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Control of Manipulators with Hyper Degrees of Freedom: Shape Control Based on Curve Parameter Estimation

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Abstract: In this paper, a new shape control law is derived as a result of introducing the parametric curve representation. This control law is based on the estimation of the curve parameters corresponding to the target joint positions and the target tip position. Estimating target curve parameters makes it possible to find, easily, a simple shape control law by the Lyapunov design method.

Key Words: Hyper Degrees of Freedom Manipulator, Shape Control, Lyapunov Design Method, Estimation of Curve Parameters

1. INTRODUCTION

A Hyper Degrees of Freedom (HDOF) manipulator is a manipulator which has extraordinarily many degrees of freedom, like a elephant trunk, a snake body or a monkey tail. If we could control such a manipulator completely, we would let the manipulator achieve dextrous tasks: for example, moving in highly constrained environment or grasping various sizes and shapes of objects and so on.

For an HDOF manipulator, the most important output to be controlled is its *shape* rather than its tip position/orientation. Therefore, we have proposed *shape control* to bring an HDOF manipulator onto a given target curve and derived a shape control law which do not need to solve the troublesome inverse kinematics [3].

In this paper, a new shape control law is derived as a result of introducing the parametric curve representation which is conventional in the classical differential geometry. The derived shape control law is based on estimating the curve parameters corresponding to the target joint positions and the target tip position on the curve. Estimating target curve parameters makes it possible to find, easily, a simple shape control law by the Lyapunov design method.

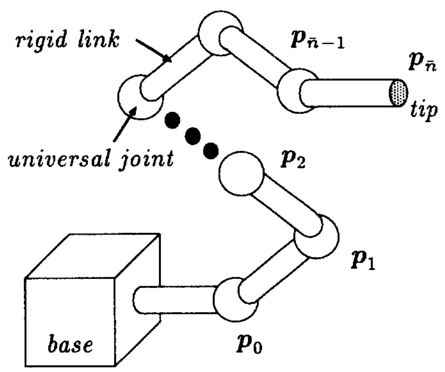

Fig.1: Kinematic Chain

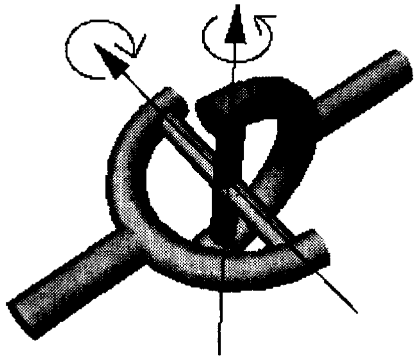

Fig.2: Universal Joint

2. PRELIMINARIES

2.1 Kinematics and Dynamics

Consider a kinematic chain of $(\bar{n}+1)$ rigid links connected by means of $\bar{n}$ *universal joints* (Fig.1) each of which has 2 orthogonal revolute axes (Fig.2). One end of the chain is connected to the base. The other end is open and referred to as the *tip* of the manipulator.

Let $p_i \in \Re^3$ $(i = 0, \cdots, \bar{n}-1)$ denote the position vector of the $(i+1)$-st universal joint, and $p_{\bar{n}} \in \Re^3$ is the position vector of the manipulator's tip.

Assume that we can generate torque u_j $(j = 1, \cdots, n)$ at each revolute axis, where $n = 2\bar{n}$ since each universal joint has 2 joint angles. Let θ_j be the rotation angle of the j-th axis and $\boldsymbol{\theta} = [\,\theta_1 \cdots \theta_n]^T \in \Re^n$. Then the dynamics of a manipulator with n degrees of freedom can be expressed as

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = u \qquad (1)$$

where $u = [\,u_1 \cdots u_n\,]^T \in \Re^n$ is the control input torque vector, $M(\theta) \in \Re^{n\times n}$ is the inertia matrix (positive definite for any θ), $C(\theta,\dot{\theta})\,\dot{\theta} \in \Re^n$ is the Coriolis and centrifugal forces vector, and $g(\theta) \in \Re^n$ is the gravitational torque vector. The matrix $\dot{M}(\theta) - 2C(\theta,\dot{\theta})$ has skew symmetric property. Throughout this paper, friction torques are neglected.

Remark: By using universal joints, some link parameters used in the manipulator dynamics (1) become zero, such as the link length between a pair of revolute joints. However, this does not effect the crucial properties of the dynamics; for example, the inertia matrix $M(\theta)$ still remains positive definite for any θ.

2.2 Representation of Curves

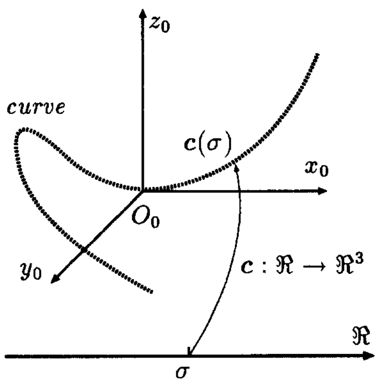

Fig.3: Representation of a Curve

To represent curves in $\Re^3$, we introduce the parametric curve representation. Consider a map

$$c : \Re \rightarrow \Re^3 \qquad (2)$$

and a curve parameter $\sigma \in \Re$. When σ moves from $-\infty$ to $+\infty$, the locus of the image $c(\sigma)$ forms a curve in $\Re^3$ (Fig.3).

Assume that the map c has the following properties.

Assumption 1 (Curves)

1. *$c(\sigma)$ is continuously differentiable w.r.t. σ.*

2. *$\exists\sigma_0 \in \Re, c(\sigma_0) = \mathbf{o}$. This means that a curve always passes through the origin. Without loss of generality, we set $\sigma_0 = 0$.*

3. *$\forall\sigma \in \Re$, $\left\|\dfrac{dc}{d\sigma}(\sigma)\right\| = 1$. That is, the length of arbitrary tangent vectors on a curve is equal to 1.*

4. *$\dfrac{dc}{d\sigma}(\sigma_0) = i_x$, where $i_x := [1\ 0\ 0\,]^T$. That is, the tangent vector at the origin is x-axis-oriented.*

5. *Define $\kappa(\sigma)$ and $\tau(\sigma)$ as the curvature and the torsion of c at σ respectively. There exist positive constants κ_M and τ_M s.t. $\forall\sigma \in \Re, \kappa(\sigma) \leq \kappa_M, \tau(\sigma) \leq \tau_M$. That is, both the curvature and the torsion of a curve are bounded from above.*

3. PROBLEM STATEMENT

In this chapter, we formulate a shape control problem.

3.3 Available Information

We make the following assumptions w.r.t. the available information.

Assumption 2 (Information)

1. *A joint angle vector θ and its velocity vector $\dot{\theta}$ are measurable.*

2. *The dynamical model of a manipulator (1) is exactly known.*

3.4 Objective Set

In this section, we clarify the objective of shape control in terms of the following 5 requirements:

1. All the joints and the tip of the manipulator are on a given target curve. That is,

$$c(\sigma_i^*) - p_i(\theta) = \mathbf{o} \qquad (i = 1, \cdots, \bar{n}) \qquad (3)$$

where $\sigma_i^* \in \Re$ is the curve parameter corresponding to the target position of the $(i-1)$-st joint. Define $e(\theta,\sigma) \in \Re^{3\bar{n}}$ as

$$e(\theta,\sigma) := \bar{c}(\sigma) - \bar{p}(\theta) \qquad (4)$$

where $\sigma \in \Re^{\bar{n}}$, and $\bar{p}(\theta)$, $\bar{c}(\sigma) \in \Re^{3\bar{n}}$ are defined as

$$\bar{c}(\sigma) := \begin{bmatrix} c(\sigma_1) \\ \vdots \\ c(\sigma_{\bar{n}}) \end{bmatrix}, \quad \bar{p}(\theta) := \begin{bmatrix} p_1(\theta) \\ \vdots \\ p_{\bar{n}}(\theta) \end{bmatrix}. \qquad (5)$$

Then we can rewrite condition (3) as

$$e(\theta,\sigma^*) = \mathbf{o} \qquad (6)$$

where $\sigma^* = [\sigma_1^* \cdots \sigma_{\bar{n}}^*]^T$.

2. The manipulator is *well-ordered* on the curve (Fig.4), which is expressed as

$$\sigma_0^* < \sigma_1^* < \cdots < \sigma_{\bar{n}}^* \quad (7)$$

where $\sigma_0^* = 0$. Or more strictly

$$\boldsymbol{H}_m \le \boldsymbol{H}(\boldsymbol{\sigma}^*) \quad (8)$$

where $\boldsymbol{H}(\boldsymbol{\sigma})$ is a diagonal matrix defined as

$$\boldsymbol{H}(\boldsymbol{\sigma}) := \mathrm{diag}\{\sigma_1 - \sigma_0, \cdots, \sigma_{\bar{n}} - \sigma_{\bar{n}-1}\} \quad (9)$$

and $\boldsymbol{H}_m \in \Re^{\bar{n}}$ is the constant matrix defined as $\boldsymbol{H}_m := \mathrm{diag}\{ l_1, \cdots, l_{\bar{n}} \}$ where l_i is the i-th link length.

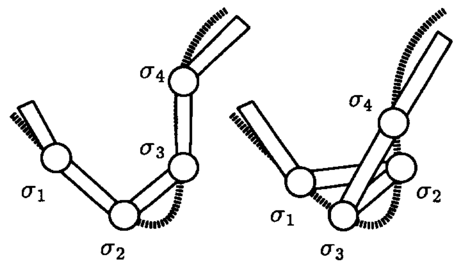

Fig.4: Well-ordered Requirement

3. The manipulator does *not take a shortcut* (Fig.5), which is expressed as

$$\boldsymbol{H}(\boldsymbol{\sigma}^*) \le \boldsymbol{H}_M \quad (10)$$

where $\boldsymbol{H}_M \in \Re^{\bar{n}}$ is the constant matrix depending upon κ_M, τ_M, l_i.

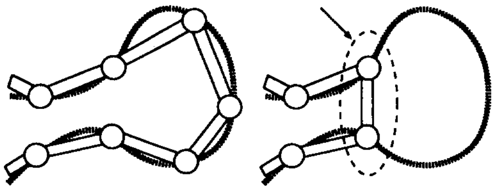

Fig.5: No-Shortcut Requirement

4. Joint angles of the manipulator are limited. For some positive constant $\theta_{M,i} < \pi$, θ_i satisfies the condition

$$|\theta_i| \le \theta_{M,i} \qquad (i = 1, \cdots, n) \quad (11)$$

or equivalently,

$$\boldsymbol{L}(\boldsymbol{\theta}) \le \boldsymbol{L}_M \quad (12)$$

where

$$\boldsymbol{L}(\boldsymbol{\theta}) = \mathrm{diag}\{|\theta_1|, \cdots, |\theta_n| \} \quad (13)$$

and $\boldsymbol{L}_M \in \Re^n$ is a constant matrix defined as $\boldsymbol{L}_M = \mathrm{diag}\{\theta_{M,1}, \cdots, \theta_{M,n} \}$.

5. The manipulator does not move on the curve. That is,

$$\dot{\boldsymbol{\theta}} = \mathbf{o}. \quad (14)$$

By the above requirements, we can describe the objective set of shape control $\mathcal{M}^* \in \Re^{2n}$, as

$$\mathcal{M}^* = \left\{ \boldsymbol{x} = \left[\boldsymbol{\theta}^T \ \dot{\boldsymbol{\theta}}^T\right]^T \ \middle| \ \boldsymbol{e}(\boldsymbol{\theta}, \boldsymbol{\sigma}^*) = \mathbf{o}, \right. \\ \left. \boldsymbol{H}_m \le \boldsymbol{H}(\boldsymbol{\sigma}^*) \le \boldsymbol{H}_M, \boldsymbol{L}(\boldsymbol{\theta}) \le \boldsymbol{L}_M, \dot{\boldsymbol{\theta}} = \mathbf{o} \right\} \quad (15)$$

Remark: The usefulness of introducing the parametric curve representation is that we can represent the desirable situation such as requirement 2 and 3 explicitly. As a result, we can say that our objective state is unique under a mild assumption shown in the next section.

3.5 Shape Control Problem

We make the following assumption:

Assumption 3 (Non-singularity)

For $\boldsymbol{\theta}^*$ and $\boldsymbol{\sigma}^*$ satisfying the requirements 1 – 4,

$$\det \boldsymbol{J}(\boldsymbol{\theta}^*, \boldsymbol{\sigma}^*) \ne 0 \quad (16)$$

where $\boldsymbol{J}(\boldsymbol{\theta}, \boldsymbol{\sigma}) \in \Re^{3\bar{n} \times 3\bar{n}}$ is defined as

$$\boldsymbol{J}(\boldsymbol{\theta}, \boldsymbol{\sigma}) := \left[\ \frac{\partial \bar{\mathbf{c}}}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}) \quad -\frac{\partial \bar{\mathbf{p}}}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta}) \ \right]. \quad (17)$$

◇

This assumption assures that there is only one objective point $\boldsymbol{x}^* \in \Re^{2n}$, which is immediately concluded from the implicit function theorem. We call $\boldsymbol{J}(\boldsymbol{\theta}, \boldsymbol{\sigma})$ *the shape Jacobian.*

Under this assumption, the shape control problem can be stated as follows.

Shape Control Problem

Under the stated assumptions, find a control input $\boldsymbol{u}$ s.t. the state of the manipulator system $\boldsymbol{x} = \left[\boldsymbol{\theta}^T \ \dot{\boldsymbol{\theta}}^T\right]^T$ converges to $\mathcal{M}^* = \{\boldsymbol{x}^*\}$ asymptotically.
◇

3.6 Curve Parameter Estimation

One of the simplest strategies to solve the shape control problem is:

1. find $\boldsymbol{x}^*$, and
2. apply the well known control laws in joint space (See [1], for example).

For the shape control problem, we need to find $\boldsymbol{x}^*$, however, the calculating $\boldsymbol{x}^*$ is troublesome. Therefore, our strategy proposed here is to control the shape of the manipulator by estimating the objective curve parameter vector σ^*, that is, we do not need to find x^*.

Let $\sigma \in \Re^{\bar{n}}$ be the estimated curve parameter vector and $\xi = \left[\theta^T \ \dot{\theta}^T \ \sigma^T\right]^T \in \Re^{5\bar{n}}$ be the extended state space vector. The extended objective set $\mathcal{M}_e^* \in \Re^{5\bar{n}}$, is expressed as

$$\begin{aligned}\mathcal{M}_e^* &= \{\xi \mid e(\theta,\sigma) = o, H_m \le H(\sigma) \le H_M, \\ &\quad L(\theta) \le L_M, \dot{\theta} = o\}. \qquad (18)\end{aligned}$$

There exists only one objective point $\xi^* \in \Re^{5\bar{n}}$. Thus, we can formulate an extended shape control problem as follows.

Extended Shape Control Problem

Under the stated assumptions, find a control input u s.t. the extended state of the manipulator system $\xi = \left[\theta^T \ \dot{\theta}^T \ \sigma^T\right]^T$ converges to $\mathcal{M}_e^* = \{\xi^*\}$ asymptotically. ◇

Clearly, it is enough to solve the extended shape control problem to meet the requirements 1 – 5.

4. SHAPE CONTROL LAW

In this chapter, we show a shape control law with a curve parameter estimation law. Our result is the following proposition.

Proposition

Assume that the given target curve is chosen to satisfy the condition

$$\forall \xi \in \Omega \subset \Re^{5\bar{n}}, \quad \det J(\theta,\sigma) \neq 0 \qquad (19)$$

for some $\Omega \supset \mathcal{M}_e^*$. Consider a shape control law

$$u = \left\{\frac{\partial \bar{p}}{\partial \theta}(\theta)\right\}^T K e(\theta,\sigma) - K_d \dot{\theta} + g(\theta) \qquad (20)$$

with a curve parameter estimation law

$$\dot{\sigma} = -\left\{\frac{\partial \bar{c}}{\partial \sigma}(\sigma)\right\}^T K e(\theta,\sigma) \qquad (21)$$

where $K \in \Re^{3\bar{n}\times 3\bar{n}}$ and $K_d \in \Re^{n\times n}$ are constant positive definite matrices. Then the state ξ converges to $\mathcal{M}_e^* = \{\xi^*\}$ asymptotically. ◇

(proof)

Consider the Lyapunov function candidate $V : \Re^{5\bar{n}} \to \Re$

$$V(e,\dot{\theta}) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + \frac{1}{2}e^T K e. \qquad (22)$$

The first term of the above function represents the kinetic energy. and the second term is the error index specifying the distance between the manipulator and the target curve, estimation error included. The time derivative of V along the trajectory becomes

$$\begin{aligned}\dot{V}(e,\dot{\theta}) &= \dot{\theta}^T M(\theta)\ddot{\theta} + \frac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} + e^T K \dot{e} \\ &= \dot{\theta}^T\left[u - \left\{\frac{\partial \bar{p}}{\partial \theta}(\theta)\right\}^T K e - g(\theta)\right] \\ &\quad + e^T K \frac{\partial \bar{c}(\sigma)}{\partial \sigma}\dot{\sigma} \\ &= -\dot{\theta}^T K_d \dot{\theta} - e^T K \frac{\partial \bar{c}}{\partial \sigma}(\sigma)\left\{\frac{\partial \bar{c}}{\partial \sigma}(\sigma)\right\}^T K e \qquad (23)\end{aligned}$$

which shows that $\dot{V}$ is only negative semi-definite w.r.t. $(e,\dot{\theta})$. Consider the set $\Omega_\gamma \subset \Omega$ such that,

$$\begin{aligned}\Omega_\gamma &:= \{\xi \in \Omega \mid V(\xi) \le \gamma, H_m \le H(\sigma) \le H_M, \\ &\quad L(\theta) \le L_M\} \qquad (24)\end{aligned}$$

for some positive constant γ. For a sufficiently small γ, the set Ω_γ is compact and positively invariant because $\dot{V} \le 0$ in Ω_γ. By LaSalle's theorem, we conclude that the state ξ starting from the interior of Ω_γ converges to $\mathcal{M}$ which is the largest invariant set satisfying $\dot{V} = 0$ in Ω_γ. The set $\mathcal{M}$ is described as

$$\mathcal{M} = \left\{\xi \in \Omega_\gamma \;\middle|\; \dot{\theta} = o, \ J^T K e = o\right\}. \qquad (25)$$

Since $\mathcal{M} \subset \Omega$, J is invertible in $\mathcal{M}$. Thus (25) is rewritten as

$$\mathcal{M} = \left\{\xi \in \Omega_\gamma \;\middle|\; \dot{\theta} = o, \ e = o\right\}. \qquad (26)$$

The equation (26) shows that $\mathcal{M}$ is equivalent to $\mathcal{M}_e^*$. Thus, we conclude that ξ converges to $\mathcal{M}_e^* = \{\xi^*\}$ asymptotically. □

5. CONCLUSION

In this paper, we showed a new shape control law based on curve parameter estimation. By this control law, we can control a serial rigid link HDOF manipulator without solving the troublesome inverse kinematics.

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