Adaptive Current Control Scheme of PM Synchronous Motor with Estimation of Flux Linkage and Stator Resistance

Kyeong-Hwa Kim*, In-Cheol Baik, Se-Kyo Chung, and Myung-Joong Youn
Department of Electrical Engineering
Korea Advanced Institute of Science and Technology

Abstract: **An adaptive current control scheme of a permanent magnet (PM) synchronous motor with the simultaneous estimation of the magnitude of the flux linkage and stator resistance is proposed. The adaptive parameter estimation is achieved by a model reference adaptive system (MRAS) technique. The adaptive laws are derived by the Popov's hyperstability theory and the positivity concept. The predictive control scheme is employed for the current controller with the estimated parameters. The robustness of the proposed current control scheme is compared with the conventional one through the computer simulations.**

I. INTRODUCTION

Permanent magnet (PM) synchronous motors have been gradually replacing DC motors in a wide range of drive applications such as machine tools and industrial robots. The advantage of using a PM synchronous motor is that many drawbacks caused by the brushes and commutators of a DC motor can be eliminated. In addition, the PM synchronous motor has high power density, large torque to inertia ratio, and high efficiency as compared with a DC motor having the same output rating[1]-[2]. The PM synchronous motor, however, has the nonlinear characteristics and inherent coupling problem, therefore to directly control the developed torque, the vector control is usually employed. With the high performance vector control system, the torque and flux current components are decoupled so that the independent torque and flux controls are possible as in DC motors. Therefore, high performance current controller is crucial for the successful implementation of the vector control[3]-[5].

The current control for an inverter-fed PM synchronous motor drive can be classified as hysteresis control, ramp comparison control, synchronous frame proportional-integral (PI) plus decoupling control, and predictive control[3]-[4]. Because of their existing disadvantages and limitations such as large current errors, irregular PWM inverter operation, and performance degradation at high speeds, the hysteresis and ramp comparison current control schemes cannot be used in a high performance vector control system for all operating conditions[3]. In a synchronous frame PI plus decoupling current control, the current response is slow and has some overshoots under the mismatch of load parameters[4]. On the other hand, in a predictive control scheme, the switching instants of the power switch are determined by calculating the required voltage to force the load currents to follow their reference values. With the space vector modulation technique, this control scheme is known to have the advantages such as constant switching frequency and lower current ripple[5]-[6]. This scheme, however, requires the full knowledge of load parameters and operating conditions, and cannot give a satisfactory response under the parameter mismatch. Among the machine parameters, the inaccuracy of the back EMF influences particulary on the steady state response. Recently, to overcome such a limitation, a current control scheme independent of the back EMF variation has been proposed[7]-[8]. In this scheme, the back EMF can be estimated by using the feedback of the delayed input voltages and currents. Thus, a robust control performance against the back EMF variation can be obtained. But, this scheme still requires other motor parameters such as the stator resistance and stator inductance. Under the mismatch of these parameters, the back EMF cannot be exactly estimated to the real value. In addition, because the developed torque is proportional to the product of the flux linkage and q-axis current, the estimated flux linkage has direct influence on the developed torque response.

In this paper, the estimation error in the flux linkage caused by the mismatched stator resistance is analytically obtained at the various operating conditions. Based on this analysis, to overcome the disadvantages of the scheme[8], a robust current control of a PM synchronous motor with the adaptive parameter estimation is proposed. Assuming that the stator inductance is constant, the stator resistance and the magnitude of flux linkage will be simultaneously estimated using the model reference adaptive system (MRAS) technique, and these estimated parameters are used for the predictive current controller. Thus, an improved robustness against the parameter variations can be obtained and the exact torque control will be possible.

II. MODELING OF PM SYNCHRONOUS MOTOR AND CURRENT OBSERVER

The PM synchronous motor considered in this paper is consists of a permanent magnet rotor and three phase stator windings which is sinusoidally distributed and displaced by $120°$. The voltage equations of a PM synchronous motor in the synchronously rotating reference frame are described as follows:

$$v_{qs} = R_s i_{qs} + L_s \frac{d}{dt} i_{qs} + L_s \omega_r i_{ds} + \lambda_m \omega_r \quad (1)$$

$$v_{ds} = R_s i_{ds} + L_s \frac{d}{dt} i_{ds} - L_s \omega_r i_{qs} \quad (2)$$

where R_s is the stator resistance, L_s is the stator inductance, ω_r is the electrical rotor angular velocity, θ_r is the electrical rotor angle, and λ_m is the amplitude of flux linkage established by permanent magnet. Using the q-axis and d-axis currents as the state variables, the state equation of a PM synchronous motor can be expressed as follows:

$$\frac{d}{dt} i_s = A i_s + B v_s + d \tag{3}$$

where $i_s = [\, i_{qs} \;\; i_{ds} \,]^T$, $v_s = [\, v_{qs} \;\; v_{ds} \,]^T$

$$A = -\frac{R_s}{L_s} I + \omega_r J, \quad B = \frac{1}{L_s} I,$$

$$d = \begin{pmatrix} -\frac{\lambda_m}{L_s}\omega_r \\ 0 \end{pmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

It is shown in the above equation that the amplitude of flux linkage λ_m appears as a disturbance term of the state equation. The full state current observer which estimates stator currents can be expressed as the following equation:

$$\frac{d}{dt} \hat{i}_s = \hat{A} \hat{i}_s + B v_s + \hat{d} + G(\hat{i}_s - i_s) \tag{4}$$

where '^' denotes the estimated quantities and G is an observer gain matrix. $\hat{A}$ and $\hat{d}$ are the matrices in which R_s and λ_m are replaced by the estimated values $\hat{R}_s$ and $\hat{\lambda}_m$, respectively. In order to assign two poles of the current observer at the specified locations on the complex plane, observer gain matrix G should consist of the symmetric part and the skew-symmetric part. In this paper, G is calculated by the following equations:

$$G = -g_1 I + g_2 J = \begin{pmatrix} -g_1 & -g_2 \\ g_2 & -g_1 \end{pmatrix} \tag{5}$$

$$g_1 = (k-1)\, \hat{R}_s \frac{1}{L_s} \tag{6}$$

$$g_2 = (k-1)\omega_r. \tag{7}$$

With this observer gain, the locations of the closed loop observer poles are k-times ($k \ge 1$) than those of the PM synchronous motor. In addition, with this type of the observer gain, the stable error dynamics guarantees the satisfaction of the strictly positive real (SPR) condition for the stability of the adaptive system.

III. ESTIMATION OF FLUX LINKAGE AND STATOR RESISTANCE

A. Adaptive scheme for parameter estimation

To estimate the magnitude of the flux linkage and the stator resistance, an MRAS technique will be used where the state equation of a PM synchronous motor is used for a reference model and the full state current observer is used for an adjustable model. The errors between the reference model and the adjustable model can be used to drive the adaptation mechanism and to update the parameters in the adjustable model in order to reduce the errors of model outputs. When the estimated values of the stator resistance and flux linkage are used in the current observer, the error dynamic equation can be obtained by subtracting (4) from (3) as follows:

$$\dot{e} = (A+G)e - W \tag{8}$$

where $e = i_s - \hat{i}_s$ and W is a nonlinear time-varying block and is defined as follows:

$$W = -\Delta A \cdot \hat{i}_s - \Delta d \tag{9}$$

where ΔA and Δd are the error matrices caused by the parameter variations and can be expressed as follows:

$$\Delta A = A - \hat{A} = -\frac{\Delta R_s}{L_s} \cdot I \tag{10}$$

$$\Delta d = d - \hat{d} = \begin{pmatrix} -\frac{\omega_r}{L_s} \\ 0 \end{pmatrix} \Delta\lambda_m \tag{11}$$

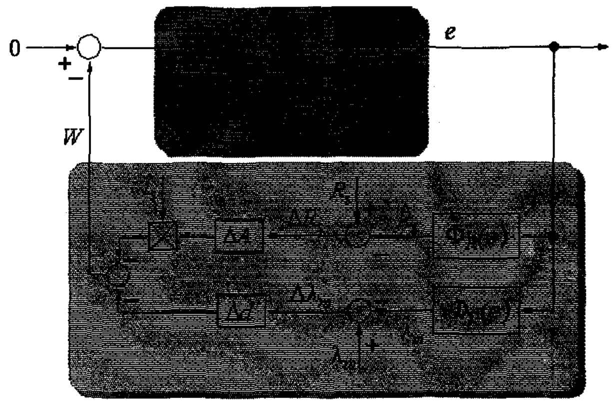

Fig. 1 MRAS for the estimation of the flux linkage and stator resistance

$$\Delta R_s = R_s - \hat{R}_s$$

$$\Delta\lambda_m = \lambda_m - \hat{\lambda}_m$$

Based on (8), an MRAS system is constructed as shown in Fig. 1, which consists of a linear time invariant forward block and a nonlinear feedback block. This system is hyperstable if the forward path transfer function matrix is SPR and the input-output inner product of nonlinear feedback block satisfies the Popov's integral inequality [9]-[10]. With the stable error dynamics which means that the diagonal term of matrix (A+G) is negative, it can be shown that the forward transfer function matrix $[sI-(A+G)]^{-1}$ is SPR[9]. In order to verify the stability of the adaptive system and to derive the adaptation mechanism, the Popov's inequality can be expressed as follows:

$$\int_0^{t_1} e^T W dt = \int_0^{t_1} \left(e^T \hat{i}_s \frac{\Delta R_s}{L_s} + e^T \begin{pmatrix} \omega_r \\ 0 \end{pmatrix} \frac{\Delta\lambda_m}{L_s} \right) dt \ge -\gamma_0^2$$

$$\text{for all } t_1 \ge 0 \tag{12}$$

where γ_0^2 is a finite positive constant. This equation has two parameter error components. One is caused by the stator resistance error and the other is caused by the flux linkage error. This can be seperated into two input-output inner products for respective parameter errors as follows:

$$\int_0^{t_1} e^T \hat{i}_s \frac{\Delta R_s}{L_s} dt \ge -\gamma_1^2 \tag{13}$$

$$\int_0^{t_1} e^T \begin{pmatrix} \omega_r \\ 0 \end{pmatrix} \frac{\Delta\lambda_m}{L_s} dt \ge -\gamma_2^2 \tag{14}$$

where γ_1^2 and γ_2^2 are finite positive constants. To estimate the stator resistance, the following adaptive law is defined

$$\hat{R}_s = \int_0^t \Phi_R(e) d\tau + \hat{R}_s(0) \tag{15}$$

where $\hat{R}_s(0)$ is the initial estimated value of the stator resistance and $\Phi_R(e)$ is the nonlinear function of the state error which is determined in order that (15) satisfies the inequality (13) as follows:

$$\int_0^{t_1} \frac{e^T \hat{i}_s}{L_s} \left(R_s - \int_0^t \Phi_R(e) d\tau - \hat{R}_s(0) \right) dt \ge -\gamma_1^2. \tag{16}$$

From (16), the stator resistance can be estimated as follows:

$$\hat{R}_s = -\left(k_{PR} + \frac{k_{IR}}{s} \right) \cdot (e_{qs}\, \hat{i}_{qs} + e_{ds}\, \hat{i}_{ds}) \tag{17}$$

where k_{PR} and k_{IR} are the PI gains for the stator resistance estimation, respectively, and the PI adaptation is used to improve the estimation performance during transient period. Similarily, using (14), the flux linkage can be estimated as

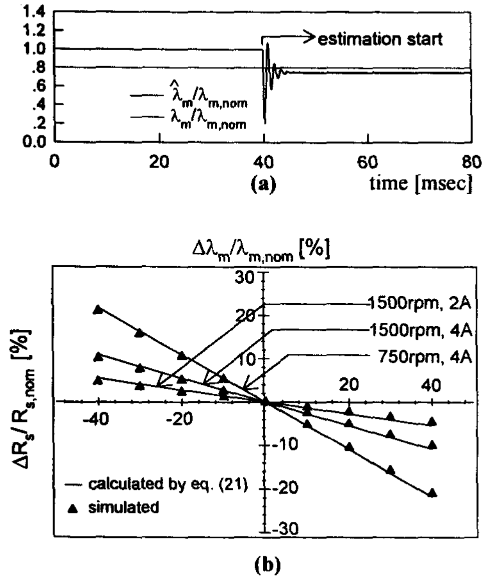

Fig. 2 Estimation error of the flux linkage under the mismatched stator resistance
(a) estimation of the flux linkage at $\omega_r = 1500$[rpm], $i_{qs}=4$[A], and $\Delta R_s = -0.2R_{s,nom}$
(b) estimation error of the flux linkage at various operating conditions

follows:

$$\hat{\lambda}_m = -\left(k_{P\lambda} + \frac{k_{I\lambda}}{s}\right)\cdot(e_{qs}\omega_r) \quad (18)$$

where $k_{P\lambda}$ and $k_{I\lambda}$ are the PI gains for the flux linkage estimation. When these estimated parameters converge to the real values, the closed loop observer error dynamics becomes

$$\dot{e} = (A+G)e = \begin{pmatrix} -k\frac{R_s}{L_s} & -k\omega_r \\ k\omega_r & -k\frac{R_s}{L_s} \end{pmatrix}\cdot e \quad (19)$$

which is k-times faster than that of the PM synchronous motor.

B. *Estimation error in flux linkage caused by mismatched stator resistance*

When $\widehat{R_s}$ in the current observer is set to $R_{s,nom}$ with no adaptation and R_s is varied from this value, the error dynamics becomes from (8) as

$$\dot{e} = \begin{pmatrix} -k\frac{R_{s,nom}}{L_s} - \frac{\Delta R_s}{L_s} & -k\omega_r \\ k\omega_r & -k\frac{R_{s,nom}}{L_s} - \frac{\Delta R_s}{L_s} \end{pmatrix}\cdot e + \Delta A\,\hat{i}_s + \Delta d \quad (20)$$

where $\Delta R_s = R_s - R_{s,nom}$. When the error dynamics is stable, the flux linkage adaptation converges to the following value in order to reduce the error in the q-axis current:

$$-\frac{\Delta R_s}{L_s}\hat{i}_{qs} - \frac{\omega_r}{L_s}\Delta\lambda_m \to 0 \quad \text{or} \quad \hat{\lambda}_m \to \lambda_m + \frac{i_{qs}}{\omega_r}\Delta R_s. \quad (21)$$

As shown in (21), the estimation error in the flux linkage depends on the current level, operating speed, and mismatched quantity of the stator resistance. Fig. 2(a) shows the estimation of λ_m without the estimation of the mismatched R_s at ω_r=1500[rpm], i_{qs}=4[A], and $\Delta R_s = -0.2R_{s,nom}$ when λ_m is varied to 80% of its nominal value at t=0. The adaptation algorithm starts at t=40 msec using (18). It can be shown that there is a steady state error in the estimated value caused by the mismatched stator resistance. In this case, the estimation error of $\hat{\lambda}_m$ is 5.04% of its nominal value. Fig. 2(b) shows the calculated and simulated estimation errors of the flux linkage at the various operating conditions where the calculated estimation errors are obtained using (21). It is shown that these errors are larger at low speed and heavy load.

TABLE 1
Specifications of PMSM

Item	*Value*	*Unit*
Rated power	750	Watts
Rated speed	3000	rpm
Number of poles	4	
Stator resistance	2.14	
Stator inductance	4.6	mH
Magnetic Flux	0.2	Wb

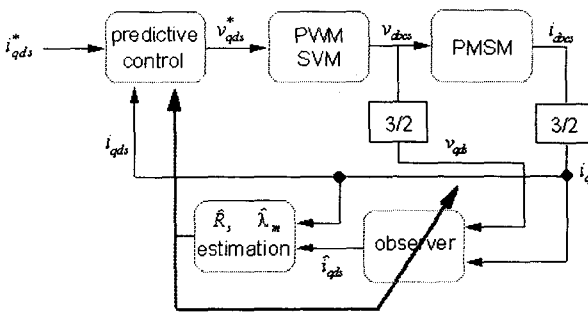

Fig. 3 Overall block diagram for the proposed scheme

IV. SIMULATION RESULTS

The overall block diagram of the proposed current control scheme is shown in Fig. 3. The control system is consists of current controller, pulse width modulator, full state current observer, and parameter estimator. The current controller is achieved by using the predictive control technique with the estimated parameters. Using the predictive control technique, in order to make the next step current errors equal to zero, the reference voltages are calculated with the estimated parameters as the following equation and are applied to a motor using the space vector modulation technique[6]:

$$v^*_{qs}(k) = \widehat{R_s}i_{qs}(k) + \frac{L_s}{T}[\,i^*_{qs}(k+1) - i_{qs}(k)] + \omega_r L_s i_{ds}(k) + \omega_r\hat{\lambda}_m \quad (22)$$

$$v^*_{ds}(k) = \widehat{R_s}i_{ds}(k) + \frac{L_s}{T}[\,i^*_{ds}(k+1) - i_{ds}(k)] - \omega_r L_s i_{qs}(k) \quad (23)$$

where T is a sampling period and k is a time index. The nominal parameters of a PM synchronous motor used for the simulation are listed on Table I. Fig. 4 shows the current responses for the proposed control scheme when the stator resistance and the magnitude of the flux linkage are varied to 80% of their nominal values at t=0 and 120% at t=60 msec. respectively. $\widehat{R_s}$ and $\widehat{\lambda_m}$ are initially set to their nominal values until the adaptation algorithm starts at t=40 msec. The q-axis current command is 4A

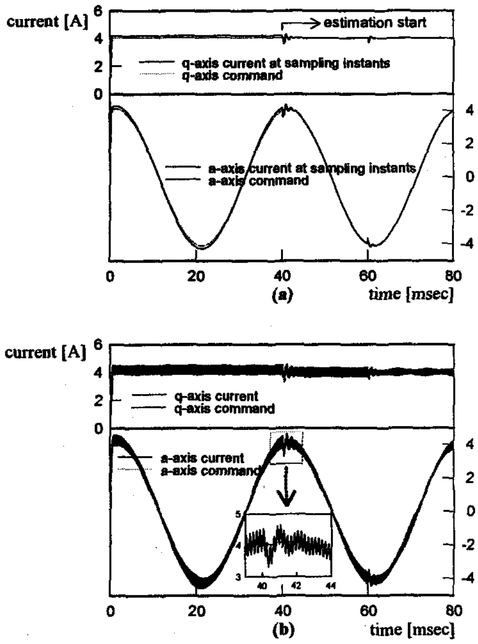

Fig. 4 Proposed control scheme when $R_s = 0.8R_{s,nom}$ and $\lambda_m = 0.8\lambda_{m,nom}$ at t=0 and $R_s = 1.2R_{s,nom}$ and $\lambda_m = 1.2\lambda_{m,nom}$ at t=60[msec]
(a) q-and a-axes currents at sampling instants
(b) q-and a-axes currents

and the sampling period is 0.2 msec. Fig. 4(a) shows the q-axis and a-axis currents at each sampling instants and Fig. 4(b) shows the q-axis and a-axis real currents. Until t=40 msec, because the magnitude of the flux linkage is smaller in a real motor than that in a controller, the magnitude of the load current is larger than that of the current command. As soon as the adaptation mechanisms start at t=40 msec, the load current tracks the command, well and current error is effectively reduced. It can be shown that the current ripple becomes small at t=60 msec. This is because that the magnitude of the flux linkage is larger to 20% and thus, the magnitude of the back EMF is larger. Fig. 5 shows the simultaneous estimations of the stator resistance and the magnitude of flux linkage under the same condition of Fig. 4. The PI gains for the parameter estimation are set as follows: k_{PR}=0.01, k_{IR}=0.22, $k_{P\lambda}$=0.01, and $k_{I\lambda}$=0.09. It can be shown that both parameters converge to real values within several miliseconds.

V. CONCLUSIONS

In this paper, an adaptive current control of a PM synchronous motor with the simultaneous estimations of the stator resistance and the magnitude of flux linkage has been proposed and verified its robust performance against the variations of the motor parameters. The adaptive parameter estimation is achieved by MRAS technique where the adaptive laws are derived by the hyperstability theory and the positivity concept. It is verified that the estimation of the magnitude of flux linkage without the

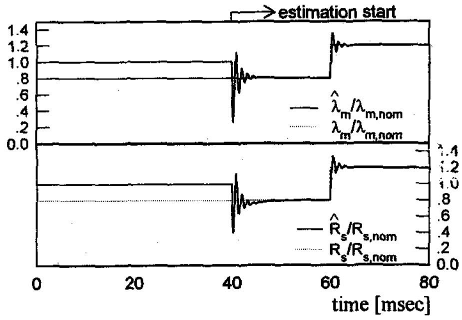

Fig. 5 Estimation of the flux linkage and stator resistance under the same condition of Fig. 4

estimation of the mismatched stator resistance may result in the estimation error of the magnitude of flux linkage and this error depends on the operating conditions. This error has been obtained at the various operating conditions Thus, by using the proposed control scheme, an improved robustness against the variations of the motor parameters will be obtained.

REFERENCES

[1] B. K. Bose, *Power Electronics and AC Drives*, Englewood Cliffs, NJ, Prentice Hall; 1986.
[2] T. H. Liu, C. M. Young, and C. H. Liu, Microprocessor-based controller design and simulation for a permanent magnet synchronous motor drive, *IEEE Trans. Industrial Electronics*, vol. IE-35, no. 4, November 1988, pp. 516-523.
[3] D. M. Brod, and D. W. Novotny, Current control of VSI-PWM inverters, *IEEE Trans. Industry Applications*, vol. IA-21, no. 4, May/June 1985, pp. 562-570.
[4] T. M. Rowan, and R. J. Kerkman, A new synchronous current regulator and an analysis of current-regulated PWM inverters, *IEEE Trans. Industry Applications*, vol. IA-22, no. 4, July/August 1986, pp. 678-690.
[5] H. L. Huy, and L. A. Dessaint, An adaptive current control scheme for PWM synchronous motor drives: analysis and simulation, *IEEE Trans. Power Electronics*, vol. PE-4, no. 4, October 1989, pp. 486-495.
[6] H. W. van der Broeck, H. C. Skudelny, and G. V. Stanke, Analysis and realization of a pulsewidth modulator based on voltage space vectors, *IEEE Trans. Industry Applications*, vol. IA-24, no. 1, Jan./Feb. 1988, pp. 142-150.
[7] D. S. Oh, K. Y. Cho, and M. J. Youn, A discretized current control technique with delayed input voltage feedback for a voltage-fed PWM inverter, *IEEE Trans. Power Electronics*, vol. PE-7, no. 2, April 1992, pp. 364-373.
[8] K. Y. Cho, J. D. Bae, S. K. Chung, and M. J. Youn, Torque harmonics minimization in permanent magnet synchronous motor with back EMF estimation, *IEE Proc. Electr. Power Appl.*, vol. 141, no. 6, November 1994, pp. 323-330.
[9] Y. D. Landau, *Adaptive Control-The Model Reference Approach*. Marcel Dekker, New York; 1979.
[10] G. Yang, and T. H. Chin, Adaptive-speed identification scheme for a vector-controlled speed sensorless inverter-induction motor drive, *IEEE Trans. Industry Applications*, vol. IA-29, no. 4, July/August 1993, pp. 820-825.