{K}_l)\|r\|^2. \tag{34}$$

By rewriting Eq.(34) in the matrix form

$$\dot{V} \leq -\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} \tag{35}$$

where

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{K}_p & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_v + \frac{1}{2}\boldsymbol{K}_l \end{bmatrix} \quad \text{and} \quad \boldsymbol{x} = \begin{bmatrix} \|\boldsymbol{e}\| \\ \|\boldsymbol{r}\| \end{bmatrix}.$$

The matrix $\boldsymbol{Q}$ defined in Eq.(35) is positive definite, therefore $\dot{V}$ will be negative semidefinite. By the *Rayleigh-Ritz Theorem*, Eq.(35) can be written

$$\dot{V} \leq -\lambda_{min}\{\boldsymbol{Q}\}\|\boldsymbol{x}\|^2. \tag{36}$$

Now, we detail the type of stability for the tracking error. First note from Eq.(36) that we can place the new upper bound on $\dot{V}$

$$\dot{V} \leq -\lambda_{min}\{\boldsymbol{Q}\}\|\boldsymbol{r}\|^2 \tag{37}$$

which can also be written as

$$\int_0^{\infty} \dot{V}(\sigma)d\sigma \leq -\lambda_{min}\{\boldsymbol{Q}\}\int_0^{\infty} \|\boldsymbol{r}(\sigma)\|^2 d\sigma. \tag{38}$$

Multiplying Eq.(38) by -1 and integrating the left-hand side of Eq.(38) yields

$$V(0) - V(\infty) \geq \lambda_{min}\{\boldsymbol{Q}\}\int_0^{\infty} \|\boldsymbol{r}(\sigma)\|^2 d\sigma. \tag{39}$$

Since $\dot{V}$ is negative semidefinite as delineated by Eq.(36), we can state that V is a nonincreasing function that is upper bounded by $V(0)$. By recalling that $\boldsymbol{M}(\boldsymbol{q})$ is lower bounded as delineated by the positive-definite property of the inertia matrix, we can state that V given in Eq.(29) is lower bounded by zero. Since V is nonincreasing, upper bounded by $V(0)$, and lower bounded by zero, we can write Eq.(39) as

$$\lambda_{min}\{\boldsymbol{Q}\}\int_0^{\infty} \|\boldsymbol{r}(\sigma)\|^2 d\sigma \leq \infty \tag{40}$$

or

$$\sqrt{\int_0^{\infty} \|\boldsymbol{r}(\sigma)\|^2 d\sigma} \leq \infty. \tag{41}$$

The bound delineated by Eq.(41) informs us that $\boldsymbol{r} \in L_2^n$, which means that the filtered tracking error $\boldsymbol{r}$ is bounded by Eq.(41). From the definition of $\boldsymbol{r}$ given in Eq.(15), we can state $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$ are bounded. Since $\boldsymbol{e}, \dot{\boldsymbol{e}}, \boldsymbol{r}$ are bounded, we can use Eq.(28) to show that $\dot{\boldsymbol{r}}$, and hence $\ddot{V}$ in Eq.(33) are bounded. Therefore V is lower bounded, $\dot{v}$ is negative semidefinite, and $\ddot{V}$ is bounded, we can use *Barbalat's lemma* to state that

$$\lim_{t\to\infty} \dot{V} = 0 \tag{42}$$

which means that by the *Rayleigh-Ritz Theorem*

$$\lim_{t\to\infty} \lambda_{min}\{\boldsymbol{Q}\}\|\boldsymbol{x}\|^2 = 0 \quad \text{or} \quad \lim_{t\to\infty} \boldsymbol{x} = \boldsymbol{0}. \tag{43}$$

To establish a stability result for the position tracking error $\boldsymbol{e}$, we establish the transfer function relationship between the position tracking error and the filtered tracking error $\boldsymbol{r}$. From Eq.(15), we can state that

$$\boldsymbol{e}(s) = \boldsymbol{H}(s)\boldsymbol{r}(s) \tag{44}$$

where s is the Laplace transform variable, and $\boldsymbol{H}(s)$ is written as

$$\boldsymbol{H}(s) = (s\boldsymbol{I} + \boldsymbol{I})^{-1}. \tag{45}$$

Since $\boldsymbol{H}(s)$ is a strictly proper, asymptotically stable transfer function matrix and $\boldsymbol{r} \in L_2^n$, we can state that

$$\lim_{t\to\infty} \boldsymbol{e} = \boldsymbol{0}. \tag{46}$$

And therefore

$$\lim_{t\to\infty} \dot{\boldsymbol{e}} = \boldsymbol{0}. \tag{47}$$

The result above informs us that the tracking error $\boldsymbol{e}$ and $\dot{\boldsymbol{e}}$ are asymptotically stable.

4. NUMERICAL SIMULATIONS

Let us consider the manipulator in Fig. 3. The link masses, inertias, lengths, and joint friction coefficients for the simulation are given in Table 1. For disturbance observer, nominal systems are selected for each link as follows,

$$P_n{}^1(s) = \frac{1}{1.0s + 1.0} \qquad P_n{}^2(s) = \frac{1}{0.4s + 1.0}. \tag{48}$$

For feedback controller, following gain matrix is selected.

$$\boldsymbol{K}_p = \begin{pmatrix} 300 & 0 \\ 0 & 300 \end{pmatrix} \qquad \boldsymbol{K}_v = \begin{pmatrix} 20 & 0 \\ 0 & 20 \end{pmatrix}. \tag{49}$$

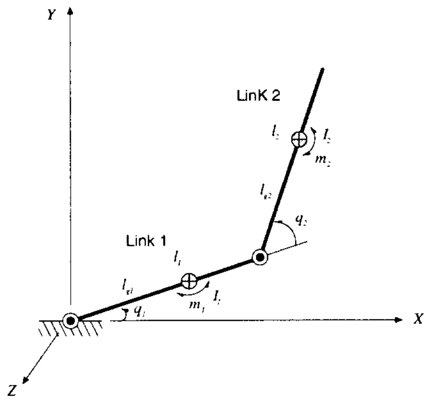
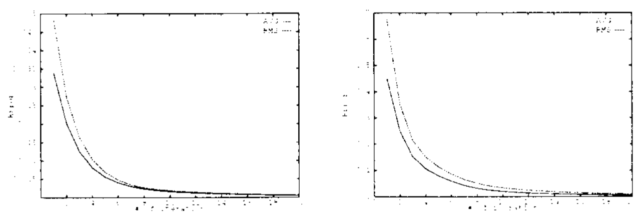


Fig. 3 Two-link manipulator

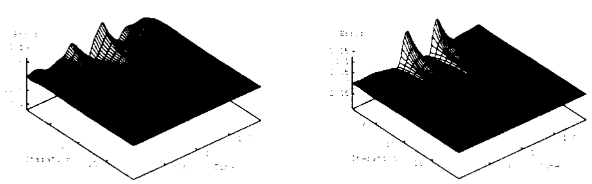
Table 1 Kinematic and dynamic parameters

	link 1	link 2
mass(m)	10 kg	10 kg
inertia(I)	0.2 $kg\ m^2$	0.15 $kg\ m^2$
length(l)	0.5 m	0.5 m
length(l_g)	0.2 m	0.2 m
friction(b)	1.5 $kg\ m^2/s$	1.0 $kg\ m^2/s$



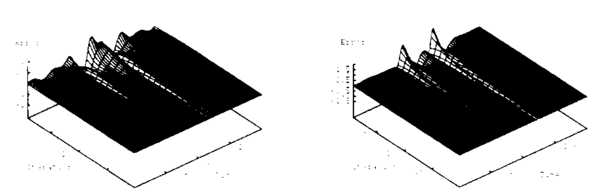
a) Position error b) Velocity error

Fig. 4 Cartesian error profile



a) X axis error a) Y axis error

Fig. 5 3-dimensional Cartesian position error profile



a) X axis error a) Y axis error

Fig. 6 3-dimensional Cartesian velocity error profile

The gain $\boldsymbol{K}_l$ for learning controller is chosen as

$$\boldsymbol{K}_l = \begin{pmatrix} 20 & 0 \\ 0 & 20 \end{pmatrix}. \tag{50}$$

Circle of radius $25cm$ is to be traced in $2sec$. 20 times of repetitive operation is commanded. In the first operation, since there is no measured data previously, the learning controller is not active in this case. Consequently, system performance is dependent on mostly the disturbance observer. The robot follows trajectory within small bounds. Fig. 4 shows RMS(root mean square) error and average error for number of iterations. Fig. 5, Fig. 6 shows 3-dimensional Cartesian position and velocity errors respectively.

5. CONCLUSION

In this paper, a new repetitive learning controller with robustness property using disturbance observer was proposed. By using disturbance observer, whole system behaved like nominal system in low frequency, and high frequency noise could be rejected. To acquire better tracking performance, the learning controller was used as a prefilter for the desired trajectory to compensate for the dynamic lag of the closed-loop system and to extend the effective tracking bandwidth.

The convergence of learning controllers was proved based on Lyapunov stability theorem. Through the numerical simulation, the proposed control scheme is verified.

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