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개선된 부가비례항법을 이용한 3차원 유도법칙의 구현

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The Realization of the Three Dimensional Guidance Law Using Modified Augmented Proportional Navigation

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Abstract

This paper deals with 3-dimensional missile guidance law. This presents the general optimal solution of the state equation which includes the target maneuvering as the Gauss-Markov processing.

The main results are about the transformation between the Cartesian coordinates on which both the guidance law and the filter are based and the polar coordinates system in real missile guidance and radar measurement information. And the extended Kalman filter and adjustment of the estimated target acceleration by triangular functions is proposed solution to this transformation problem.

It is shown that this proposed transformation is valid in real 3-dimensional guidance problem by the computer simulation.

1. Introduction

For a maneuvering target, APNG is known *to be more effective than pure* proportional navigation guidance (PPNG) and APNG is derived from the *linear quadratic gaussian (LQG) control method in Cartesian coordinates* [1].

It is worthwhile to pay attention to the researches on the guidance law *using guidance* law in two ways.

First, Paul[2] introduced the four kinds of target maneuvering model and constructed the guidance law using one of model in the form of the extended Kalman filter and linear optimal guidance input including the information not only of the position and velocity but also of the target maneuver acceleration. Since, however, the Kalman filter and optimal guidance law are presented in Cartesian coordinate system, the result is not applicable to the real guidance system which needs the normal acceleration command to steer the missile.

Second, Tang[3] dealt with this problem directly in polar coordinate system using the extended Kalman filter for simplified model and time-varying optimal guidance law. The shortages of this solution are that this is limited to 2-dimensional pitch plane and that the optimal guidance law is the direct solution for the only one case of finite time LQ problem, so if applied to another case this solution falls into a suboptimal one.

The control of missile contains originally *the polar nature: axial thrust and* two angular rates. In addition, the filtering problem also does from the fact that the information from *the radar consists of range, elevation angle and* azimuth angle. In dealing with the three dimensional guidance problem, the *relative dynamic equation in polar* coordinates for guidance law and filter is so complicated that many appended problems arise, though missile and target are supposed to be point masses.

The compromise between these two accessing ways can be the one using guidance law and filter in the form of Cartesian coordinates and then transforming them to the polar coordinates form. This method has a merit of easier manageability and realizability compared with the former candidate.

This paper concentrates on this topic, and shows that this access can guarantee a good result without handling the complicated nonlinear control and filtering technique.

2. Review on Optimal Guidance Law

The *relative dynamic* equation in Cartesian coordinates between missile and target is given by

$$\dot{x} = \begin{bmatrix} 0 & I_3 & 0 \\ 0 & 0 & I_3 \\ 0 & 0 & -\alpha \end{bmatrix} x + \begin{bmatrix} 0 \\ -I_3 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ w_T \end{bmatrix}, \tag{2.1}$$

$$x = [\, x_r, y_r, z_r, \dot{x}_r, \dot{y}_r, \dot{z}_r, A_{Tx}, A_{Ty}, A_{Tz} \,]^T$$

$$u = [\, A_{mx}, A_{my}, A_{mz} \,]^T$$

where

x_r, y_r, z_r denote the relative target position in inertial coordinates, and

A_{Tx}, A_{Ty}, A_{Tz} are target maneuvering acceleration,

I_3 *is the identical matrix of dimension* 3, A_{mx}, A_{my}, A_{mz} *are the missile* acceleration, α denotes the target bandwidth, and w_T is Gaussian white noise.

For the system given by (2.1), the optimal input minimizing the performance index

$$J = x^T(t_f) S_f x(t_f) + \int_{t_0}^{t_f} u^T R u \, dt \tag{2.2}$$

where

$$S_f = diag\begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

and R is a appropriate positive definite matrix,
is given by

$$u_x = \Lambda t_g^{-2} \left[x_r + \dot{x}_r t_g + \frac{1}{2} A_{Tx} t_g^2 \right]. \tag{2.3}$$

where t_g denotes time-to-go defined by

$$t_g = t_f - t, \tag{2.4}$$

and Λ is the navigation ratio defined by

$$\Lambda \equiv \left(\frac{1}{3} I + C R^{-1} t_f^{-3} \right)^{-1}. \tag{2.5}$$

In the procedure of derivation of (2.3), the assumption of the small correlation of the target maneuver white noise i.e.

$$\alpha \to 0 \tag{2.6}$$

is needed.

3. APNG in Polar Coordinates

Figure 1 shows the relative position of the two objects. Point M denotes the origin of inertial coordinates which corresponds to missile position and *T is relative target position during the guidance phase.*

Now let us consider the horizontal plane geometry of missile-and-target in the inertial coordinates.

The azimuth angle is given by

$$\phi = \tan^{-1} \frac{y}{x}. \tag{3.1}$$

Differentiate (3.1) to get

$$\dot{\phi} = \frac{x \dot{y} - y \dot{x}}{x^2 + y^2}. \tag{3.2}$$

Modifying the above equation without harming the equality gives

$$\begin{aligned} \dot{\phi} &= \frac{t_g x \dot{y} + x y - t_g y \dot{x} - x y}{(x^2 + y^2) t_g} \\ &= \frac{1}{(x^2 + y^2) t_g} \left\{ x(t_g \dot{y} + y) - y(t_g \dot{x} + x) \right\} \end{aligned} \tag{3.3}$$

Substitute the optimal solutions obtained in Section 2 to get

$$\begin{aligned} \dot{\phi} &= \frac{1}{(x^2 + y^2) t_g} \frac{t_g^2}{\Lambda} \left\{ x \left[u_y - \frac{\Lambda}{2} A_{Ty} \right] - y \left[u_x - \frac{\Lambda}{2} A_{Tx} \right] \right\} \\ &= \frac{1}{\Lambda \sqrt{x^2 + y^2} / t_g} \left\{ \frac{x}{\sqrt{x^2 + y^2}} \left[u_y - \frac{\Lambda}{2} A_{Ty} \right] - \frac{y}{\sqrt{x^2 + y^2}} \left[u_x - \frac{\Lambda}{2} A_{Tx} \right] \right\} \end{aligned} \tag{3.4}$$

To make this equation neater, define the ground crossing velocity as

$$V_{gc} \equiv \frac{\sqrt{x^2+y^2}}{t_g}, \tag{3.5}$$

then (3.4) can be changed into

$$\dot{\phi} = \frac{1}{\Lambda V_{gc}}\left\{\left[u_y - \frac{\Lambda}{2}A_{Ty}\right]\cos\phi - \left[u_x - \frac{\Lambda}{2}A_{Tx}\right]\sin\phi\right\} \tag{3.6}$$

Since we need the lateral steering command, let

$$u_\phi \equiv u_y\cos\phi - u_x\sin\phi. \tag{3.7}$$

Then

$$u_\phi = \Lambda V_{gc}\dot{\phi} + \frac{\Lambda}{2}(A_{Ty}\cos\phi - A_{Tx}\sin\phi) \tag{3.8}$$

It is similar to the previous procedure to derive the equation for lateral plane geometry.
First,

$$\theta = \tan^{-1}\frac{z}{\sqrt{x^2+y^2}} \tag{3.9}$$

Differentiate (3.9) to get

$$\dot{\theta} = \frac{-zx\dot{x} - zy\dot{y} + (x^2+y^2)\dot{z}}{\sqrt{x^2+y^2}\,(x^2+y^2+z^2)}. \tag{3.10}$$

Modifying the above equation by adding the null term, we obtain

$$\dot{\theta} = \frac{1}{t_g\gamma^2\sqrt{x^2+y^2}}\left\{-t_g zx\dot{x} - t_g zy\dot{y} - z(x^2+y^2) + t_g(x^2+y^2)\dot{z} + z(x^2+y^2)\right\} \tag{3.11}$$

where

$$\gamma \equiv \sqrt{x^2+y^2+z^2}. \tag{3.12}$$

Arranging (3.11), we get

$$\dot{\theta} = \frac{1}{t_g\gamma^2\sqrt{x^2+y^2}}\left\{-zx(x+t_g\dot{x}) - zy(y+t_g\dot{y}) + (x^2+y^2)(z+t_g\dot{z})\right\} \tag{3.13}$$

Substituting the optimal results into (3.13) we get

$$\dot{\theta} = \frac{1}{\Lambda V_c}\left\{-\left[u_x - \frac{\Lambda}{2}A_{Tx}\right]\tan\theta\cos\phi - \left[u_y - \frac{\Lambda}{2}A_{Ty}\right]\tan\theta\sin\phi + \left[u_z - \frac{\Lambda}{2}A_{Tz}\right]\cos\theta\right\} \tag{3.14}$$

where V_c is closing velocity defined by

$$V_c = \gamma / t_g. \tag{3.15}$$

Like the way used in the derivation of (3.8), we define the lateral steering command

$$u_\theta \equiv -(u_x\cos\phi + u_y\sin\phi)\tan\theta + u_z\cos\theta. \tag{3.16}$$

Then we obtain the lateral steering command for APNG in inertial polar coordinates as

$$u_\theta = \Lambda V_c\dot{\theta} + \frac{\Lambda}{2}(-(A_{Tx}\cos\phi + A_{Ty}\sin\phi)\tan\theta + A_{Tz}\cos\theta). \tag{3.17}$$

Whereas to implement the result of Section 2 one must know the exact or estimated time-to-go information, (3.8) and (3.17) can be applied without it.

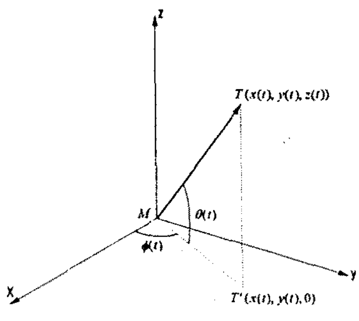

Figure 1 Relative Position in Inertial Coordinates

4. Extended Kalman Filter

In real implementation, the radar seeker gives the target information in the form of polar coordinates such as range between missile and target, elevation LOS angle and azimuth LOS angle, so it is not avoidable that the linear states are combined in nonlinear form in the measurement as follows:

$$\begin{bmatrix}\gamma\\ \theta\\ \phi\end{bmatrix} = \begin{bmatrix}\sqrt{x_r^2+y_r^2+z_r^2}\\ \tan^{-1}\left(z_r/\sqrt{x_r^2+y_r^2}\right)\\ \tan^{-1}(y_r/x_r)\end{bmatrix} \tag{4.1}$$

On this account, the nonlinear filtering technique must be included in the guidance law design, and we can think of the *extended Kalman filter* as the easiest method with optimality.

The noise model used in this simulation follows Paul[4] and some modification are annexed. The covariance of the modeled measurement noise is

$$R_k = E\{v_k^T v_k\} = diag\left[10 \quad \left(\frac{0.25}{\gamma_k^2} + 5.6\times10^{-7}\right)\times 5 \quad \left(\frac{0.25}{\gamma_k^2} + 5.6\times10^{-7}\right)\times 5\right] \tag{4.2}$$

where diagonal terms denote the noises applied in the range, elevation angle and azimuth angle, respectively.

5. Simulation

The number of scenarios used in this simulation is two, and in each scenario we assume that the target flies with velocity of 300 m/sec and that the missile velocity is constantly kept 700 m/sec. The launching of missile is omitted. The detailed scenario are arranged in Table 1, where the 'N', 'E' mean 'north of', 'east of' the origin, and 'H' means the height of the object. The 'El', 'Az' stand for the 'elevation' angle and 'azimuth' angle with respect to the north, respectively.

In Section 2, we did not consider the time delay effect of airframe in the course of derivation of the optimal control law. In this simulation the airframe dynamics including the missile autopilot is modeled in the case of STT missile. Since we have not concerned about the detailed missile aerodynamics and the calculated control inputs are given in the form of pitch and yaw normal accelerations independently at the beginning, the BTT missile whose dynamics for yaw and pitch channels are coupled is not appropriate.

In [11], the aerodynamics including accelerometer is proposed, which will be adapted in this simulation with the same parameters and the transfer function from the command normal acceleration to the real output acceleration is given by

$$\frac{a_z}{a_{zcomm}} = \frac{-108.3(s+2)(s+34.41)(s-34.41)}{(s+1.96)(s+13.58)(s^2+37.48s+4830.81)}. \tag{5.1}$$

From final value theorem, the steady-state gain of (5.1) is 2. This problem can be solved simply by applying the half of calculated command input.

The results of 30-times Monte Carlo simulation are listed in Table 2. Since we set the missile and the target as point mass, thinking of the sizes of the both, the miss distance within fifteen meters can hurt the target seriously.

In Figure 2, the plots of RMS error of the estimated target acceleration in Cartesian coordinates and the error of LOS rates between the missile and the target are listed. Since the errors in each scenario do not much differ, only the ones for scenario 2 are selected. Though the EKF is a approximate filter of the nonlinear system, it shows the reasonable results.

Figure 3 show the trajectories of the two objects in three dimensional space, where the thiner lines represent the projections of trejectories. It can be easily known from Table 1 which line represents missile or target.

Figure 4 show the relative position trajectory in APNG that is the one thought of by the missile. We can get the fact that while APNG tries to keep the missile flying straight toward the target which is the cause of smaller miss distance in APNG.

Table 1 Two Scenarios for Simulation

Parameters	Scenario 1	Scenario 2
Initial Missile Position (km)	H : 1	H : 1
Initial Missile Velocity Angle	El : 40°, Az : -26°	El : 30°, Az : -26°
Initial Target Position (km)	N : 10, E : 5, H : 7	N : 10, E : 5, H : 5
Initial Target Velocity Angle	El : 0°, Az : 180°	El : 10°, Az : 210°
Target Maneuver Acceleration	70 m/sec²	60 m/sec²
Target Maneuver Angle	El : 20°, Az : -120°	El : 40°, Az : 100°
Target Maneuver Time	from 4 to 8 seconds	from 3 to 12 seconds

6. Conclusion

The solution for the missile guidance problem which includes the target maneuver and the existence of noise has been proposed using LQG control method. To cope with not only a constant velocity target but also a target with acceleration, it is needed for the target maneuver acceleration to be included in guidance computer. It is shown that one of the possible and easy way is using APNG in the sense of optimality.

One of the obstacles that hinder a straight access is the problem of transformation between two coordinate system: Cartesian and polar. While the output of the seeker and the input exerted on missile is given in the form of polar coordinates, the implementation of optimal filter and the solution of APNG, as already known, is easily solvable if the problem is based on the Cartesian coordinates.

Instead of tackling nonlinear optimal control and filtering subject, the modification of the results in Cartesian into the polar form has been suggested throughout this thesis. One for filtering is the usage of the Extended Kalman filter, and another for optimal control, the main result of this thesis, is the adjustment of target maneuver acceleration using sinusoidal functions to get normal maneuver accelerations with respect to LOS. In this stage, whereas the former result requires the time-to-go information which cannot be easily obtained, the proposed guidance law is readily realizable with the Extended Kalman filter.

The simulation shows that APNG guarantee the miss distance less than twenty meters for the two scenarios.

For the Extended Kalman filter, though it is a approximate of a nonlinear system, the EKF shows a reasonable estimates the errors of LOS and target maneuver acceleration.

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Table 2 Table of RMS Miss Distance

Methods	Proposed APNG
Scenario1	12.76 m
Scenario2	16.19 m

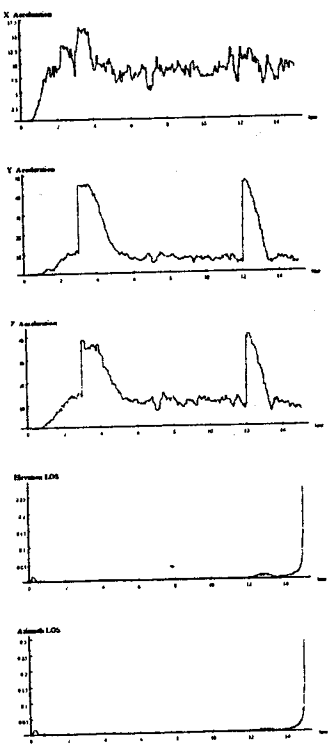

Figure 2 RMS Error of Estimates for Scenario2

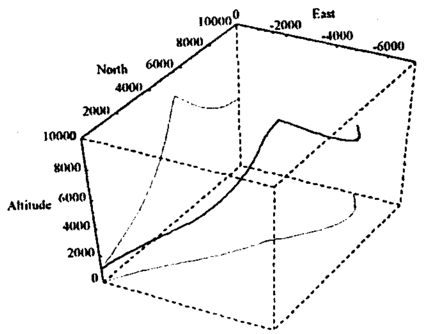

Figure 3 Trajectories of Missile and Target in Scenario2

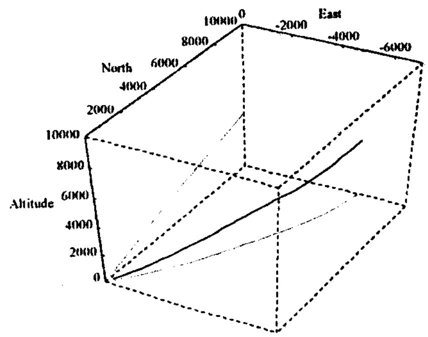

Figure 4 The Relative Trejectory in Scenario2