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CRX: A Characteristic Transport Theory Code for Cell and Assembly Calculations in Reactor Core Design

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Abstract - A characteristic transport theory code CRX is developed and tested for cell and assembly calculations. Since the characteristic method treats explicitly (analytically) the streaming portion of the transport equation, CRX treats strong absorbers well and has no practical limitations placed on the geometry of the problem. To test the code, it was applied to three benchmark problems which consist of complex meshes and compared with other codes.

I. Introduction

For lattice calculations in reactor core design, the integral transport method[1,2] is popular due to their capability in treating complicated geometries. But the integral transport method has several drawbacks due to its formulation in which the transport equation is integrated for the angular variable. First, the discretized matrices are almost full and asymmetric. Second, in constructing the matrix elements by the ray tracing procedure, large computing time is consumed. Third, the exact treatment of the reflective boundary condition is very difficult and the white boundary condition is usually used. Fourth, the treatment of the anisotropic scattering is very difficult. On the other hand, the discrete ordinate (S_N) method uses very simple calculation procedure, its formulation is very simple, and large matrices are not used. But its simple formulation leads to three main drawbacks, that is to say, the ray effect, the occurrence of the negative flux, and the restriction in the mesh geometry it can treat.

Similarly to the streaming rays method[3], the characteristic method[4] first proposed by Askew combines the desirable features of the above two methods by solving the differential form of the transport equation with arrays of the characteristic lines or tracks as used in the collision probability calculations. In this method, the outgoing flux is calculated by integrating the transport equation along the characteristic lines with the known incoming flux and the source for each discretized direction, and thus its solution is highly accurate in streaming dominant problems. And the desirable features of the S_N method are retained in this method. Therefore, this method has, in theory, no limitations in the complexity of geometry and

the reflective condition is easily treated. In CRX, any shape of mesh which consists of straight lines and circular arcs are easily treated and the homogenization procedure before the assembly calculation is not required. And the code always provides positive solution due to the characteristic formulation, if the incoming flux and source are positive.

II. Theory and Methodology

The derivation of the characteristic method starts with the differential form of the multigroup transport equation. The within group (g) equation for the discretized direction (n, m) is given as follows :

$$\sin\theta_n \frac{d\Psi^g_{m,n}}{dp} + \sigma^g_t \Psi^g_{m,n} = q^g_{m,n} \tag{1}$$

where p is the projected coordinate on $x-y$ plane of the coordinate along the neutron trajectory for the direction (m, n) and θ is the polar angle. In the above equation, m and n represent the azimuthal angle index and the polar angle index, respectively. The above equation can be analytically integrated along the characteristic line to obtain the outgoing flux. The equation is given as follows :

$$\Psi^g_{m,n}(\vec{r}) = \Psi^g_{m,n}(\overrightarrow{r_{\text{in}}}) e^{-\sigma^g_t L/\sin\theta_n} + \int_0^{L/\sin\theta_n} e^{-\sigma^g_t v} q(\vec{r} - v\,\vec{\Omega}_{m,n}) dv \tag{2}$$

where we assumed that the material property is uniform within the integral region and L is the track length. The equation for computational mesh (i, j) with flat source approximation is simply given by the following expression :

$$\Psi^{g,out}_{i,j,m,n,l} = \Psi^{g,\text{in}}_{i,j,m,n,l} \tau^g_{i,j,m,n,l} + \frac{q^g_{i,j,m,n}}{\sigma^g_{t,i,j}} (1 - \tau^g_{i,j,m,n,l}) \tag{3}$$

where l is the ray index and $\tau^g_{i,j,m,n,l}$ is given by

$$\tau^g_{i,j,m,n,l} = e^{-\sigma^g_{t,i,j} L_{i,j,m,n,l}/\sin\theta_n} . \tag{4}$$

The average angular flux for the direction (m, n) along the l'th ray is obtained by integrating Eq.(1). The equation is given as follows :

$$\overline{\Psi}^g_{i,j,m,n,l} = \frac{q^g_{i,j,m,n}}{\sigma_{t,i,j}} + \frac{\Psi^{g,\text{in}}_{i,j,m,n,l} - \Psi^{g,out}_{i,j,m,n,l}}{\sigma_{t,i,j} L_{i,j,m,n,l}} . \tag{5}$$

However, to perform the scattering source iteration, the average angular flux over the computational mesh is required for the generation of the source. The equation for the average flux over the computational mesh is obtained by summing the average fluxes (Eq.(5)) over rays that pass

through the mesh. The equation is given by the following expression :

$$\overline{\Psi}^{g}_{i,j,m,n} = \frac{q^{g}_{i,j,m,n}}{\sigma^{g}_{t,i,j}} + \frac{\sin\theta_n}{A_{i,j}\sigma_{t,i,j}} \sum_{l \in ij\ cell} \delta_m (\Psi^{g,\mathrm{in}}_{i,j,m,n,l} - \Psi^{g,out}_{i,j,m,n,l}) \qquad (6)$$

where $A_{i,j}$ represents the area of the (i,j) mesh and δ_m represents the spacing between adjacent rays for the m'th azimuthal direction. See Fig. 1. In CRX, the spacing for each azimuthal angle is uniform. For the finite spacing (δ_m), Eq.(6) is approximate but sufficiently accurate with a sufficient number of rays. The accuracy of Eq.(6) is improved by the renormalization of the track lengths using the following formula :

$$L'_{i,j,m,n,l} = L_{i,j,m,n,l} \frac{A_{i,j}}{\sum_{m,l \in ij\ cell} L_{i,j,m,n,l} \delta_m w_m} \quad . \qquad (7)$$

The complete formulations are now established for the scattering source iteration. These equations are solved iteratively by the conventional scattering iteration method for inner iteration and the power method for outer iteration.

III. Applications and Results

For verification of the code, three benchmark problems are selected. The first is the fixed source problem that was proposed by Kavenoky and described by Sanchez[5] and comprises of reflected 3x3 heterogeneous pin cells with an internal burnable absorber rod. A spatially constant and isotropically emitting source of 1 n/cm^3sec is present in the moderator. The configuration is shown in Fig. 2. The errors in the scalar flux are compared with the results of the CARCINOMA code[5]. In the CARCINOMA code, the (E,3) approximation was used and the mesh division of the approximation is also shown in Fig. 2. The results are summarized in Table 2. In this calculation, the results of the two-dimensional collision probability code CLUP77[6] is used as the reference. As shown in Fig. 2, only eight meshes are used in our code. The results show that the increase of the polar angles mitigates the error.

The second benchmark problem consists of homogeneous 3x3 cells with reflective boundary. The center cell is the fuel that has non-zero fission cross section. Therefore, this problem is an eigenvalue problem. The configuration is given in Fig. 3(a) and the results are summarized in Table 3. In the results, it is noted that more meshes per cell are required in the CRX code to reduce the error than in the TWODANT code. But this requirement in CRX can be compensated by its capability of the heterogeneous calculation. In the TWODANT code, much more meshes would be required to describe the heterogeneities of a cell and practically impossible to describe the heterogeneities of an assembly.

The third is the NEACRP2 benchmark problem[7] but the original boundary condition is replaced by reflective condition. The benchmark problem consists of a mini BWR lattice with four fuel pin cells, water gaps, and a cruciform control rod. It is a six group problem with a severe flux gradient across the system generated by the control rod. The configuration is given in Fig. 3(b). The scalar flux distribution along the uppermost row of meshes is shown in Fig. 5. The figure reveals a severe flux gradient near the interface between water gap and cruciform control rod.

IV. Conclusions

In this paper, a characteristic transport theory code CRX is described and tested. The code was developed to accurately analyze the heterogeneous assembly with complicated mesh shapes. To test its accuracy and applicability to practical problems, the code was applied to three benchmark problems. The numerical results show that the code provides accurate solutions in the scalar flux distribution in comparison with the collision probability method codes with a sufficient number of angles. Therefore, it is expected that CRX can be used for the realistic analysis of heterogeneous cells and assemblies of complex geometry.

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References

1. Nam Zin Cho, "Overview of Transport Methods in Lattice Calculations," KAIST Internal Report, NURAD-94-02 (1994).
2. Nam Zin Cho *et el.*, "Development of a Code System for Fuel Assembly Calculation," prepared by Korea Advanced Institute of Science and Technology for Korea Atomic Energy Research Institute, KAERI/CM-063 /94 (1995). *(in Korean)*
3. N. Z. Cho, S. G. Hong, "The Method of Streaming Rays for Neutron Transport Problems," *Proceedings of the Korean Nuclear Society Spring Meeting, Vol.1, pp.67-72, May 1994, Pohang.*
4. M. J. Halsall, "CACTUS - A Characteristic Solution to the Neutron Transport Equation in Complicated Geometries," AEEW-R 1291 (1980).
5. R. Sanchez, "Approximation Solutions of the Two-Dimensional Transport Equation by Collision Probability Methods," *Nucl. Sci. Eng.*, **64**, 384-404 (1977).
6. K. Tsuchihashi, "CLUP77: A Fortran Program of Collision Probabilities for Square Clustered Assembly," JAERI-1196, JAERI(1971).
7. G. Ball, Z. Weiss, "STYX-1: A Benchamrking Program for Neutron Transport Calculations in Fuel-Assembly Type Geometries," *Ann. Nucl. Energy*, **20**, 59-70 (1993).

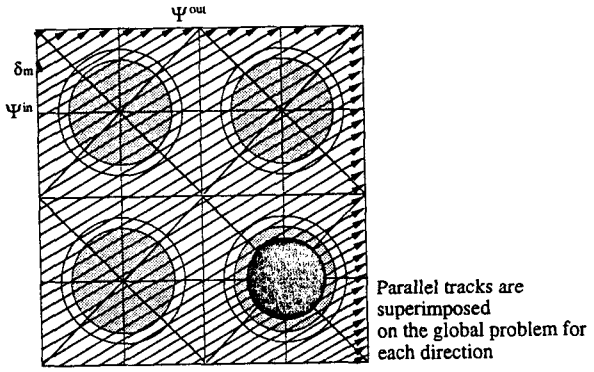


Fig. 1. Ray tracing in the CRX code

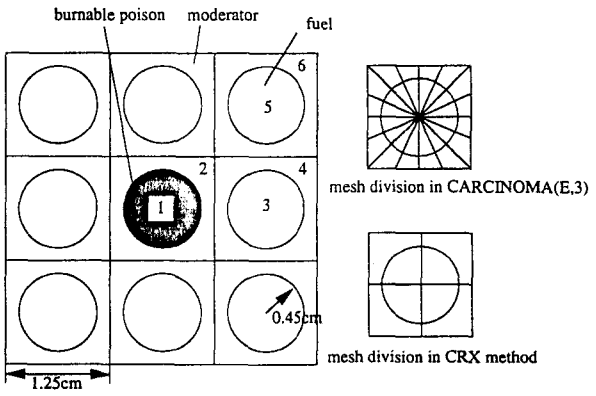


Fig. 2. Configuration for the benchmark problem I

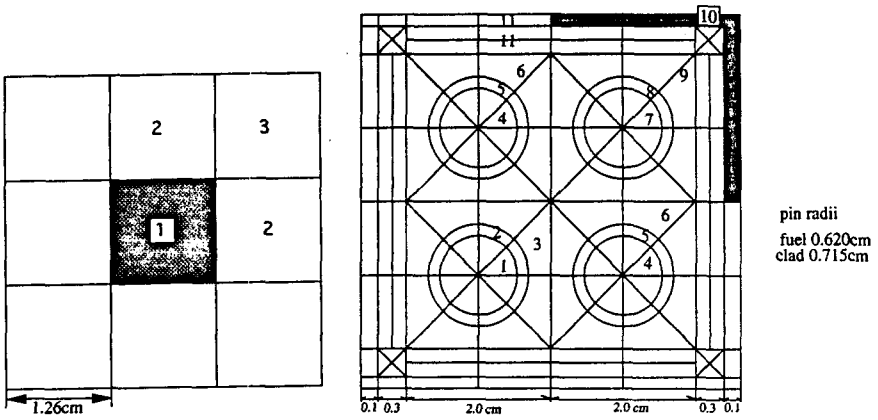
Table 1 Cross sections (cm^{-1}) for the benchmark problem I,II

	problem 1			problem 2		
material	moderator	fuel	BP	material 1	material 2	material 3
σ_t	1.25	0.625	14.00	1.000	1.000	2.000
σ_a	0.008	0.270	14.00	0.200	0.500	0.000
$\nu\sigma_f$	0.000	0.000	0.000	0.450	0.250	0.000

Table 2 Results of the benchmark problem I

	region 1	region 2	region 3	region 4	region 5	region 6
Reference flux	0.3312	3.226	3.612	4.081	3.832	4.313
CRX(10,2,400) * relative error(%)	0.452	2.358	0.077	2.505	0.634	2.190
CRX(10,4,400) relative error(%)	1.690	0.923	0.744	1.146	1.244	0.892
CARCINOMA(E,3) relative error(%)	3.290	0.055	0.058	0.833	2.948	2.249

*10: number of azimuthal angles per octant, 2: number of polar angles per octant, 400: number of rays for each direction



(a) Benchmark problem II (b) Benchmark problem III

Fig. 3. Configurations for the benchmark problem II, III

Table 3 Relative errors(%) in the scalar flux and multiplication factor in the benchmark problem II

	region 1	region 2	region 3	k_∞
TWODANT(2x2)	0.20877(1.74%)	0.59338(2.03%)	1.05143(0.25%)	1.016550(0.93%)
TWODANT(3x3) *	0.20520(ref.)	0.58152(ref.)	1.05509(ref.)	1.026123(ref.)
CRX(4x4) **	0.20735(1.05%)	0.58914(1.31%)	1.05273(0.22%)	1.019823(0.61%)
CRX(6x6) **	0.20507(0.06%)	0.58468(0.54%)	1.05411(0.09%)	1.024395(0.17%)

*TWODANT(S_{32},3x3 meshes/cell) solution is used as reference.

**(10,4) angles/octant are used.

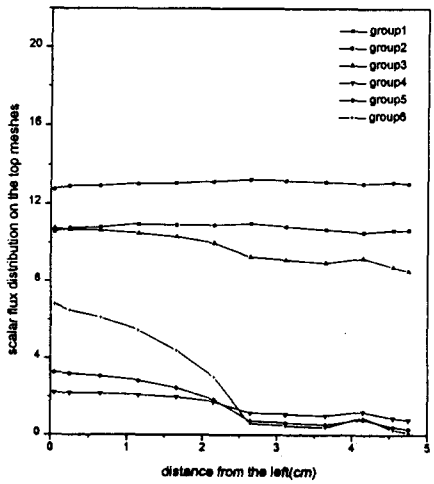


Fig. 4. Results of the benchmark problem III (k_∞=0.816465)