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Nonlinear H_∞ Control for Linear Systems using Nonlinear Weight

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Abstract This study deals with the nonlinear H_∞ control problem of linear system using nonlinear weight. Generally the solvable condition of nonlinear H_∞ control problem is given by the Hamilton Jacobi equality or inequality, but it is very difficult to solve. In this study, some constraints of nonlinear weight reduce the solvable condition to linear Riccati equation. Some examples of the control system design using nonlinear weight are shown.
Keywords : *nonlinear H_∞ control, Hamilton-Jacobi inequality, nonlinear weight*

1 Introduction

In this paper, we consider nonlinear H_∞ control problems for linear system with nonlinear weight. In other words, the state equation of the generalized plant is linear, and the equation of controlled output is nonlinear. It is knows that the solvable condition for the nonlinear H_∞ control problem is given by the Hamilton-Jacobi equation or inequality. [1, 2, 3, 4]. Since the Hamilton-Jacobi equation, which is a partial differential equation, is very difficult to solve. On the other hand it is known that local version of Hamilton-Jacobi equation for linear system is expressed as Algebraic Riccati equation, that is easy to calculate. We derives some conditions of nonlinear weight for reducing the solvability condition of a Hamilton-Jacobi equation to a Algebraic Riccati equation.

We use the following notation. $\boldsymbol{x} \in \boldsymbol{R}^n$ denotes an n-dimensional real vector, and Euclidean norm is defined by $\|\cdot\|$. $\boldsymbol{u}(t) : \boldsymbol{R} \rightarrow \boldsymbol{R}^n$ on $[t_0, \infty]$, let $\boldsymbol{L}_2$ be a set of measurable function on $[t_0, \infty]$ with $\int_{t_0}^{\infty} \|\boldsymbol{u}(t)\|^2 dt < \infty$, its norm is defined by $\|\boldsymbol{u}\|_2 \stackrel{\text{def}}{=} (\int_{t_0}^{\infty} \|\boldsymbol{u}(t)\|^2 dt)^{\frac{1}{2}}$, and $\|\boldsymbol{u}\|_2 \neq 0$. $\boldsymbol{L}_2/\{0\}$ denotes $\boldsymbol{L}_2$ with $\boldsymbol{u} \in \boldsymbol{L}_2$.

Consider the following nonlinear system $\boldsymbol{S}_{zw}$.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{w} \\ \boldsymbol{z} &= \boldsymbol{h}(\boldsymbol{x}) \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{w} \in \boldsymbol{R}^m, \boldsymbol{z} \in \boldsymbol{R}^p$ is a state vector, an external input and a control output, respectively. $\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{h}(\boldsymbol{x})$ are smooth vector functions with $\boldsymbol{f}(\boldsymbol{o}) = \boldsymbol{o}, \boldsymbol{h}(\boldsymbol{o}) = \boldsymbol{o}$. $\boldsymbol{g}(\boldsymbol{x})$ is matrix function. $\boldsymbol{L}_2$-gain for $\boldsymbol{S}_{zw}$ is defined as follows.

$$\|\boldsymbol{S}_{zw}\|_{L2c} = \sup_{\boldsymbol{w} \in \boldsymbol{L}_2/\{0\}} \frac{\|\boldsymbol{z}\|_2}{\|\boldsymbol{w}\|_2}$$

It is known that following theorem.

Theorem 1 *The system $\boldsymbol{S}_{zw}$(1) is exponential stable and $\|\boldsymbol{S}_{zw}\|_{L2c} < \gamma$, if and only if there exist a positive constant ε, and a positive definite solution $\phi(\boldsymbol{x})$ of the following inequality. [1, 2, 3, 4].*

$$\frac{\partial \phi}{\partial \boldsymbol{x}^T}\boldsymbol{f} + \frac{1}{4\gamma^2}\frac{\partial \phi}{\partial \boldsymbol{x}^T}\boldsymbol{g}\boldsymbol{g}^T\frac{\partial \phi}{\partial \boldsymbol{x}} + \boldsymbol{h}^T\boldsymbol{h} + \varepsilon \boldsymbol{x}^T\boldsymbol{x} \leq 0 \tag{2}$$

2 Problem Definition

In this paper, we consider the following system.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{w} + \boldsymbol{B}_2\boldsymbol{u} \\ \boldsymbol{z} &= (1 + l(\boldsymbol{x}))\boldsymbol{C}_1\boldsymbol{x} + \boldsymbol{D}_{12}(\boldsymbol{x})\boldsymbol{u} \end{aligned} \tag{3}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n, \boldsymbol{w} \in \boldsymbol{R}^{m_1}, \boldsymbol{u} \in \boldsymbol{R}^{m_2}, \boldsymbol{z} \in \boldsymbol{R}^p$ is a state vector, an external input, a control input, a control output, respectively. $\boldsymbol{A}, \boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{C}_1$ are constant matrices. We assume the following condition. $\boldsymbol{D}_{12}^T\boldsymbol{C}_1 = \boldsymbol{O}, \boldsymbol{D}_{12}^T\boldsymbol{D}_{12} = a^2(\boldsymbol{x})\boldsymbol{I}$, where $l(\boldsymbol{x}), a(\boldsymbol{x})$ is a scalar function with $1 + l(\boldsymbol{x}) > 0, a(\boldsymbol{x}) > 0, l(\boldsymbol{o}) = 0, a(\boldsymbol{o}) = 1$

Notice that the state equation of the system (3) is linear, but the equation of control output is nonlinear. From assumption, we can consider that $l(\boldsymbol{x}), a(\boldsymbol{x})$ is a nonlinear weight for the state and a control input, respectively.

3 Solvability Condition of Nonlinear H_∞ Control Problems

Let γ be a positive constant. The sufficient condition for which the system $\boldsymbol{S}_{zw}$(3) is internally stable and has $\boldsymbol{L}_2$-gain such that $\|\boldsymbol{S}_{zw}\|_{L2c} < \gamma$, and a state feedback control law which satisfies it are given by following theorem.

Theorem 2 *Consider the system (3). Assume that $l(\boldsymbol{x}), a(\boldsymbol{x})$ are satisfy the following condition.*

$$\begin{aligned} &\left(1 - \frac{1}{a^2}\right) \cdot \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x} \\ &\quad + \left\{(1 + l(\boldsymbol{x}))^2 - 1\right\} \cdot \boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x} \leq 0 \end{aligned} \tag{4}$$

If there exists positive definite symmetric matrix $\boldsymbol{P}$ which satisfies the following Riccati equation.

$$\begin{aligned} \boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} + \frac{1}{\gamma^2}\boldsymbol{P}\boldsymbol{B}_1\boldsymbol{B}_1^T\boldsymbol{P} + \boldsymbol{C}_1^T\boldsymbol{C}_1 + \varepsilon \boldsymbol{I} \\ - \boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P} = \boldsymbol{O} \end{aligned} \tag{5}$$

then the state feedback nonlinear H_∞ control problem is solvable. Where ε is a positive constant. Then a state feedback law is given by

$$\boldsymbol{u} = \boldsymbol{K}(\boldsymbol{x}) = -\frac{1}{a^2}\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x} \tag{6}$$

Proof: If $\boldsymbol{u}$ is expressed by $\boldsymbol{K}(\boldsymbol{x})$, then the closed loop system as follows

$$\begin{aligned} \dot{\boldsymbol{x}} &= (\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{K}(\boldsymbol{x})) + \boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{z} &= (1 + l(\boldsymbol{x}))\boldsymbol{C}_1\boldsymbol{x} + \boldsymbol{D}_{12}(\boldsymbol{x})\boldsymbol{K}(\boldsymbol{x}) \end{aligned} \tag{7}$$

The condition that the above closed loop system is internally stable and has L_2-gain such that $\|\boldsymbol{S}_{zw}\|_{L2c} \leq$

γ, is given by theorem 1. Therefore, we consider a Hamilton-Jacobi inequality(2) for this closed loop system(7). If a positive definite solution ϕ of (2) is expressed as $\phi = \boldsymbol{x}^T \boldsymbol{P} \boldsymbol{x}$, where $\boldsymbol{P}$ is positive definite symmetric matrix, then

$$\frac{\partial \phi}{\partial \boldsymbol{x}} = \boldsymbol{P}\boldsymbol{x} + \boldsymbol{P}^T\boldsymbol{x}, \quad \frac{\partial \phi}{\partial \boldsymbol{x}^T} = \boldsymbol{x}^T\boldsymbol{P} + \boldsymbol{x}^T\boldsymbol{P}^T.$$

The inequality(2) can be written as

$$\begin{aligned} H &= \boldsymbol{x}^T\Big(\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} + \frac{1}{\gamma^2}\boldsymbol{P}\boldsymbol{B}_1\boldsymbol{B}_1^T\boldsymbol{P} + \boldsymbol{C}_1^T\boldsymbol{C}_1 \\ &\quad + \varepsilon \boldsymbol{I} - \boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\Big)\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x} \\ &\quad + 2\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{K} + a^2(\boldsymbol{x})\boldsymbol{K}^T\boldsymbol{K} \\ &\quad + \left\{(1 + l(\boldsymbol{x}))^2 - 1\right\}\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x} \le 0. \end{aligned}$$

If we choose the $\boldsymbol{P}$ as a solution of the Riccati equation(5), then

$$\begin{aligned} H &= a^2(\boldsymbol{x})\boldsymbol{K}^T\boldsymbol{K} + 2\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{K} + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x} \\ &\quad + \left\{(1 + l(\boldsymbol{x}))^2 - 1\right\}\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x} \le 0. \end{aligned}$$

We have by 'completing the squares',

$$\begin{aligned} H &= a^2(\boldsymbol{x})\left(\boldsymbol{K} + \frac{1}{a^2}\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}\right)^T\left(\boldsymbol{K} + \frac{1}{a^2}\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}\right) \\ &\quad + \left(1 - \frac{1}{a^2}\right)\cdot \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x} \\ &\quad + \left\{(1 + l(\boldsymbol{x}))^2 - 1\right\}\cdot\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x} \le 0 \end{aligned}$$

Therefore, a state feedback control law which minimize H is (6). If $a(\boldsymbol{x}), l(\boldsymbol{x})$ satisfy (4), then the Hamilton-Jacobi inequality is satisfied, and the nonlinear H_∞ control problem is solvable.

Note: The first order approximation of the system(3) around the origin is considered as following linear system.

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{w} + \boldsymbol{B}_2\boldsymbol{u} \\ \boldsymbol{z} &= \boldsymbol{C}_1\boldsymbol{x} + \bar{\boldsymbol{D}}_{12}\boldsymbol{u} \end{aligned}$$

$$\bar{\boldsymbol{D}}_{12}^T\boldsymbol{C}_1 = \boldsymbol{O}, \quad \bar{\boldsymbol{D}}_{12}^T\bar{\boldsymbol{D}}_{12} = \boldsymbol{I}$$

The necessary and sufficient condition that this linear system is internally stable and H_∞-norm from $\boldsymbol{w}$ to $\boldsymbol{z}$ is less than γ (In other words, linear H_∞ control problem is solvable.) is given by Riccati equation(5). Then a state feedback law is given by $\boldsymbol{u} = -\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}$ It denotes that the solvability condition of the nonlinear H_∞ control problem for system(3) around origin is determined by Riccati equation(5).

In other words, The solvability condition (5) around the origin is the global solvability condition under the assumption(4) in theorem2. A state feedback control law(6) is same as $\boldsymbol{u} = -\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}$ around the origin.

4 Determination of Nonlinear Weight

For designing control system using theorem 2, The nonlinear weight $l(\boldsymbol{x}), a(\boldsymbol{x})$ has to satisfy the assumption(4). We take notice of the term $l(\boldsymbol{x}), a(\boldsymbol{x})$ and consider two problems. One is $l(\boldsymbol{x}) \ge 0$, the other is $a(\boldsymbol{x}) \ge 1$. In (4),If $l(\boldsymbol{x})$ is greater than or equals to 0, then the second term of left hand side of (4). ($\{(1+l(\boldsymbol{x}))^2 - 1\}\cdot\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}$) is nonnegative. To satisfy (4), the first term ($(1- \ 1/a^2)\cdot\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}$) has to be semi-negative, i.e : $0 < a(\boldsymbol{x}) \le 1$.

The other side, if $a(\boldsymbol{x})$ is greater than or equals to 1, the first term of left hand side of (4) is nonnegative, so the second term has to be semi-negative, i.e. : $-1 < l(\boldsymbol{x}) \le 0$.

It is considered as one of tradeoff. For system (3), we can consider that $l(\boldsymbol{x})$ is nonlinear weight for state $\boldsymbol{x}$, $a(\boldsymbol{x})$ is for control input $\boldsymbol{u}$.

We consider two problems, which the disturbance attenuation problem ($l(\boldsymbol{x}) \ge 0$) and the input limitation problem($a(\boldsymbol{x}) \ge 1$), to determine nonlinear weight $l(\boldsymbol{x}), a(\boldsymbol{x})$ which satisfy (4).

4.1 Disturbance Attenuation Problem

First, we consider the condition $l(\boldsymbol{x}) \ge 0$.

Theorem 3 *If the nonlinear weight $l(\boldsymbol{x}), a(\boldsymbol{x})$ have following form,*

$$l(\boldsymbol{x}) = \sqrt{1 + m_0\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}} - 1 \qquad (8)$$

$$a(\boldsymbol{x}) = \frac{1}{\sqrt{1 + m_0\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}}} \qquad (9)$$

then the nonlinear H_∞ control problem is solvable. Where $\boldsymbol{P}$ is a positive definite solution of Riccati equation (5). and m_0 is an any scalar function such that $m_0 \ge 0$. Then a state feedback control law is given by

$$\boldsymbol{K}(x) = -\left(1 + m_0\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}\right)\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}. \qquad (10)$$

Proof: If $l(\boldsymbol{x}) \ge 0$, then the second term of left hand side of the equation(4) is not negative. So we have to choose $a(\boldsymbol{x})$ which satisfy (4). But when $\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}$ equals to 0 , the first term of (4) equals to 0 regardless $a(\boldsymbol{x})$.

Therefore, the second term has to be

$$\left\{(1 + l(\boldsymbol{x}))^2 - 1\right\}\cdot\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x} = 0.$$

To satisfy this condition, we choose a weight $l(\boldsymbol{x})$ as

$$(1 + l(\boldsymbol{x}))^2 - 1 = m_0\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x},$$

where m_0 is an any scalar function which satisfy $m_0 \ge 0$. Then $l(\boldsymbol{x})$ becomes (8). We rewrite (4), then we have

$$\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{B}_2\boldsymbol{B}_2^T\boldsymbol{P}\boldsymbol{x}\cdot\left(1 - \frac{1}{a^2} + m_0\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}\right) \le 0.$$

So, we have

$$a \le \frac{1}{\sqrt{1 + m_0\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}}}.$$

Then a state feedback control law is given such that (10) by substituting $a = \frac{1}{\sqrt{1+m_0\boldsymbol{x}^T\boldsymbol{C}_1^T\boldsymbol{C}_1\boldsymbol{x}}}$

Note: In theorem 3,we can choose a scalar function m_0 unrestrictedly. In other words, it is free parameter. Moreover, the first order approximation of the controller(10) around the origin is $K = -B_2^T Px$. It is same as linear one. The other side, The further the state from the origin, which implies $m_0 x^T C_1^T C_1 x$ is large , the $K(x)$ has large gain.

4.2 Input Limitation Problem

Next, we consider the condition $a(\boldsymbol{x}) \geq 1$.

Theorem 4 *If the nonlinear weight $l(\boldsymbol{x}), a(\boldsymbol{x})$ have following form,*

$$l(x) = -1 + \sqrt{\frac{1 + m_1 x^T C_1^T C_1 x}{1 + m_1 x^T C_1^T C_1 x + m_1 x^T P B_2 B_2^T P x}} \quad (11)$$

$$a(x) = \sqrt{\frac{1 + m_1 x^T C_1^T C_1 x + m_1 x^T P B_2 B_2^T P x}{1 + m_1 x^T P B_2 B_2^T P x}} \quad (12)$$

then the nonlinear H_∞ control problem is solvable. Where $\boldsymbol{P}$ is a positive definite solution of Riccati equation (5), and m_1 is an any scalar function such that $m_1 \geq 0$. Then a state feedback control law is given by

$$K(x) = -\frac{1 + m_1 x^T P B_2 B_2^T P x}{1 + m_1 x^T C_1^T C_1 x + m_1 x^T P B_2 B_2^T P x} B_2^T P x \quad (13)$$

Proof: It can be proved by the similarly way in theorem (3).

Note: In theorem 4, we can choose a scalar function m_1 unrestrictedly. In other words, it is free parameter. Moreover, the first order approximation of the controller(13) around the origin is $K = -B_2^T Px$. It is same as linear one. The other side, The further the state from the origin, the $K(x)$ has small gain.

5 Numerical Example

In this section, we consider vibration control for 2-degree-of-freedom structure as an example of designing controller with nonlinear weight.

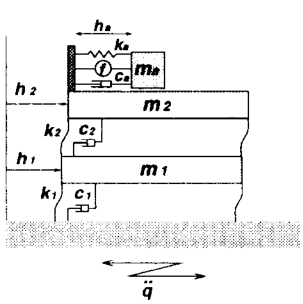

Figure 1:Model of 2-degree-of-freedom structure

5.1 Model

The 2-degree-of-freedom structure is given by Fig:1. When a disturbance $\ddot{q}$ is inputed from the earth, the

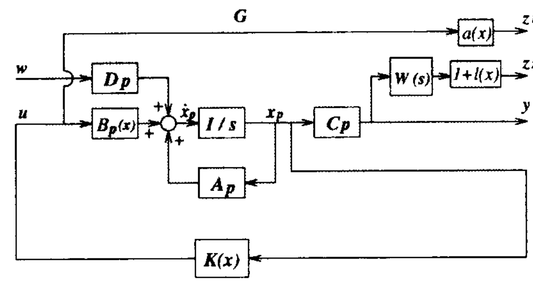

Figure 2:Block diagram of generalized plant

dynamic vibration absorber (DVA), which are installed above the top floor, controls the vibration of the building. This problem is considered as a disturbance attenuation problem. Damping coefficient and Spring constant of the DVA are c_a and k_a, respectively. The control input u is an actuator force f. The DVA like this is called active, and the state equation of this system is linear.

Parameters of the structure on Fig:1 is given as follows $m_1 = m_2 = 2.0, m_a = 0.138$ (kg), $c_1 = c_2 = 0.08, c_a = 0.78$ (N s/m), $k_1 = k_2 = 2600.0, k_a = 62.2$ (N/m) Natural frequency of this structure are 22.2 (rad/sec) which is the first mode and 58.3 (rad/sec) which is the second mode. We choose c_a and k_a, so that in $u = f = 0$ (passive control), the system can be stabilized and the DVA can control the first mode of vibration optimal.

We define the state variables x_p, outputs y such that

$$\begin{aligned} \boldsymbol{x}_p &= \begin{bmatrix} h_1 & h_2 & h_a & \dot{h}_1 & \dot{h}_2 & \dot{h}_a \end{bmatrix}^T \\ \boldsymbol{y} &= \begin{bmatrix} h_1 & (h_2 - h_1) & h_a \end{bmatrix}^T \end{aligned}$$

Then we have the state equation of this system,

$$\begin{aligned} \dot{\boldsymbol{x}}_p &= \boldsymbol{A}_p \boldsymbol{x}_p + \boldsymbol{B}_p u + \boldsymbol{D}_p \ddot{q} \\ \boldsymbol{y} &= \boldsymbol{C}_p \boldsymbol{x}_p \end{aligned} \quad (14)$$

5.2 Generalized Plant

The block diagram of the nonlinear H_∞ state feedback control system is shown by Fig:2. The disturbance w is $\ddot{q}$, and $\boldsymbol{W}$ denotes the linear weight for the output $\boldsymbol{y}$.

$\boldsymbol{W}(s) = diag[W_s, W_s, W_a]$ is given by

$$W_s = \frac{2450.0}{s + 70.0}, \quad W_a = \frac{70.0}{s + 70.0}$$

Express by the state equation,

$$\begin{aligned} \dot{\boldsymbol{x}}_W &= \boldsymbol{A}_W \boldsymbol{x}_W + \boldsymbol{B}_W \boldsymbol{y} \\ \boldsymbol{z}_2 &= \boldsymbol{C}_W \boldsymbol{x}_W + \boldsymbol{D}_W \boldsymbol{y} \end{aligned}$$

The state variables of the generalized plant G is given by $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_p^T & \boldsymbol{x}_W^T \end{bmatrix}^T$, then we have

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1 w + \boldsymbol{B}_2 u \\ \boldsymbol{z} &= (1 + l(\boldsymbol{x}))\boldsymbol{C}_1 \boldsymbol{x} + a(\boldsymbol{x})\boldsymbol{D}_{12} u \\ \boldsymbol{y} &= \boldsymbol{C}_2 \boldsymbol{x} \end{aligned} \quad (15)$$

where $\boldsymbol{A}, \boldsymbol{B}_1, \boldsymbol{B}_2(\boldsymbol{x}), \boldsymbol{C}_1, \boldsymbol{C}_2, \boldsymbol{D}_{12}$

$$A = \begin{bmatrix} A_p & O \\ B_W C_p & A_W \end{bmatrix}, B_1 = \begin{bmatrix} D_p \\ O \end{bmatrix}, B_2 = \begin{bmatrix} B_p \\ O \end{bmatrix}$$

$$C_1 = \begin{bmatrix} O & O \\ D_W C_p & C_W \end{bmatrix}, D_{12} = \begin{bmatrix} 1 \\ o \end{bmatrix}$$

$$C_2 = \begin{bmatrix} C_p & O \end{bmatrix}.$$

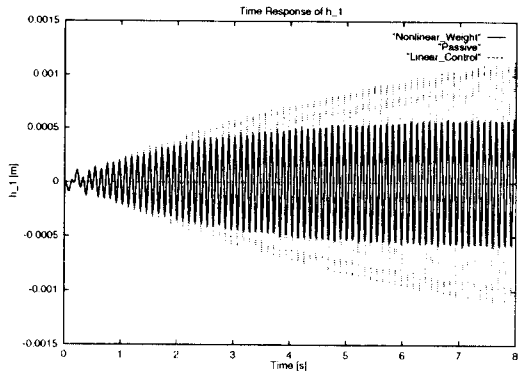

Figure 3:Time responses of h_1:$\ddot{q} = 0.1 \sin 58.3t$

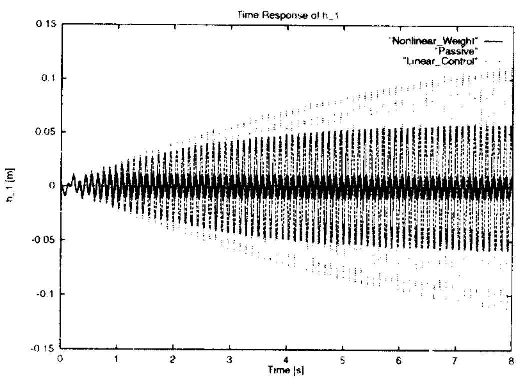

Figure 4:Time responses of h_1:$\ddot{q} = 10.0 \sin 58.3t$

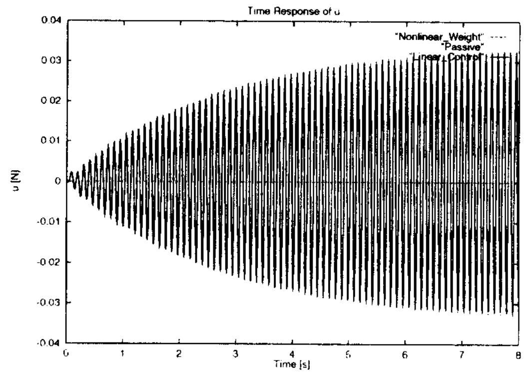

Figure 5:Time responses of u:$\ddot{q} = 0.1 \sin 58.3t$

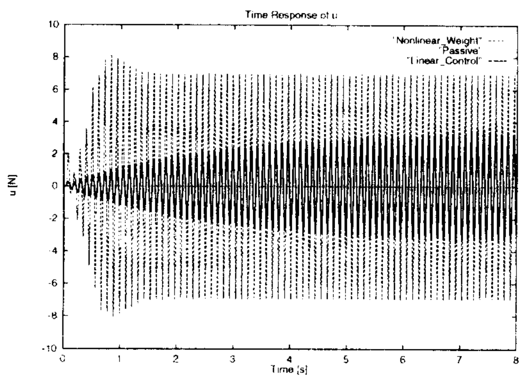

Figure 6:Time responses of u:$\ddot{q} = 10.0 \sin 58.3t$

5.3 Simulation

We show results of simulation from Fig:3 to Fig:6
We use following control inputs.

- passive controller ($u = 0$)
- linear H_∞ controller
  ($u = \boldsymbol{K}(\boldsymbol{x}) = -\boldsymbol{B}_2^T \boldsymbol{P}\boldsymbol{x}$)
- nonlinear H_∞ controller
  ($u = \boldsymbol{K}(\boldsymbol{x}) = -\frac{1}{a^2}\boldsymbol{B}_2^T \boldsymbol{P}\boldsymbol{x}$)

When $l(\boldsymbol{x}) = 0$ and $a(\boldsymbol{x}) = 1$, the nonlinear H_∞ control coincide with the linear H_∞ state feedback control. We choose $\gamma = 0.5, \varepsilon = 0.01$ in Riccati equation(5), and parameter of the nonlinear weight is $m_0 = 100.0$.

First, when the small sine wave disturbance $\ddot{q}$ of second mode frequency inputs, response of h_1 is shown in Fig:3 The responses of the linear control and the nonlinear one have less amplitude than passive one.

Second, when the large sine wave disturbance inputs,response of h_1 is shown in Fig:4. The relation of passive and linear control does not change, but nonlinear control has less amplitude.

If the disturbance has small amplitude, the response of h_1 does not have large difference between linear and nonlinear control, because $l(\boldsymbol{x})$ almost equals to 0. The other hand, the disturbance has large one, the effect of $l(\boldsymbol{x})$ is larger, so the effect of nonlinear controller comes up. It is evident, because the nonlinear controller is same as linear one around the origin.

Finally, the response of u is shown Fig:5 as small disturbance of second mode frequency inputs, Fig:6 as large. When the small disturbance inputs, the response of u of the nonlinear controller is almost same as linear one. But the disturbance become large, u of nonlinear controller is greater than linear one.

6 Conclusion

In this paper, we consider a nonlinear H_∞ control problem for linear system with nonlinear weight, and derive the solvability condition. The simulation results show the efficiency of nonlinear weight. some constraint to nonlinear weight, the solvability condition is given by Riccati equation(5). It is same as the solvability condition of linear H_∞ state feedback control problem. In other words, If linear H_∞ control problem is solvable, then nonlinear H_∞ control problem is solvable by choosing nonlinear weight, suitably, and a state feedback control law is designed simplicity.

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