多品目 多手段 輸送網模型

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1. Introduction

The World Health Organization defines a disaster as any occurrence that causes damage, ecological disruption, loss of human life, deterioration of health and health services on a scale sufficient to warrant an extraordinary response from outside the affected community or area (WHO Manual, January 1989). Natural disasters such as earthquakes, hurricanes, floods, drought, volcanic eruption, famine, etc. are part of our daily life. They have significant devastating effects in terms of human injuries and property damage.

In disaster relief operations, many local, state, and federal emergency management officials face critical questions related to the safety and well being of the people who are affected by these emergencies (Lewis, 1985). The public officials are also faced with many other critical questions after a natural disaster occurs. The most important of these questions is how to respond to these emergencies in the most efficient manner to minimize the loss of life and maximize the efficiency of the rescue operations. In case of these emergencies various organizations often face significant problems of transporting large amounts of many different commodities including food, clothing, medicine, medical supplies, machinery, and personnel from different points of origin to different destinations in the disaster areas. The transportation of supplies and relief personnel must be done quickly and efficiently to maximize the survival rate of the affected population and minimize the cost of such operations. Often, many different modes of transportation are available for the purpose of shipping the supplies and personnel. All of these modes of transportation may not be suitable for every commodity. Some commodities may change the type of mode in the middle of transportation from origin to destination. Time plays a crucial role in managing the response to a particular emergency.

The basic underlying logistical problem for disaster relief management is to move a number of different commodities using a number of different modes of transportation, from a number of origins to one or more destinations over a transportation network in a timely manner effectively and efficiently. This is a multi-commodity, multi-modal network flow problem with time windows which is one of the most complex network flow problems in operations research.

In such cases as emergency response, this transportation network may or may not be damaged as a result of the natural disaster. This is a transportation decision making process over time. The basic goal is to deliver the required demand within the required time if such time is specified, or as soon as possible. This decision making process must also be responsive to the shipping requirements so that the commodities and personnel are carried by the most preferred mode as much as possible. At the same time, the process must be responsive to the changes in the transportation network configuration which may have resulted from the cause of the emergency. These transportation decisions must be made so that the overall cost or time of the operation is optimized.

The logistical problem as defined above is often constrained by the ability to load, store, and unload cargo at the origins and destinations, and the availability of the transportation modes over time. These multicommodity, multimodal network flow problems in the real world are extremely large and very difficult to solve because of such factors as the existence of hundreds of potential routes, multiple commodities and transportation modes, the nature of the demand requirements and their variation over time, and the variation of delivery time requirements. These problems are often formulated and solved as complex large scale network optimization problems.

This paper aims at developing a decision making tool which can be potentially be used by emergency response managers in planning for disaster relief operations. In particular, the paper deals with the problem of determining the detailed routing and scheduling of the available transportation modes, delivery schedules of the various commodities at their destinations, and the load plans for each of the transportation modes. In this effort our ultimate goal is to develop a decision support system which can aid the Federal and State authorities in emergency response management. Such comprehensive planning tools currently do not exist.

The organization of this paper is as follows. Section 2 reviews the state-of-the-art related to the multicommodity, multimodal network flow problem. Section 3 presents the formulation of the multicommodity, multimodal network flow problem as a large scale mixed integer linear programming model. Section 4 discusses two solution algorithms which have been developed specifically to solve the problem presented in Section 3. Section 5 presents the results of model testing. Finally, Section 6 presents the conclusions and directions for future research.

2. Review of the Literature

Although the literature in logistics management is extensive, and in fact, some of this research addresses the transportation issues involved following urban disasters (Hobeika et al., 1987; 1988; Ardekani, 1991, 1992), the particular problem which is the subject of this research has received little attention. This is primarily because of the complexities which arise when we are dealing with multiple commodities and multiple modes of transportation. In this section we focus our attention on the literature review which is relevant to the disaster relief operations and the multicommodity, multimodal network flow problem.

A number of authors have addressed the problem of emergency response management. Kemball-Cook and Stephenson (1984) addressed the need for logistics management in relief operations for the increasing refugee population in Somalia. Ardekani and Hobeika (1988) addressed the need of logistics management in relief operations for the 1985 Mexico City earthquake. Knott (1987) developed a linear programming model for the bulk food transportation problem and the efficient use of the truck fleet to minimize the transportation cost or to maximize the amount of food delivered (single commodity, single modal network flow problem). In another article, Knott (1988) developed a linear programming model using expert knowledge for the vehicle scheduling of bulk relief of food to a disaster area. Ray (1987) developed a single-commodity, multi-modal network flow model on a capacitated network over a multi-period planning horizon to minimize the sum of all costs incurred during the transport and storage of food aid. Brown and Vassiliou (1993) developed a real-time decision support system which uses optimization methods, simulation, and the decision maker's judgement for operational assignment of units to tasks and for tactical allocation of units to task requirements in repairing major damage to public works following a disaster.

The literature in the multicommodity, multimodal network flow problem is relatively sparse. Crainic and Rousseau (1986) developed an optimization algorithm based on decomposition and column generation principles to minimize the total operating and delay cost for multicommodity, multimodal freight transportation when a single organization controls both the service network and the transportation of goods.

Guelat et al. (1990) presented a multicommodity, multimodal network assignment model for the purpose of strategic planning to predict multicommodity flows over a multimodal network. The objective function to be minimized was the sum of total routing cost and total transfer cost. The solution algorithm exploited the natural decomposition by commodity and resulted in a Gauss-Seidel-like linear approximation (GSLA). The model test was implemented for the Brazil transportation network with 211 origins and destinations, six commodities, and ten modes.

Crainic et al. (1990) presented and analyzed the rail component of the network optimization model which was mentioned in Guelat et al. (1990) through the application to the Sao Francisco River corridor that is one of the strategic export corridors of Brazil. Their goal was to obtain a strategic modeling framework of rail freight transportation that is typical of the rail mode in the temporal-spatial and economic relations. Drissi-Kaitouni (1991) suggested some variants of the solution algorithm (GSLA) developed in Guelat et al. (1990) for a multicommodity, multimodal assignment model. He improved the implementation of the proposed GSLA algorithm by identifying some unnecessary time consuming computation in the evaluation of marginal costs. The algorithm suggested is in the class of the Restricted Simplicial Decomposition. Gedeon et al. (1990) considered a normative transshipment flow problem in a multimodal network. The model was tested for the transportation of coal in Finland. A test network included 2572 nodes (10 origins and 23 destinations) and 10 modes.

The above review indicates the importance of the general multicommodity, multimodal network flow problem and the diversity of its application in real world situations. Most of literature shown in the previous section deal only with a rail network and the various modes mean different types of train service. The models have nonlinear convex differentiable objective functions and linear constraints, and they do not contain any capacity constraints on the links or nodes. None of the previous model allows any mode transfer during operations.

This research extends the state-of-the-art by presenting a model at the operational level which allows for transfer of commodities between modes operations, and predicts the detailed routing and scheduling. Therefore, the model formulation at the operational level and the development of efficient solution heuristics are the major contribution of this research.

3. Model Formulation

This section presents the formulation of the multicommodity, multimodal network flow problem as a single objective linear programming model on a time - space network.

3.1 Time - Space Network

A physical network is converted into a time-space network to account for the dynamic decision process. In the context of the problem which is dealt with in this research, nodes in the time-space network represent the physical locations of the supply and demand points for each mode and over time, while the arcs represent the connecting routes between these points. Each node in the physical network is represented by the number of mode types at each time period of the planning horizon. In a sense the time-space network in this context can be thought of as an overlay of several physical networks, one for each mode, which are represented over time. These overlaid networks are connected to each other by the transfer links which make it possible for the commodities to be transferred between modes.

There are three types of traffic flow on the physical network. The first type is the routing traffic that moves from one node to another node by a certain type of mode. The second type is the transfer traffic that changes mode type from one mode to another mode at a certain node. The third type is the supply or demand carry-over that is carried over to the next time period at a certain node.

The duration of one time period should be based on the link travel time for each mode. It must be small enough so that the amount of slack time on the routing links is not excessive. However, the planning horizon should not be too short in order for the time-space network to be meaningful. Also, it should not be too long as it will increase the dimension of the time-space network and make the problem difficult to solve.

The movements of commodities and personnel on a physical network over time are represented by the links in the time-space network. Routing Links represent the physical movement of commodities in space. Transfer Links represent the transfer of traffic between the available modes. Finally, Supply or Demand Carry-Over Links represent the commodity supply or demand carry over from one period to the next.

Figure 1 shows the physical network that has 4 nodes, 5 two-way arcs, and 3 modes. Node A represents the origin and nodes C and D denote the destinations. The travel time over the arc in each mode type is shown in terms of time periods. Figure 2 shows the time-space network generated from Figure 1 with 10 time units of planning horizon. The length of one time period is assumed to be one time unit. In Figure 2, all transfer time is assumed to be one time period. The carry-over links that are created at node A and B represent the supply carry-over links. On the other hand, the carry-over links that are shown at node C and D denote the demand carry-over links.

3.2 Model Formulation

3.2.1 Assumptions and Limitations

The following is a summary list of assumptions for and limitations of the model.

1. Transfer is only allowed at the origin nodes that also have a role of transshipment nodes and at the transshipment nodes.
2. All the cost functions are assumed linear.
3. All the commodity quantities at supply and demand nodes are known.
4. Vehicles can be recycled.

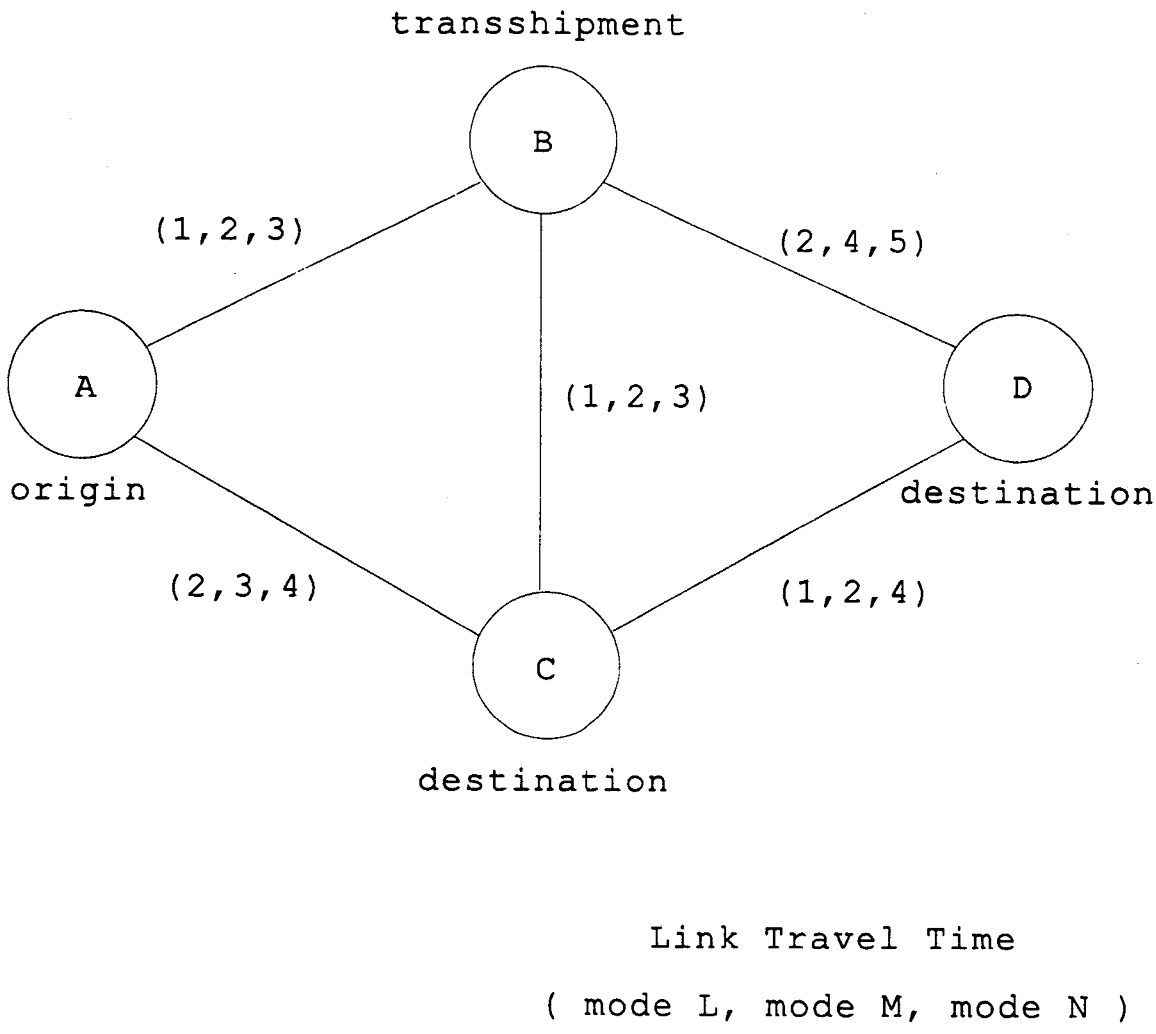

Figure 1. A Physical Network

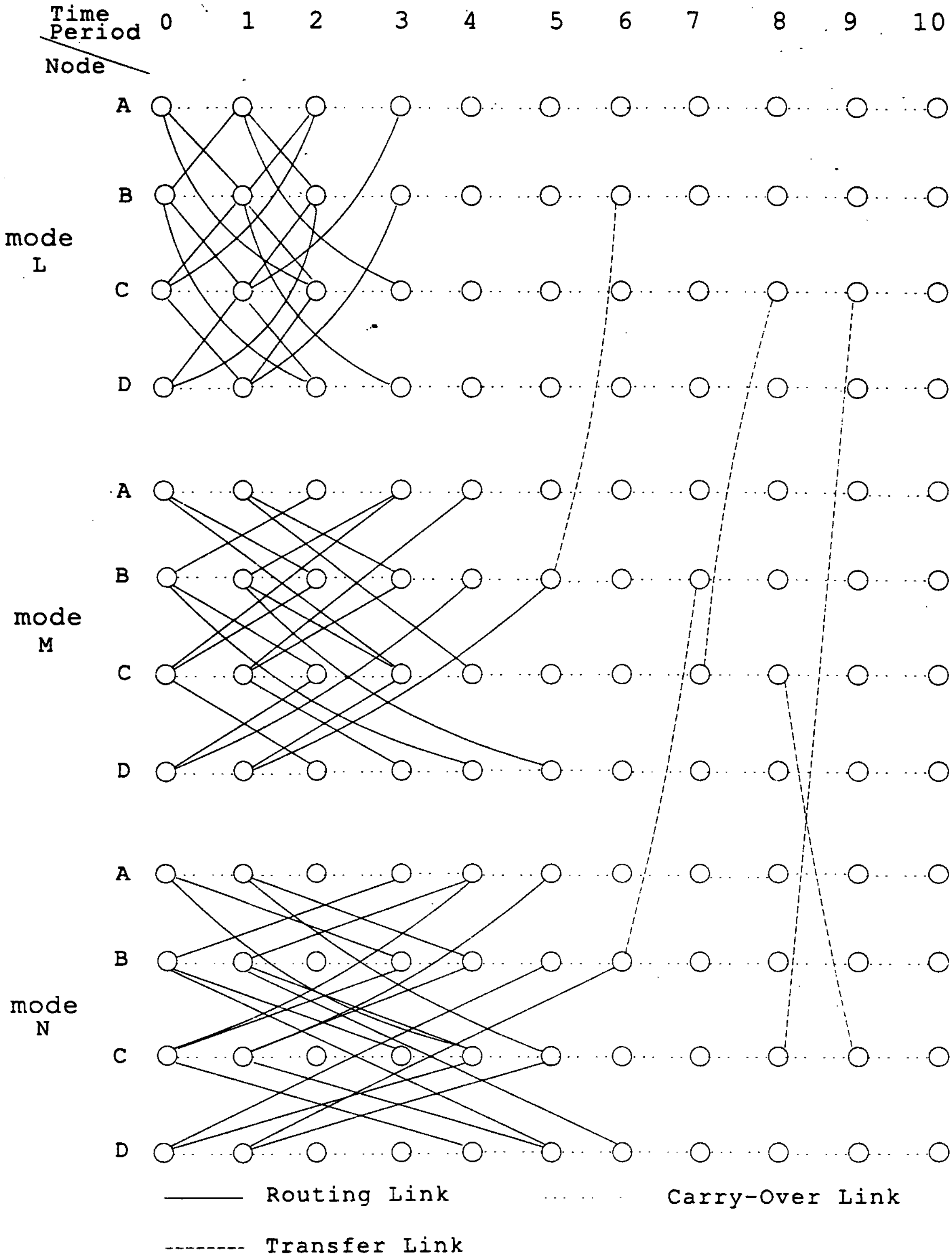

Figure 2. Conversion To A Time-Space Network

5. A number of different modes are available.

6. An average size vehicle is assumed for each mode.

7. Mode shift is allowed.

3.2.2 Notation and Definitions

Before the mathematical model is presented, the following notation is defined.

N = set of nodes, $i,j \in N$ are indices ($U \cup UV \cup V \cup VW \cup W = N$)

U = set of pure origin nodes, $u \in U$ is an index

UV = set of origin nodes that also have a role of transshipment nodes, $uv \in UV$ is an index

V = set of transshipment nodes, $v \in V$ is an index

VW = set of destination nodes that also have a role of transshipment nodes, $vw \in VW$ is an index

W = set of destination nodes, $w \in W$ is an index

A = set of links in the physical network, $a \in A$ is an index

M = set of modes, $m \in M$ is an index

G = set of goods or commodities, $g \in G$ is an index

EPT_{gi} = earliest pick-up time period of commodity g at node i

SE_{git} = amount of exogenous supply of commodity g at node i at time period t

EDT_{gi} = earliest delivery time period of commodity g at node i

DE_{git} = amount of exogenous demand of commodity g at node i at time period t

YE^{m}_{it} = number of vehicles of mode m which are available at node i at time period t

YCA^{m} = vehicle capacity of mode m

$ACA^{m}_{itjt'}$ = arc capacity between node i at time period t and node j at time period t' in number of vehicles for mode m

$CVR^{m}_{itjt'}$ = unit cost of moving vehicle of mode m from node i at time period t to node j at time period t'

$CGR^{gm}_{itjt'}$ = unit cost of shipping of commodity g by mode m from node i at time period t to node j at time period t'

CSC_{git} = unit cost for carrying over the supply for commodity g at node i from time period t to time period t+1

CDC_{git} = unit cost for carrying over the demand for commodity g at node i from time period t to time period t+1

$CGT^{gmm'}_{it(t+Kmm')}$ = unit cost of transfer of commodity g from mode m to mode m' at node i at time period t. Kmm' represents the number of time periods required for this transfer.

All of the above terms define the inputs to the model. The following decision variables are used in the formulation:

$Y^{m}_{itjt'}$ = flow of vehicles of mode m from node i at time period t to node j at time period t'

YC^{m}_{it} = no. of vehicles of mode m which is carried over from time period t to time period t+1 at node i

$X^{gm}_{itjt'}$ = flow of commodity g by mode m from node i at time period t to node j at time period t'

SC_{git} = amount of supply of commodity g which is carried over from time period t to time period t+1 at node i

DC_{git} = amount of demand of commodity g which is carried over from time period t to time period t+1 at node i

$XT^{gmm'}_{it(t+Kmm')}$ = amount of commodity g which is transferred from mode m to mode m' at node i at time period t

Among the 6 decision variables, $Y^{m}_{itjt'}$ and YC^{m}_{it} are integer variables. The rest are continuous variables. Hence, the problem is a mixed integer problem. The following internal decision variables are also used in the formulation:

SE^{m}_{git} = amount of exogenous supply of commodity g assigned to mode m at node i at time period t

SC^{m}_{git} = amount of supply of commodity g which is carried over by mode m from time period t to time period t+1 at node i

DE^{m}_{git} = amount of exogenous demand of commodity g delivered by mode m at node i at time period t

DC^{m}_{git} = amount of demand of commodity g which is carried over by mode m from time period t to time period t+1 at node i

All of the internal decision variables are continuous variables. The mathematical formulation of the model is as follows.

$$\text{Minimize} \quad \sum_{i}\sum_{j}\sum_{t}\sum_{t'}\sum_{m} CVR^{m}_{itjt'} \times Y^{m}_{itjt'} \tag{1}$$

$$+ \sum_{i}\sum_{j}\sum_{t}\sum_{t'}\sum_{g}\sum_{m} CGR^{gm}_{itjt'} \times X^{gm}_{itjt'} \tag{2}$$

$$+ \sum_{i \in U,UV,V}\sum_{t}\sum_{g} CSC_{git} \times (\sum_{m} SC^{m}_{git}) + \sum_{i \in VW,W}\sum_{t}\sum_{g} CDC_{git} \times (\sum_{m} DC^{m}_{git}) \tag{3}$$

$$+ \sum_{i \in UV,V}\sum_{t}\sum_{g}\sum_{m}\sum_{m'} CGT^{gmm'}_{it(t+Kmm')} \times XT^{gmm'}_{it(t+Kmm')} \tag{4}$$

Subject to

$$\begin{aligned}
&\sum_{j}\sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} + SC^{m}_{gi(t-1)} + SE^{m}_{git} \\
&= \sum_{j}\sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} + SC^{m}_{git} \qquad (i \in U, UV, V) \\
&\sum_{j}\sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} - DC^{m}_{gi(t-1)} \\
&= \sum_{j}\sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} - DC^{m}_{git} + DE^{m}_{git} \qquad (i \in VW, W) \\
&\qquad \text{for all } m, g, i, t
\end{aligned} \tag{5}$$

$$\begin{aligned}
SE_{git} &= \sum_{m} SE^{m}_{git} \qquad (i \in U, UV) \\
DE_{git} &= \sum_{m} DE^{m}_{git} \qquad (i \in VW, W)
\end{aligned} \tag{6}$$

$$DC^{m}_{git} = 0 \qquad \text{for all } t < EDT_{gi} \tag{7}$$

$$\begin{aligned} &X^{gm}_{itjt'} \geq 0, \qquad XT^{gmm'}_{it(t+Kmm')} \geq 0, \\ &SE^{m}_{git} \geq 0, \qquad SC^{m}_{git} \geq 0, \qquad DE^{m}_{git} \geq 0, \qquad DC^{m}_{git} \geq 0 \\ &\qquad\qquad \text{for all } g,\ m,\ m',\ i,\ t,\ j,\ t' \end{aligned} \tag{8}$$

$$YE^{m}_{it} + \sum_{j}\sum_{t'} Y^{m}_{jt'it} + YC^{m}_{i(t-1)} = \sum_{j}\sum_{t'} Y^{m}_{itjt'} + YC^{m}_{it} \qquad \text{for all } m,\ i,\ t \tag{9}$$

$$Y^{m}_{itjt'} \leq ACA^{m}_{itjt'} \qquad \text{for all } m,\ i,\ t,\ j,\ t' \tag{10}$$

$$Y^{m}_{itjt'} \geq 0 \text{ and integer}, \qquad YC^{m}_{it} \geq 0 \text{ and integer} \qquad \text{for all } m,\ i,\ t,\ j,\ t' \tag{11}$$

$$YCA^{m} \times Y^{m}_{itjt'} - \sum_{g} X^{gm}_{itjt'} \geq 0 \qquad \text{for all } m,\ i,\ t,\ j,\ t' \tag{12}$$

The objective function to be minimized is the sum of the vehicular flow costs, the commodity flow costs, the supply or demand carry-over costs, and the transfer costs over all time periods. Although disaster relief operations can be multi-objective in nature, and minimizing the response time can be as important as minimizing costs, we have chosen to use cost minimization as the proper objective function. The main reason for this choice is that costs are easier to quantify and the timeliness of emergency response can be accomplished by defining appropriate penalty costs for late deliveries. In this model demand carry-over costs represent the penalty for late delivery and are embedded in the objective function.

The beginning of the planning horizon is considered as the earliest time among earliest pick-up time periods for all commodities and nodes. The vehicular flow costs represent the cost of using vehicles of all modes for transporting commodities. The vehicular flow costs are represented in Eq.(1). The commodity flow costs represent the cost of transporting commodities with available modes. Eq.(2) represents the commodity flow costs.

The supply or demand carry-over costs reflect the cost incurred by the carry-over of commodities from one time period to the next time period. They have two components. One is the supply carry-over costs due to scheduling and consolidation considerations. The other is the demand carry-over costs due to the late delivery of demand. The supply or demand carry-over costs are represented in Eq.(3). The transfer costs account for the cost incurred by the shifting of a commodity from one mode to another mode type. Eq.(4) represents the transfer costs.

There are three sets of constraints that form the mathematical formulation in a broad sense. These are:

A) Commodity flow constraints

1) Flow conservation constraints
2) Flow definition constraints
3) Time window constraints
4) Nonnegativity constraints

B) Vehicular flow constraints

1) Flow conservation constraints
2) Arc capacity constraints
3) Integrality constraints

C) Linkage constraints between vehicular and commodity flow.

The commodity flow constraints operate only on the real-valued commodity flow decision variables. We have divided these constraints into four sets of constraints. The first set of constraints represent the conservation of flow through all time periods for each commodity and for each node. The first set of constraints are mathematically represented by Eq.(5). These constraints in general ensure that the commodity flow which enters node i at time period t is equal to the commodity flow which leaves this node. The second set of constraints define the exogenous supply of and demand for commodities. Eq.(6) represents these constraints. The third set of constraints, shown in Eq.(7), represent the time window constraints for commodities at destination nodes. These constraints state that each commodity g can not be delivered to the destination node i before the earliest delivery time period (EDT_{gi}). The last set of constraints restrict all of the commodity related variables to be greater than or equal to zero. The last constraints are represented in Eq.(8).

The vehicular flow constraints operate only on integer-valued vehicular flow decision variables. These constraints are classified into three sets of constraints. The first set of constraints represent the vehicular flow conservation for each mode, at each node, and at each time period. These constraints are expressed in Eq.(9). These constraints in general ensure that the vehicular flow which enters node i at time period t is equal to the vehicular flow which leaves this node. The second set of constraints represent the capacity constraints for each mode and on each arc. These constraints are represented in Eq.(10) and state that the vehicular flow of mode m on an arc that starts at node i at time period t and ends at node j at time period t' should be less than or equal to the capacity of that arc at that time period. The third set of constraints ensure the integrality of the vehicle related decision variables. These are represented in Eq.(11).

The linkage constraints represent the relationship between vehicular flow and commodity flow variables. These constraints are shown by Eq.(12) and operate on both the vehicular flow and commodity flow variables. These constraints determine the minimum number of vehicles needed to move the commodity assigned on arc (i,j) for each mode type.

4. Solution Procedure

Two solution algorithms are proposed for the multicommodity, multimodal network flow problem formulated in Section 3. The first solution algorithm decomposes the model into subproblems based on the relaxation of linkage constraints to exploit the special structure of the model. The second solution algorithm is an ad-hoc method that fixes integer variables gradually at every iteration until all integer variables are fixed to integer values. Both of the solution procedures use a linear programming package, LINDO.

4.1 Solution Algorithm I

In Section 3, we formulated the model as a mixed integer linear minimization problem. Let us call this minimization problem the original problem (OP). An investigation of this formulation indicates the complexity of the problem and its special structure. This problem structure is shown in Figure 3. In Figure 3, the commodity flow cost is the sum of commodity flow costs, supply or demand carry-over costs, and transfer costs. By examining the problem structure, it is clear that the problem could be solved relatively easily if the linkage constraints are relaxed. Once the linkage constraints are relaxed, we can decompose the problem into smaller subproblems (commodity flow subproblem and vehicular flow subproblem) which can be solved more easily. This relaxation and decomposition facilitates the use of the Lagrangian relaxation approach which is a powerful heuristic.

The first proposed heuristic is based on Lagrangian relaxation and uses LINDO (Linear, INteractive, and Discrete Optimizer) to solve the subproblems (Schrage, 1991). First, the Lagrangian problem is created by relaxing the complicating constraints which are the linkage constraints between vehicular and commodity flow (Eq. 12) and incorporating a penalty term in the objective function. The penalty term which is added to the objective function is as follows:

$$- \gamma \left(YCA^{m} \times Y^{m}_{itjt'} - \sum_{g} X^{gm}_{itjt'} \right) \qquad \textbf{(13)}$$

where γ is a positive Lagrangian multiplier vector. The Lagrangian problem is summarized in Appendix A.

When we solve the Lagrangian problem (LGP(γ)) to set a lower bound, the LGP(γ) can be decomposed separately into two subproblems, LGP1(γ) (the vehicular flow subproblem) and LGP2(γ) (the commodity flow subproblem). Two subproblems can be solved independently rather than solving LGP(γ) alone, which may be very costly.

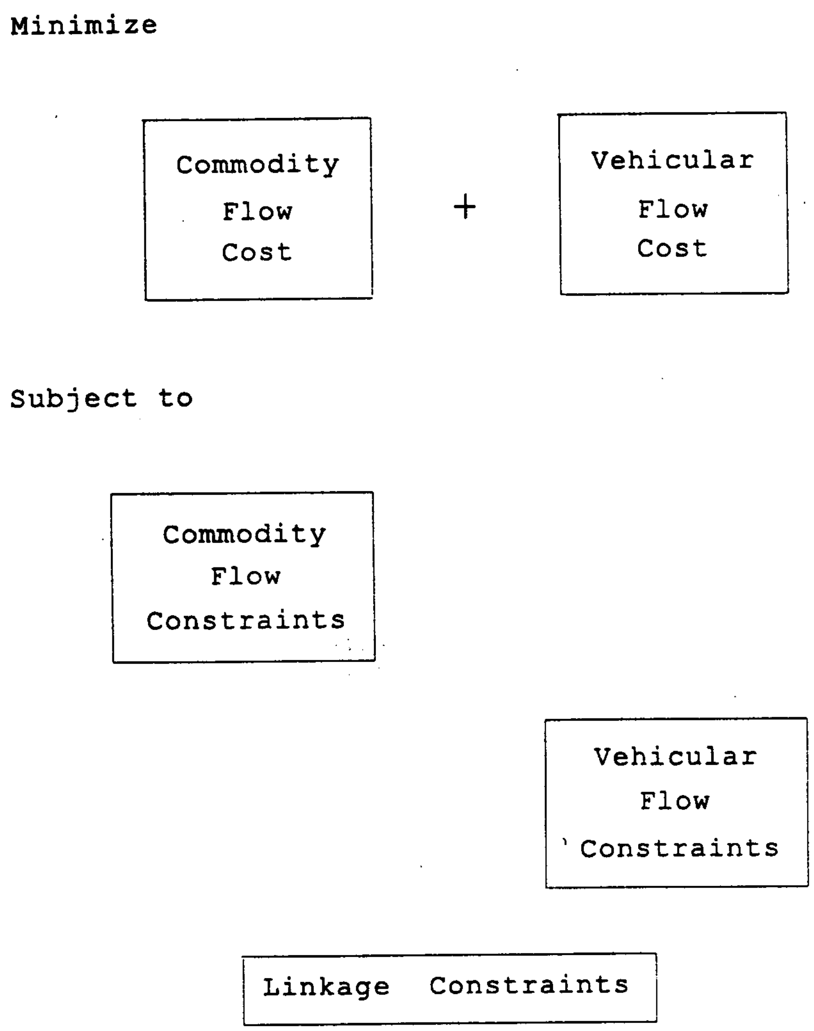

Figure 3. Problem Structure

Moreover an integer solution can be guaranteed whenever we solve the LGP1(γ) with LP relaxation because LGP1(γ) has the network problem structure (Phillips and Garcia-Diaz, 1981). The LGP1(γ) and LGP2(γ) are also summarized in Appendix A without the nonnegativity constraints. In the formulation of LGP2(γ), we add other constraints which are not shown in the original problem as follows:

$$\sum_{g} X^{gm}_{itjt'} \leq YCA^{m} \times ACA^{m}_{itjt'} \quad \text{for all } m, i, t, j, t' \qquad \textbf{(14)}$$

These constraints are an alternative expression for arc capacity constraints shown in Eq.(10). The capacity of each arc is expressed in terms of commodities which flow over that arc instead of the vehicular flow.

By adding Eq.(14) in the LGP2(γ), we can prevent the commodity flow in each arc from being over the arc capacity, which may happen by relaxing the linkage constraints. A feasible solution to the OP which provides an upper bound for the solution of the original problem can be obtained by solving OP with the added constraints. These added constraints set the value ofvehicular flow decision variables($Y^{m}_{itjt'}$) to the value obtained by solving LGP1(γ).

The detailed steps of the solution algorithm I are as follows:

Step 1. Solve OP with LP relaxation by LINDO. If a feasible solution exists, set $\alpha = 1$, $LB = -\infty$, $OP^* = \infty$, choose γ^1 arbitrarily, and go to Step 2. α is an iteration number, LB is a global lower bound so far, OP^* is a global upper bound and also the best solution to the OP so far, and γ^1 is an initial Lagrangian multiplier vector. If not, terminate because this problem is infeasible.

Step 2. Formulate and solve LGP(γ^α) by decomposing LGP1(γ^α) and LGP2(γ^α) using LINDO and set objective function value of LGP(γ^α) to LB^α. If $LB^\alpha > LB$, update $LB = LB^\alpha$ and go to Step 3. If not, go to Step 3.

Step 3. Solve OP by LINDO with the added constraints of $Y^{m}_{itjt'}$ = {solution of LGP1(γ^α)} to get a feasible solution to the OP and set the objective function value to $Ub\alpha$. If $UB^\alpha < OP^*$, update $OP^* = UB^\alpha$ and go to Step 4. If not, go to Step 4.

Step 4. If $(OP^* - LB) / |OP^*| \leq \delta$ or $\alpha = \alpha 1$, terminate. δ is a convergence ratio and $\alpha 1$ is an iteration limit. If not, go to Step 5.

Step 5. Let $\alpha 2$ be a prespecified number of iterations, β be a step size, λ be a scalar generally chosen between 0 and 2, and $Y^m_{itjt'}{}^\alpha$ and $X^{gm}_{itjt'}{}^\alpha$ be optimal solutions to $LGP(\gamma^\alpha)$. Then, if LB does not change after $\alpha 2$ iterations, set $\lambda^\alpha = \lambda^{\alpha-1}/2$. If it does, set $\lambda^\alpha = \lambda^{\alpha-1}$. Compute $\beta^\alpha = \lambda^\alpha(OP^* - LB^\alpha) / (YCA^m \times Y^m_{itjt'}{}^\alpha - \sum_g X^{gm}_{itjt'}{}^\alpha)^2$,

$$\gamma^{\alpha+1} = \max \{ 0, \gamma^\alpha - \beta^\alpha (YCA^m \times Y^m_{itjt'}{}^\alpha - \sum_g X^{gm}_{itjt'}{}^\alpha) \}.$$

Set $\alpha = \alpha + 1$ and go to Step 2.

4.2 Solution Algorithm II

The second solution algorithm is an interactive fix-and-run process. The idea of the fix-and-run process is as follows:

- First, a mixed integer linear problem is solved with the relaxation of integer variables.
- Second, the values of some integer variables are fixed in an orderly manner and the problem is solved again with the relaxation of the remaining integer variables iteratively.
- Finally, when all integer variables are fixed, the process is terminated.

The step-by-step procedure of the interactive fix-and-run process is as follows:

Step 1. Relax all integer variables and solve OP using LINDO. Set $k = 1$.

Step 2. Check all $Y^m_{itjt'}$ variables in which t is equal to k. If all $Y^m_{itjt'}$ variables in which $t = k$ are integer, then if $k = k1$, terminate. Otherwise, set $k = k+1$ and start Step 2. k1 is the final time period in the planning horizon.

Step 3. Fix all $Y^m_{itjt'}$ variables in which $t = k$ to nearest integer values while making sure that the vehicular flow conservation constraints in which they appear are not violated (Eq.9).

Step 4. Create a new problem by adding $\{ Y^m_{itjt'} = $ the fixed values in Step 3 $\}$ constraints for $t \leq k$ to the previous problem.

Step 5. Relax the rest of the integer variables and solve the new problem using LINDO. Set $k = 1$ again and go to Step 2.

In Step 3, the value of YC^m_{it} in Eq.(9) has been used as an offset to decide whether to round up or round down $Y^m_{itjt'}$ variables. The round up or round down procedure is as follows:

Step 3.1. Determine the value of YC^m_{it} from the LINDO output.

Step 3.2. Pick one $Y^m_{itjt'}$ variable out of several $Y^m_{itjt'}$ variables and round up that variable.

Step 3.3. Calculate an initial offset (YC^m_{it} - (1 - decimal value of chosen $Y^m_{itjt'}$)).

Step 3.4. Pick the next $Y^m_{itjt'}$ variable and If (previous offset - (1 - decimal value of chosen $Y^m_{itjt'}$)) ≥ 0, round up that $Y^m_{itjt'}$ variable. If not, round down.

Step 3.5. If rounding up, calculate the new offset (previous offset - (1 - decimal value of chosen $Y^m_{itjt'}$)). If rounding down, calculate new offset (previous offset + decimal value of chosen $Y^m_{itjt'}$). Go to Step 3.4.

Flow charts for both solution methods are given in Appendix B.

5. Model Implementation

This section describes the findings of the model implementation. The performance of proposed model is evaluated using a variety of summary measures including vehicular flow costs, commodity flow costs, supply and demand carry-over costs, transfer costs, total operating costs, amount of late delivered commodity, and amount of commodity delivered by each mode.

5.1 Environment of Model Implementation

For the solution algorithm I, a computer code was developed to solve the problem iteratively. This code writes an input file (objective function and constraints) readable for LINDO, reads an output file generated by LINDO, and updates an input file after checking the convergency criteria.

The model was implemented by using two 486/66 personal computers at the same time. Two PC's connected with a parallel port cable were used, such that the two PC's could interchange data. Recall that the detailed steps of the solution algorithm I was presented in Section 4. In the solution algorithm I, we solve three different linear programs by LINDO software at every iteration. The three linear programs are LGP1, LGP2, and OP. To solve OP, the results of LGP1 are needed. However, LGP2 can be solved separately from LGP1 and OP. Therefore two PC's could be utilized simultaneously to speed up the process. The first PC conducts all of the steps of the solution algorithm except Step 3. The second PC handles Step 3 which is the solution of OP with vehicular flow variables ($Y^{m}_{itjt'}$) fixed to the value of the solution of LGP1.

Through test runs, it ibecame clear that the computation time of LGP1 is relatively short. The computation time of LGP2 and OP is longer than that of LGP1. Furthermore, the computation time of OP takes longer than that of LGP2. We found out that the computation time could be reduced by approximately 30 percent when we used two PC's at the same time rather than using just one PC.

For the solution algorithm II, a computer code was also developed to solve the problem interactively. This code writes an input file readable for LINDO, reads an output file generated by LINDO, and updates the input file by fixing some integer variables based on the related flow conservation constraints. This solution was implemented in one 486/66 personal computer.

5.2 Data Characteristics

For the model implementation, a data set was artificially generated in a medium sized physical network especially for the application to the disaster relief operations. In generating these data, many assumptions were introduced.

Recall that the inputs to the model consist of the physical network inputs, the time-space network inputs, the supply and demand inputs, and the unit costs as mentioned in Section 4. For the physical network inputs, it is assumed that three modes are available. The modes are labeled by letters L, M, and N. Therefore, we need three physical networks to represent the whole network. The physical networks for modes L and M consist of seven nodes and ten two-way arcs. The physical network for mode N, however, consists of five nodes and four two-way arcs.

The physical network for each mode L, M, and N is presented in Figure 4a, Figure 4b, and Figure 4c, respectively. The nodes of these network are labeled by letters A, B, C, D, E, F, and G. The arc travel time for each mode is shown beside the arc by the number of time periods. The right-hand-side number of the arc travel time in Figures 4a, 4b, and 4c represents the arc capacity. In general, it is possible that the arc capacities are different at different time periods. However, the arc capacities are assumed to be the same throughout the entire planning horizon in this example. It is assumed that nodes A, B, and C are origins and nodes F and G are destinations. The transfer time from one mode to another mode is assumed to be one time period, regardless of modes and commodities.

Regarding the time-space network inputs, the planning horizon has a length of 15 equal time periods. Therefore, we have 16 time periods in the corresponding time-space network. For the supply and demand inputs, it is assumed that two types of commodities (P and Q) are to be delivered from 3 origins(A, B, C) to 2 destinations(F, G) by three modes L, M, and N which have a vehicle loading capacity of 4, 1, and 16 units, respectively. In this model implementation, it is assumed that we have only one time window for the pick up and delivery of each commodity.

5.3 Empirical Study

5.3.1 Selection of Parameters in the Solution Algorithm I

In the solution algorithm I presented in Section 4, parameters are to be determined initially to run that algorithm. Those parameters are an initial Lagrangian multiplier vector (γ^1), a convergence ratio (δ), an iteration limit ($\alpha 1$), a prespecified number of iterations ($\alpha 2$), and an initial scalar (λ^0).

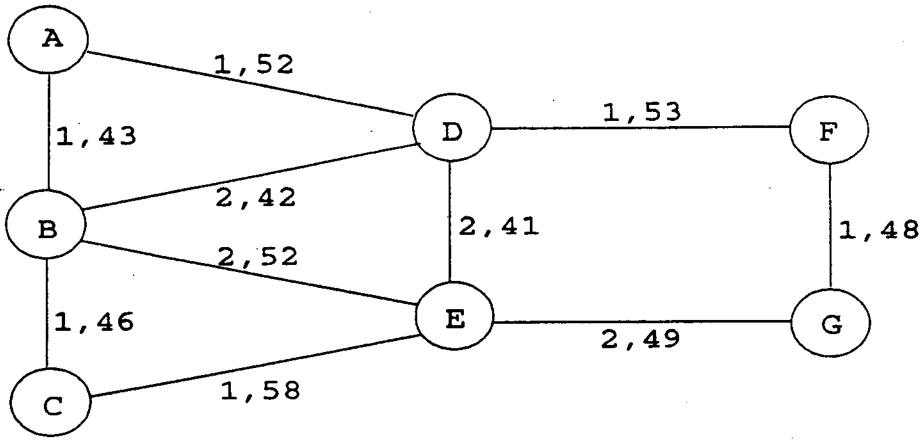

Figure 4a. A Physical Network of Mode L

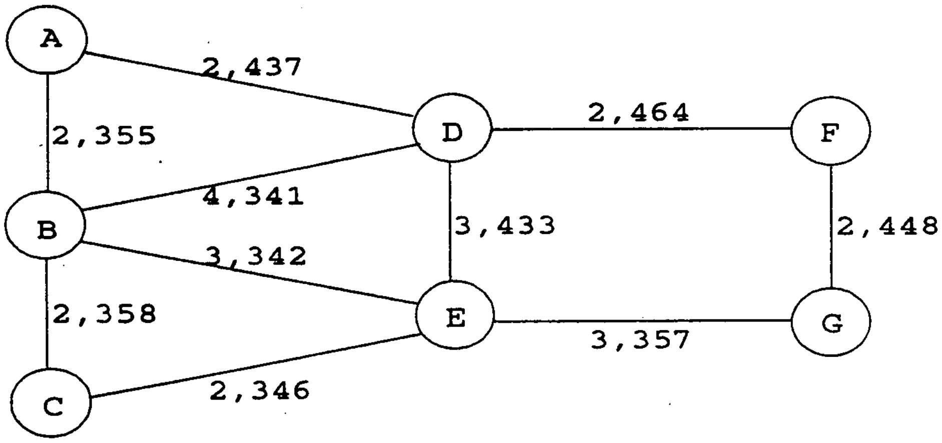

Figure 4b. A Physical Network of Mode M

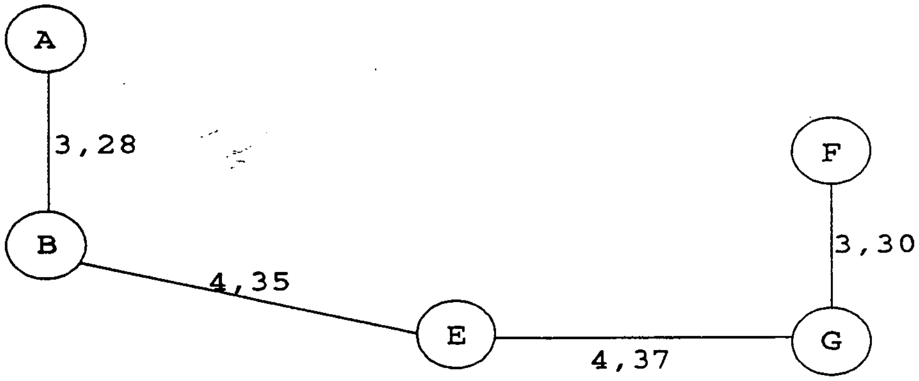

Figure 4c. A Physical Network of Mode N

The model was implemented with various parameters to choose the best combination of parameters. We used two sets of value for γ^1. In the first set all components of vector γ^1 were set to 0. In the second set all components of vector γ^1 were set to the dual price of the corresponding constraint as shown on the solution of the LP relaxation of the original problem. For the value of $\alpha 2$, we selected three values, that is, 5, 10, and 15. For the value of λ^0, four values, 0.5, 1, 1.5, and 2, were used. Therefore, we implemented 24 runs (2 x 3 x 4) to select the best combination of parameters. We tested all runs with a value of $\alpha 1$ set to 100. We selected the value of γ^1 as the dual price shown on the solution of the original problem with LP relaxation, the value of $\alpha 2$ as 10, and the value of λ^0 as 1. The values of the selected parameters correspond to the values of these parameters used by other researchers (Fisher, 1985).

5.3.2 Performance Evaluation and Results

For the performance evaluation, the problem with the given data set was solved using both solution algorithm I and II. Outputs of algorithm I and II were compared and summarized in Table 1. As we can see from Table 1, we get a nearly optimal solution which has a gap of 2.79% for the algorithm I and 0.27% for the algorithm II. The run time was estimated as 215 minutes for the algorithm I and 32 minutes for the algorithm II.

For the solution by algorithm I, among the total operating cost of $30,307,010 (100%), we have the vehicular flow cost of $16,152,000 (53.3%), the commodity flow cost of $3,669,490 (12.1%), the supply carry-over cost of $1,032,080 (3.4%), the demand carry-overcost of $9,410,400 (31.0%), and the transfer cost of $43,040 (0.2%). A total of 6000 units of commodities were needed at the two destinations. However, only 5996 units were delivered during a planning horizon. Among the 5996 units of commodities, a total of 3200 units were delivered one or more time periods late. Among the 3200 units of commodities delivered late, 2296 units were delivered one time period late and 904 units were delivered two time periods late.

Summary Measures	Classified by		Alg. I	Alg. II
	Mode	Commodity		
Vehicular flow cost	L		4770000	4590000
	M		6474000	6350400
	N		4908000	4744800
Commodity flow cost	L	P	618080	586260
	M		1718160	1707200
	N		675610	704390
	L	Q	92280	112220
	M		158320	193300
	N		407040	435500
Supply carry-over cost		P	732000	724860
		Q	300080	269880
Demand carry-over cost		P	5780000	5868000
		Q	3630400	3230400
Transfer cost		P	35600	32910
		Q	7440	11160
Total cost			30307010	29561280
Amount of late delivered commodity		P	1928	1972
		Q	1272	1288
Utilization factor	L		0.1504	0.1451
	M		0.5557	0.5481
	N		0.6323	0.6112
Amount of commodity delivered	L	P	696	674
	M		2100	2088
	N		1204	1238
	L	Q	132	114
	M		464	476
	N		1400	1410

Table 1. Performance Evaluation

For the solution by algorithm II, among the total operating cost of $29,561,280 (100%), we have the vehicular flow cost of $15,685,200 (53.1%), the commodity flow cost of $3,738,870 (12.5%), the supply carry-over cost of $994,740 (3.4%), the demand carry-over cost of $9,098,400 (30.8%), and the transfer cost of $44,070 (0.2%). All 6000 units of commodities were delivered during a planning horizon. Among the 6000 units of commodities, a total of 3260 units were delivered late. Among the 3260 units of commodities delivered late, 2372 units were delivered one time period late, 880 units two time periods late, and 8 units three time periods late.

A wide range of sensitivity analyses were implemented to examine the performance of the model under many different input parameter values by both solution procedures. Even though the solution algorithm II generated a better solution than the solution algorithm I, all the sensitivity analyses were implemented for both solution algorithms. The sensitivity analyses were divided into three groups. The first group tested the sensitivity of the model with respect to the physical network inputs, and in particular, the arc capacity. We ran the model with increased (+50%, +25%) and decreased (-25%, -50%) arc capacities. The first group of runs included 12 computer runs.

The second group tested the sensitivity of the model with respect to the supply and demand inputs, and in particular, the amount of commodity supply, the amount of commodity demand, the number of vehicle supply, and the vehicle loading capacity. For each input parameter, we ran the model with increased (+50%, +25%) and decreased (-25%, -50%) values. This group of runs involved 44 computer runs.

The third group tested the sensitivity of the model with respect to the unit costs which were the unit vehicular flow costs, the unit commodity flow costs, the unit carry-over supply costs, the unit carry-over demand costs, and the unit transfer costs. The model with increased (+50%, +25%) and decreased(-25%, -50%) values of each input parameter was run. This resulted in 48 more computer runs, completing the sensitivity analyses with a total of 104 runs. The results of these analyses are reported elsewhere (Oh, 1993). It suffices to say that all of these runs confirm that both solution algorithm I and II give nearly-optimal solution in all applications, and the performance of the solution algorithms is relatively robust.

5.4 Extension of Empirical Study

This section extends the empirical study to further test the applicability of the model. The model was implemented in two other networks.

5.4.1 Application to Small Network

The model was tested in a small network which has 4 nodes (1 origin and 2 destinations) and 5 two-way arcs. To evaluate the model performance more, ten cases of sensitivity analyses were run. Ten cases were selected randomly out of 104 runs of sensitivity analyses mentioned in the previous section. The results are summarized in Table 2. In Table 2, run number 1 represents the base case and others represent 10 cases of sensitivity analyses.

For the solution of the base case by algorithm I, among the total operating cost of \$8,978,390 (100%), we have the vehicular flow cost of \$6,874,470 (76.6%), the commodity flow cost of \$1,520,580 (16.9%), the supply carry-over cost of \$84,240 (1.0%), the demand carry-over cost of \$389,600 (4.3%), and the transfer cost of \$109,500 (1.2%). All 2850 units of commodities are delivered during a planning horizon. Among the 2850 units of commodities, 532 units (18.7%) are delivered by mode L, 1048 units (36.8%) by mode M, and 1270 units (44.5%) by mode N. Among the 1747 units of commodities delivered late, 1217 units are delivered one time period late and 530 units two time periods late. The run time was estimated as 21 minutes 54 seconds.

For the solution of the base case by algorithm II, among the total operating cost of \$8,726,280 (100%), we have the vehicular flow cost of \$6,621,900 (75.9%), the commodity flow cost of \$1,505,750 (17.2%), the supply carry-over cost of \$84,530 (1.0%), the demand carry-over cost of \$407,800 (4.7%), and the transfer cost of \$106,300 (1.2%). All 2850 units of commodities are delivered during a planning horizon. Among the 2850 units of commodities, 520 units (18.3%) are delivered by mode L, 1050 units (36.8%) by mode M, and 1280 units (44.9%) by mode N.

Among the 1755 units of commodities delivered late, 1223 units are delivered one time period late and 532 units two time periods late. The run time was estimated as 5 minutes 46 seconds.

Run No.	LP Solution	Alg. I Solution	Gap (%)	Alg. II Solution	Gap (%)
1	8687170	8978390	3.35	8726280	0.45
s1	8687170	9127250	5.07	8728600	0.48
s2	210464300	211639950	0.56	210491580	0.01
s3	165097970	167943530	1.72	165137090	0.02
s4	7335870	7617610	3.84	7365710	0.41
s5	31922682	32597360	2.11	32162412	0.75
s6	9372670	9903060	5.66	9399860	0.29
s7	8386170	8996050	7.27	8432100	0.55
s8	8729125	8876150	1.69	8768545	0.45
s9	8600780	9490750	10.35	8640350	0.46
s10	8728720	9099410	4.25	8766730	0.44

Table 2. Empirical Study in Small Network

5.4.2 Application to Large Network

The model was implemented in a large network which has 10 nodes (3 origins and 1 destination) and 20 two-way arcs. This network is the appropriate size in the real-world application. Ten cases of sensitivity analyses were randomly chosen from 104 runs of sensitivity analyses in the prior section. Those were tested to evaluate the performance of the model in a broad sense. The results are summarized in Table 3. In Table 3, run number 1 corresponds to the base case and others correspond to 10 cases of sensitivity analyses.

For the solution of the base case by algorithm I, among the total operating cost of $32,911,350 (100%), we have the vehicular flow cost of $18,119,100 (55.0%), the commodity flow cost of $4,346,490 (13.2%), the supply carry-over cost of $851,820 (2.6%), the demand carry-over cost of $9,480,000 (28.8%), and the transfer cost of $113,940 (0.4%). All 6000 units of commodities are delivered during a planning horizon. Among the 6000 units of commodities, 1216 units (20.3%) are delivered by mode L, 3000 units (50.0%) by mode M, and 1784 units (29.7%) by mode N. Among the 2888 units of commodities delivered late, 2320 units are delivered one time period late and 568 units two time periods late. It took 8 hours 34 minutes to solve the problem.

Run No.	LP Solution	Alg. I Solution	Gap (%)	Alg. II Solution	Gap (%)
1	32235770	32911350	2.10	32312950	0.24
s1	31810610	32068120	0.81	31824440	0.04
s2	32070770	32730150	2.06	32153800	0.26
s3	208937600	218694600	4.67	208956880	0.01
s4	32082010	32661090	1.80	32170190	0.27
s5	40373530	41897320	3.77	40460020	0.21
s6	33602980	34447600	2.51	33632060	0.09
s7	31177910	32077260	2.88	31184450	0.02
s8	32661330	33314040	2.00	32759250	0.30
s9	29409850	30696030	4.37	29463540	0.18
s10	32235770	33344460	3.44	32286900	0.16

Table 3. Empirical Study in Large Network

For the solution of the base case by algorithm II, among the total operating cost of $32,312,950 (100%), we have the vehicular flow cost of $16,630,000 (51.5%), the commodity flow cost of $4,186,040 (12.9%), the supply carry-over cost of $903,540 (2.8%), the demand carry-over cost of $10,532,000 (32.6%), and the transfer cost of $61,370 (0.2%). All 6000 units of commodities are delivered during a planning horizon. Among the 6000 units of commodities, 1016 units (16.9%) are delivered by mode L, 3000 units (50.0%) by mode M, and 1984 units (33.1%) by mode N. Among the 3088 units of commodities delivered late, 2284 units are delivered one time period late and 804 units two time periods late.. It took 3 hours 13 minutes to solve the problem.

All of these runs indicate that the model performance is relatively robust and except for the running time, other performance measures are not affected by the network size.

6. Conclusions and Directions for Future Research

The overall conclusions of this research can be outlined as follows. A complex multi-commodity, multi-modal network flow problem with time windows in the context of disaster relief operations is formulated and solved relatively easily. This model can be incorporated into a desision support system to support emergency response managers in planning for disaster relief operations. The proposed solution techniques perform well and provide fairly good results for this problem. The solution algorithm II performs better than algorithm I in terms of solution accuracy and run time.

The sensitivity analysis reveals that the model is sensitive to the arc capacity, the vehicle supply quantity, the vehicle capacity, the unit vehicular flow costs, and the unit demand carry-over costs. The model is moderately sensitive to the commodity supply and demand quantity within a range where the supply or demand quantity is satisfied. It is very sensitive beyond that range. Also the model is moderately sensitive to the unit commodity flow costs and the unit supply carry-over costs. The model is insensitive to the unit transfer costs in the range of the tested values.

The model performance is relatively robust and except for the running time, other performance measures are not affected by the network size. Finally, although the model and solution procedures are intended as a preliminary planning tool for disaster relief logistics management, one can actually use it in real-time provided that it is tied to a database which can be updated in real-time.

Two general directions for future research can be identified. One is the exploration of the model formulation and the other is the extension and development of the solution procedure.

In the model formulation, all of the cost functions are assumed to be linear. Non-linear cost functions which are dependent on flows can be explored to represent the model more realistically. In this case, a non-linear optimization problem will result which would be difficult to solve. Developing innovative solution procedures for such a model is an important area for future research.

Another avenue for further model improvement involves the introduction of uncertainty into the model. There exists uncertainties in forecasting supply and demand of commodities and in predicting supply of vehicles for each mode. Incorporation of uncertainties to the model formulation will result in a more sophisticated model which may not have an easy solution.

In terms of extension and development of solution methodology, we need to speed up the current solution procedures to be used in the real-world operation. For this task, faster computers and other powerful linear programming software such as CPLEX can be used. Another avenue is to improve the solution algorithm I using the solution from the solution algorithm II as a bound. We may reduce the run time and improve the solution optimality of the algorithm I.

Another area is to develop an integrated model in a software package which can access, manipulate, and update a large scale logistical database. This logistical database could be a Geographic Information System (GIS) for the region under consideration. Finally, it is suggested that the model be tested with real-world data rather than artificially generated data.

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APPENDIX A

$$\text{Minimize} \sum_i \sum_j \sum_t \sum_{t'} \sum_m \left(CVR^m_{itjt'} - \gamma^m_{itjt'} \times YCA^m \right) \times Y^m_{itjt'}$$
$$+ \sum_i \sum_j \sum_t \sum_{t'} \sum_g \sum_m \left(CGR^{gm}_{itjt'} + \gamma^m_{itjt'} \right) \times X^{gm}_{itjt'}$$
$$+ \sum_{i \in U,UV,V} \sum_t \sum_g CSC_{git} \times \left(\sum_m SC^m_{git}\right) + \sum_{i \in VW,W} \sum_t \sum_g CDC_{git} \times \left(\sum_m DC^m_{git}\right)$$
$$+ \sum_{i \in UV,V} \sum_t \sum_g \sum_m \sum_{m'} CGT^{gmm'}_{it(t+Kmm')} \times XT^{gmm'}_{it(t+Kmm')}$$

Subject to

$$\sum_j \sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} + SC^m_{gi(t-1)} + SE^m_{git}$$
$$= \sum_j \sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} + SC^m_{git} \qquad (\, i \in U,\ UV,\ V\,)$$
$$\sum_j \sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} - DC^m_{gi(t-1)}$$
$$= \sum_j \sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} - DC^m_{git} + DE^m_{git} \qquad (\, i \in VW,\ W\,)$$

for all m, g, i, t

$$SE_{git} = \sum_m SE^m_{git} \qquad (\, i \in U,\ UV\,)$$
$$DE_{git} = \sum_m DE^m_{git} \qquad (\, i \in VW,\ W\,)$$

$$DC^m_{git} = 0 \qquad \text{for all } t < EDT_{gi}$$

$$YE^m_{it} + \sum_j \sum_{t'} Y^m_{jt'it} + YC^m_{i(t-1)} = \sum_j \sum_{t'} Y^m_{itjt'} + YC^m_{it}$$

for all m, i, t

$$Y^m_{itjt'} \le ACA^m_{itjt'} \qquad \text{for all } m, i, t, j, t'$$

$$X^{gm}_{itjt'} \ge 0, \quad XT^{gmm'}_{it(t+Kmm')} \ge 0, \quad Y^m_{itjt'} \ge 0, \quad YC^m_{it} \ge 0,$$
$$SE^m_{git} \ge 0, \quad SC^m_{git} \ge 0, \quad DE^m_{git} \ge 0, \quad DC^m_{git} \ge 0$$

for all g, m, m′, i, t, j, t′

Summary of the Lagrangian Problem, LGP(γ)

$$\textit{Minimize} \quad \sum_{i}\sum_{j}\sum_{t}\sum_{t'}\sum_{m} (CVR^{m}_{itjt'} - \gamma^{m}_{itjt'} \times YCA^{m}) \times Y^{m}_{itjt'}$$

Subject to

$$YE^{m}_{it} + \sum_{j}\sum_{t'} Y^{m}_{jt'it} + YC^{m}_{i(t-1)} = \sum_{j}\sum_{t'} Y^{m}_{itjt'} + YC^{m}_{it}$$

for all m, i, t

$$Y^{m}_{itjt'} \leq ACA^{m}_{itjt'} \qquad \textit{for all } m, i, t, j, t'$$

Summary of the Formulation of LGP1(γ)

$$\text{Minimize} \quad \sum_i \sum_j \sum_t \sum_{t'} \sum_g \sum_m \left(CGR^{gm}_{itjt'} + \gamma^m_{itjt'} \right) \times X^{gm}_{itjt'}$$

$$+ \sum_{i \in U,UV,V} \sum_t \sum_g CSC_{git} \times \left(\sum_m SC^m_{git}\right) + \sum_{i \in VW,W} \sum_t \sum_g CDC_{git} \times \left(\sum_m DC^m_{git}\right)$$

$$+ \sum_{i \in UV,V} \sum_t \sum_g \sum_m \sum_{m'} CGT^{gmm'}_{it(t+Kmm')} \times XT^{gmm'}_{it(t+Kmm')}$$

Subject to

$$\sum_j \sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} + SC^m_{gi(t-1)} + SE^m_{git}$$

$$= \sum_j \sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} + SC^m_{git} \qquad (\, i \in U,\ UV,\ V \,)$$

$$\sum_j \sum_{t'} X^{gm}_{jt'it} + \sum_{m'} XT^{gm'm}_{i(t-Km'm)t} - DC^m_{gi(t-1)}$$

$$= \sum_j \sum_{t'} X^{gm}_{itjt'} + \sum_{m'} XT^{gmm'}_{it(t+Kmm')} - DC^m_{git} + DE^m_{git} \qquad (\, i \in VW,\ W \,)$$

$$\text{for all } m,\ g,\ i,\ t$$

$$SE_{git} = \sum_m SE^m_{git} \qquad (\, i \in U,\ UV \,)$$

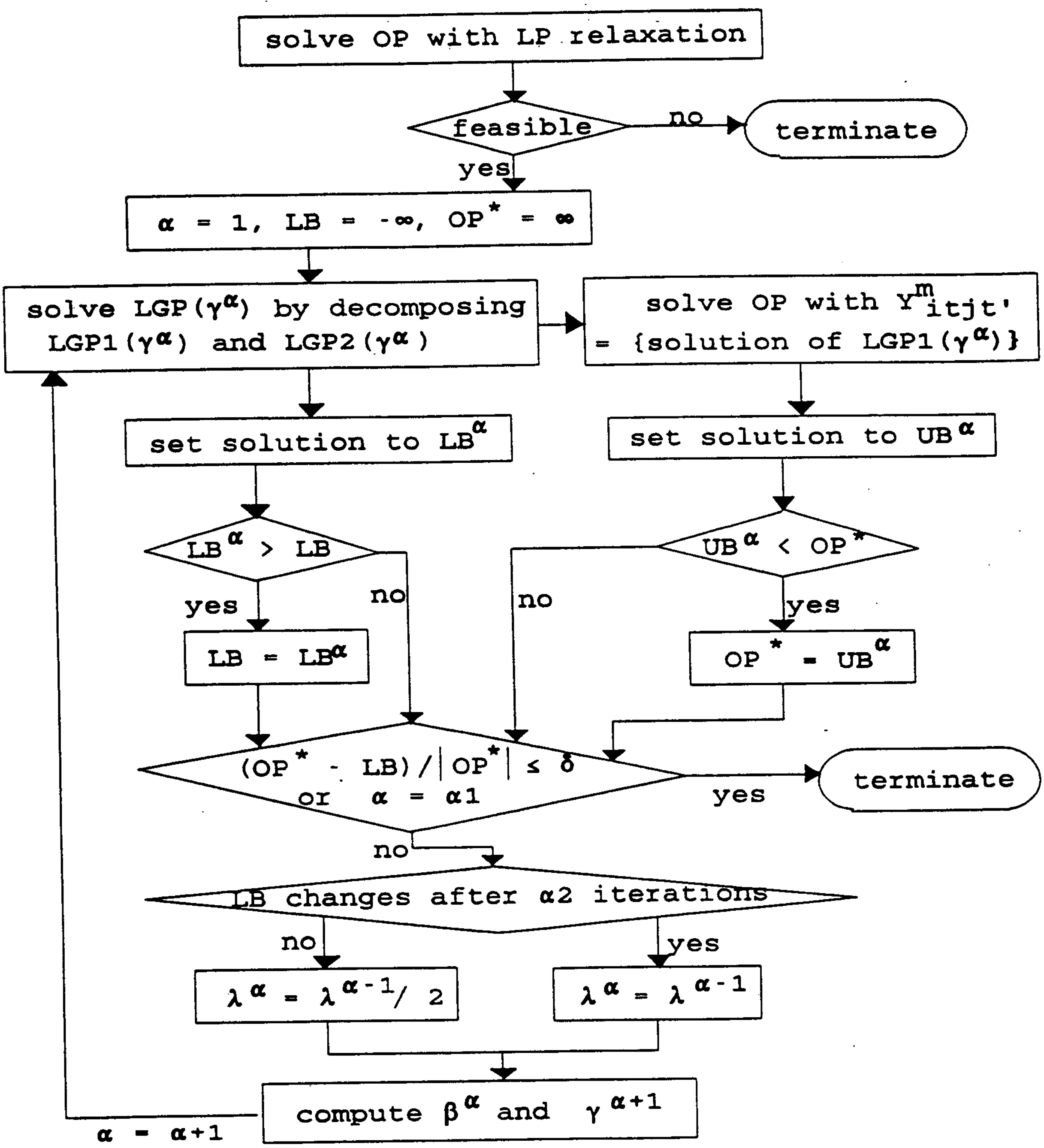
$$DE_{git} = \sum_m DE^m_{git} \qquad (\, i \in VW,\ W \,)$$

$$DC^m_{git} = 0 \qquad \text{for all } t < EDT_{gi}$$

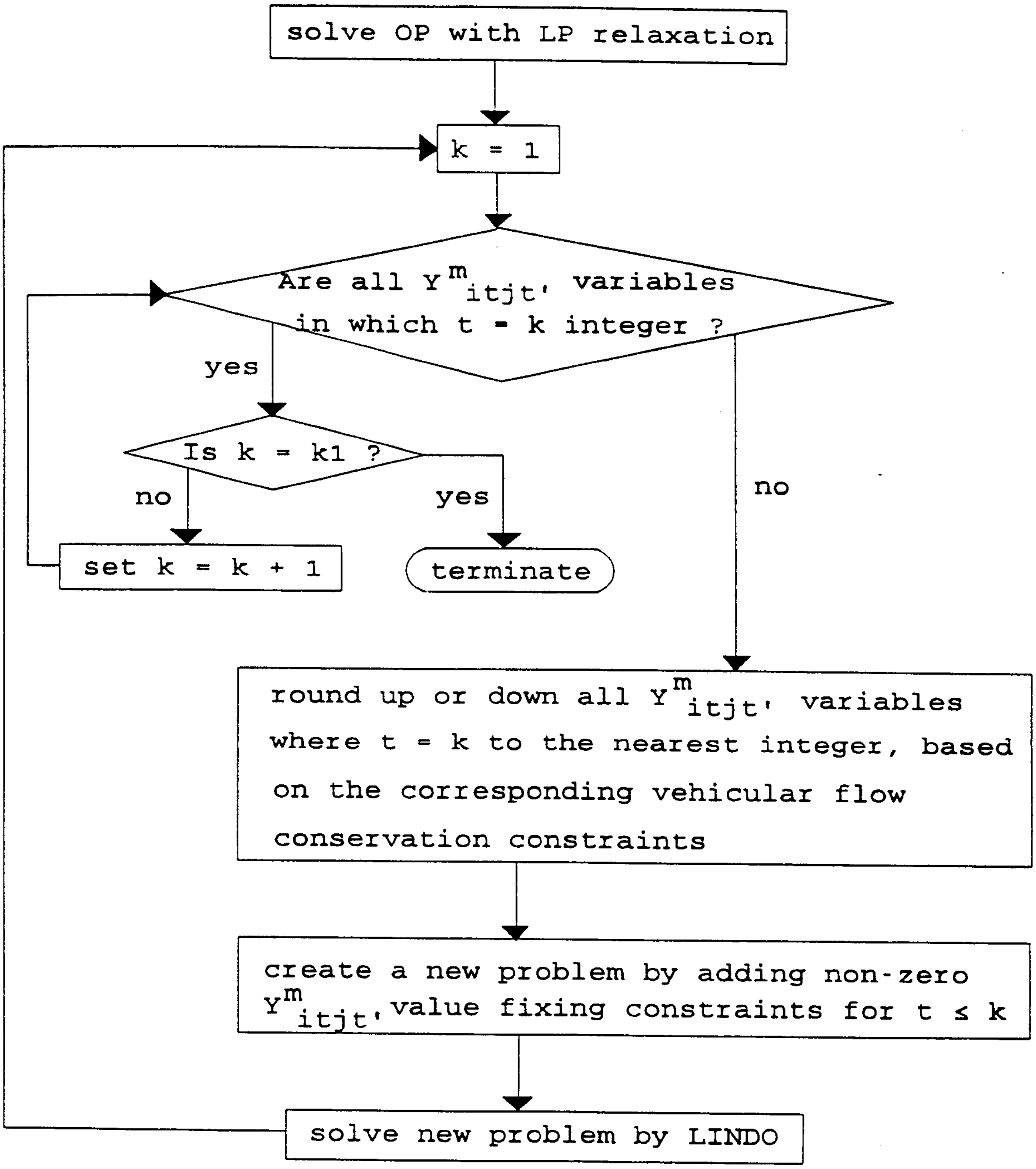
$$\sum_g X^{gm}_{itjt'} \le YCA^m \times ACA^m_{itjt'} \qquad \text{for all } m,\ i,\ t,\ j,\ t'$$

Summary of the Formulation of LGP2(γ)

APPENDIX B



Flowchart of Solution Algorithm I



Flowchart of Solution Algorithm II